Parameterized Multisurface Fitting for Multi-Frame Superresolution

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SUMMARY We propose a parameterized multisurface fitting method for multi-frame super-resolution (SR) processing. A parameter assumed for the unknown high-resolution (HR) pixel is used for multisurface fitting. Each surface fitted at each low-resolution (LR) pixel is an expression of the parameter. Final SR result is obtained by fusing the sampling values from these surfaces in the maximum a posteriori fashion. Experimental results demonstrate the superiority of the proposed method.

key words: super-resolution, Taylor series, intensity estimation, parameter multi-surface fitting

1. Introduction

SR has been studied for decades [1]. Among various SR methods, the interpolation-based approach is the most intuitive one. A pixelwise average algorithm [2] is implemented in the maximum-likelihood sense with Gaussian additive noise. A median average algorithm [3] is adopted which is robust to errors in motion and blur estimation. In [4], an interpolation-based approach using Delaunay triangulation models each triangle patch as a bivariate polynomial. Recently, a method based on interpolation by using multi-surface fitting is presented in [5], which takes local spatial structures into account. However, all the methods mentioned above only use the LR pixels in the neighborhood to estimate the HR pixel.

The only information that has not been used is that of the HR pixel. If we use the information of the HR pixel in the process of multisurface fitting, we have more information to form the surface and make the result more accurate. So it would be beneficial to use the unknown HR pixels as a parameter in paradigm of interpolation-based SR. In spired by this, we propose an image SR method named parameter multi-surface fitting. Specifically, we fit one surface at each LR pixel using the value of HR pixel as a parameter and the multi-surface fitting. Specifically, we fit one surface at each LR pixel using the value of HR pixel as a parameter and the multi-surface fitting.

2. Methodology

Suppose that each HR pixel $p_H$ has a set of LR pixels in the neighborhood after subpixel registration [6], denoted by $p_{L1} \cdots p_{Lj} \cdots p_{LK}$, where $K$ is the number of LR pixels in the neighborhood. As shown in Fig. 1, pixels from different LR images are positioned in an HR grid after subpixel registration. The size of neighborhood is chosen as $1 \times 1$ HR pixel. If no pixel exists in the chosen neighborhood, we enlarge its size in a stepwise manner. And then we can determine the number of LR pixels in the neighborhood and use those pixels to form the surfaces. We fit one surface at each LR pixel and obtain $K$ values $f_{Li}(p_H)$ by resampling each surface, i.e.,

$$f_{Li}(p_H) = S(x_{iH}, y_{iH}, \Gamma_{Li}), \quad 1 \leq i \leq K$$ (1)

where $\Gamma_{Li}$ is the fitted surface for LR pixel $p_{Li}$. And the intensity of $p_H$ can be obtained by MAP estimation [5]:

$$\hat{f}(p_H) = \arg \max_{f(p_H)} q(f(p_H)|f_{L1}(p_H), \cdots, f_{LK}(p_H))$$

$$= \arg \max_{f(p_H)} q(f_{L1}(p_H), \cdots, f_{LK}(p_H)|f(p_H)) q(f(p_H))$$ (2)

where $q(\cdot)$ is the probability density function.

We use the 2-D Taylor series to fit each surface. Suppose that $p_{Lj}$ has $M_j$ LR pixels in the neighborhood and the value of the HR pixel $p_{H}$ is assumed to be the parameter $p_{H}$. The actual values of $K$ and $M_j$ are different in different neighborhood and depend on subpixel registration and positional relationship of LR pixels. With the method in the first paragraph in Sect. 2, we can determine the actual value. Then we have $M_j+1$ equations:

![Fig. 1 Illustration of subpixel registration and neighborhood.](image-url)
where \( \Delta x_{i,j} \), \( \Delta y_{i,j} \), \( \Delta x_{j,i} \), \( \Delta y_{j,i} \), \( \Delta x_{M,i} \), \( \Delta y_{M,i} \), \( \Delta x_{H,i} \), and \( \Delta y_{H,i} \) are the LR and HR pixels that are used to fit \( f_i \). Subsequently, we can calculate the fitting error \( \Delta f_i \), and \( \Delta f_i \) can be regarded as constant dependent on intensities and positions of the LR pixels.

