**Protocol Inheritance Preserving Soundizability Problem and Its Polynomial Time Procedure for Acyclic Free Choice Workflow Nets**

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**SUMMARY** A workflow may be extended to adapt to market growth, legal reform, and so on. The extended workflow must be logically correct, and inherit the behavior of the existing workflow. Even if the extended workflow inherits the behavior, it may be not logically correct. Can we modify it so that it satisfies not only behavioral inheritance but also logical correctness? This is named behavioral inheritance preserving soundizability problem. There are two kinds of behavioral inheritance: protocol inheritance and projection inheritance. In this paper, we tackled protocol inheritance preserving soundizability problem using a subclass of Petri nets called workflow nets. Limiting our analysis to acyclic free choice workflow nets, we formalized the problem. And we gave a necessary and sufficient condition on the problem, which is the existence of a key structure of free choice workflow nets called TP-handle. Based on this condition, we also constructed a polynomial time procedure to solve the problem.

**key words:** workflow net, Petri net, behavioral inheritance, soundness, soundizability, polynomial time procedure

1. **Introduction**

A workflow may be extended to adapt to market growth, legal reform, and so on. Business continuity requires the extended workflow to inherit the behavior of the existing workflow. The extended workflow must be logically correct, but it may be not logically correct even if it inherits the behavior.

Workflows can be modeled as a subclass of Petri nets [1], called workflow nets [2] (WF-nets for short). Van der Aalst [3] has proposed a criterion of logical correctness for WF-nets, called soundness. He has also shown that many actual workflows can be modeled as a subclass of WF-nets, called free choice WF-nets (FC WF-nets for short) [4], and that the soundness of FC WF-nets can be verified in polynomial time. Moreover, Van der Aalst et al. [5] have proposed a concept of behavioral inheritance between WF-nets. There are two kinds of behavioral inheritance: protocol inheritance and projection inheritance. Yamaguchi et al. [6] have proposed a necessary and sufficient condition on protocol inheritance between an acyclic FC WF-net and its extended net, and have shown that the protocol inheritance can be verified in polynomial time. The necessary and sufficient condition suggests that there exists a non-sound extended net even if it satisfies protocol inheritance. Can we modify the non-sound extended net so that it satisfies not only protocol inheritance but also soundness? This is named protocol inheritance preserving soundizability problem. Unfortunately, there has been no research tackling this problem.

In this paper, we tackle the protocol inheritance preserving soundizability problem. Limiting our analysis to acyclic FC WF-nets, we formalize the problem. Next we propose a necessary and sufficient condition on the problem, which is the existence of a key structure of FC WF-nets called TP-handle. Based on this condition, we also construct a polynomial time procedure to solve the problem. After the introduction in Sect. 1, Sect. 2 gives the definition and properties of WF-nets. In Sect. 3, we give the definition and the necessary and sufficient condition on the problem. In Sect. 4, we present the polynomial time procedure for solving the problem. We also illustrate the procedure with examples. Section 5 gives the conclusion and future work.

2. **WF-Net and Its Properties**

(1) **WF-Net**

A (labeled) Petri net [1] is a four tuple \((P, T, A, \ell)\), where \(P\) and \(T\) are respectively disjoint finite sets of places and transitions, \(A \subseteq (P \times T) \cup (T \times P)\) is a set of arcs, and \(\ell : T \rightarrow A\) is a labelling function of transitions, where \(A\) denotes a set of labels. Let \(x\) be a node. \(\bullet x\) and \(\circ x\) respectively denote \([y \in \{y \in A \mid \ell(y) = x\}]\) and \([y \in \{y \in A \mid \ell(y) = x\}]\).

**Definition 1** ([WF-net [3]]): A Petri net \(N = (P, T, A, \ell)\) is a (labeled) WF-net iff (i) \(N\) has a single source place \(p_r \in N\) and \(\forall p \in \{p \in P \mid \ell(p) = \bullet p\} : p \neq 0\) and a single sink place \(p_o \in N\) \(\forall p \in \{p \in P \mid \ell(p) = \circ p\} : p \neq 0\); and (ii) Every node is on a path from \(p_r\) to \(p_o\).

A marking of a WF-net \(N\) is a mapping \(M : P \rightarrow \mathbb{N}\). We represent \(M\) as a bag over \(P\), and write \([p : M(p)]_{p \in P}, M(p) > 0\) for it. \([p_{r}]\) and \([p_{o}]\) are respectively the initial and final markings. Let \(M_{X} = M_{Y}\) denotes that \(\forall p \in P : M_{X}(p) = M_{Y}(p)\). \(M_{X} \geq M_{Y}\) denotes that \(\forall p \in P: M_{X}(p) \geq M_{Y}(p)\). A transition \(t\) is said to be firable in a marking \(M\) if \(M_{X} > t\). This is denoted by \(M[N, t]\). Firing \(t\) in \(M\) results in a new marking \(M' = M + t\). This is denoted by \(M[N, t] M'\). A marking \(M'\) is said to be reachable from a marking \(M\) if there exists a firing sequence of transitions transforming \(M\) to \(M'\). The set of all possible markings reachable from \(M\) is denoted by \(R(N, M)\). \(R(N, M)\) can be represented as a graph, called reachability graph \(G = (V, E)\). The vertices represent markings generated from \(M\) and its
successors, and each arc represents a transition firing.