Thus, we obtain the expression of each surface and the sampling value on the surface. It is easy to prove that the sampling value is also a polynomial of \( ph \).

\[
\hat{f}(ph) = \arg \min_{f(ph)} \left[ \sum_{i=1}^{K} \sigma_i + \sum_{i=1}^{K} \lambda \frac{(f_i(ph) - f(ph))^2}{\sigma_i^2} \right] + \lambda (f_0(ph) - f(ph))^2
\]  

where \( f_0(ph) \) is the prior estimation of \( f(ph) \) and \( \lambda \) is an empirical parameter. \( f_0(ph) \) can be obtained in many ways, such as B-spline interpolation. Substituting (5) and (11) to (12), we have

\[
\hat{f}(ph) = \arg \min_{ph} \left[ \sum_{i=1}^{K} \left( a_i + b_i ph + c_i ph^2 \right)^2 \right] + \sum_{i=1}^{K} \left( d_i + e_i ph - ph \right)^2
\]  

where \( a_i, b_i, c_i, d_i, e_i \) are the same with those in (5) and (11).

Essentially, the second item of (12) is like the form of weighted sum. The weight is the fitting error, and the surface with smaller noise and error has greater contribution to the final HR pixel value. The first item of (12) limits the fitting error in order to guarantee the surface formed more accurately and less noisy.

### 3. Experiments

We utilize 25 LR images to reconstruct HR images with Gaussian noise of 15 dB. The LR images are generated by sub-sampling with the factor of 4 in each direction and the positions of sampling are random. To eliminate the effect of prior knowledge on the final performance, we set the value of \( \lambda \) in (13) to 0.

We adopt several image quality assessment (IQA) methods to quantify the results, including visual information fidelity index (VIF) [7], feature-similarity index (FSIM) [8], and peak signal to noise ratio (PSNR). The results are shown in Table 1. Larger VIF, FSIM and PSNR values mean better results of reconstruction. Comparing to other methods, our method achieves a better result. Information of the HR pixels makes the estimated HR pixels more accurate.

Moreover, we provide visual examples in Fig. 2 to compare with other methods intuitively. From Fig. 2, we can observe that other methods fails to reconstruct the details accurately and generate fewer artifacts than other methods.

| Table 1 Comparisons based on IQA. |
|-------------------------------|-------------------------------|-------------------------------|
| Methods/Metrics               | VIF                          | FSIM                          | PSNR                          |
| Elad [2]                      | 0.3757                       | 0.7863                        | 35.3119                       |
| Farsiu [3]                    | 0.3621                       | 0.8238                        | 35.6890                       |
| Lertattanapanich [4]          | 0.4678                       | 0.8088                        | 35.0998                       |
| Zhou [5]                      | 0.4629                       | 0.8175                        | 36.2953                       |
| Our method                    | 0.5855                       | 0.8726                        | 37.8399                       |

It can be proved that \( \sigma_i^2 \) is a quadratic polynomial of \( ph \).

\[
\sigma_i^2 = a_i + b_i ph + c_i ph^2, \quad 1 \leq i \leq K
\]  

where \( a_i, b_i \) and \( c_i \) are constants related with intensities and positions of the LR pixels. Once we have (5) and (11), under
4. Conclusion

We present a SR method by considering the unknown HR pixel as a parameter of a fitted surface. The proposed method can reduce fitting errors in the manner of the MAP approach. Experiments show that our method can achieve better performance with lower reconstruction error.

Acknowledgments

The research leading to this work was supported in part by the Natural Science Foundation of China under Grant No.61271393 and 61301183 and China Postdoctoral Science Foundation under Grant 2013M540947, and in part by the Special Foundation for the Development of Strategic Emerging Industries of Shenzhen under Grant JCYJ20120619151228556.

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