$N$ is said to be FC if $\forall p_1, p_2 \in P$: $p_1 \not\preceq p_2 \Rightarrow [p_1] = [p_2] = [\emptyset]$. If we add a transition $t'$ to $N$ which connects $P_O$ with $P_I$, then the resulting net $N'$ is strongly connected. We call $N'$ the short-circuited net of $N$. There are key structures that characterize FC: handle and bridge. A path (a circuit) is said to be elementary if no node appears more than once in the path (the circuit). Let $p$ be an elementary path from a node $x$ to another node $y$, and $c$ an elementary circuit in $N$. $p$ is called a handle [1], [7] of $N$. We call $N'$ a bridge between $N$.

Definition 3 (branching bisimilarity [9]): Let $G_{N_X}$ and $G_{N_Y}$ be respectively the reachability graphs of a WF-net $(N_X, [p^X])$ and another WF-net $(N_Y, [p^Y])$. A binary relation $R \subseteq \mathbb{R}(N_X, [p^X]) \times \mathbb{R}(N_Y, [p^Y])$ is branching bisimulation iff (i) if $M_X = \exists M_Y$ and $M_X(N_X, \alpha) \rho M_Y$, then $\exists M_Y' \in R(N_Y, [p^Y])$: $M_Y(N_Y, \tau') \rho M_Y''$, $M_Y''(N_Y, (\alpha)) \rho M_Y'$, $M_X' \rho M_Y''$, and $M_X''(N_X, \tau') \rho M_Y''$; and (ii) if $M_X \not\rho M_Y$ then $(M_X = [p^X_0] = M_Y(N_Y, \tau') [p^Y_0])$ and $(M_Y = [p^Y_0] \Rightarrow M_X(N_X, \tau') [p^X_0])$: $(N_X, [p^X])$ and $(N_Y, [p^Y])$ are said to be branching bisimilar, denoted by $(N_X, [p^X]) \sim_b (N_Y, [p^Y])$, iff there exists a branching bisimulation $R$ between $G_{N_X}$ and $G_{N_Y}$.

To give the formal definition of protocol inheritance, we use encapsulation operator $\partial$. For a set $H$ of observable labels, $\partial H$ removes transitions whose labels are included in $H$. Formally, $H : \partial H : (P', T', A', \ell')$ such that $\tau' = [\ell(t) \mid \ell(t) \in H]$, $A' = A \cap ((P \times T') \cup (T' \times P))$, and $\ell' : t \in T'$.

Definition 4 (protocol inheritance [5]): A WF-net $N_X$ is a subclass of another WF-net $N_Y$ under protocol inheritance iff $\exists H: (\partial_H(N_X), [p^X]) \sim_b (N_Y, [p^Y])$.

Protocol inheritance relation is partial-order [5]. This means that protocol inheritance relation is transitive.

The following is the necessary and sufficient condition to verify protocol inheritance between an acyclic FC WF-net and its subnet [6]. It can be checked in polynomial time.

Property 2: Let $N_X = \mathcal{E}(P_X, T_X, A_X, \ell_X)$ and $N_Y = \mathcal{E}(P_Y, T_Y, A_Y, \ell_Y)$ be acyclic FC WF-nets, where $N_X$ is a sound subnet of $N_Y$; and every transition in $N_X (N_Y)$ represents a unique observable transition. $N_Y$ is a subclass of $N_X$ under protocol inheritance iff (i) $p^X_1 = p^Y_1$ and $p^X_0 = p^Y_0$; (ii) $\partial_{\ell_Y} (T_Y - T_X)$ equals $N_X$ with 0 or more TT-handles with length 2; and (iii) $\partial_{\ell_Y} (T_Y - T_X)$ is sound, where isolated places in $\partial_{\ell_Y}(T_Y - T_X)$ are removed.

This necessary and sufficient condition suggests that there exists a non-sound extended net even if it satisfies protocol inheritance. Let us consider an extension of an FC WF-net, which is shown in Fig. 1. Figure 1 (a) shows a

(a) A sound acyclic FC WF-net $N_1$.

(b) A non-sound acyclic FC WF-net $N_2$ obtained by extending $N_1$. Note that $N_2$ is a subclass of $N_1$ under protocol inheritance.

Fig. 1 An instance of protocol inheritance preserving soundizability problem.
sound acyclic FC WF-net $N_1$. Figure 1 (b) shows an acyclic FC WF-net $N_2$ obtained by extending $N_1$. $N_2$ is a subclass of $N_1$ under protocol inheritance but is not sound. The detail is described later.

3. Protocol Inheritance Preserving Soundizability Problem and Its Necessary and Sufficient Condition

In this section, we give the formal definition of protocol inheritance preserving soundizability problem of acyclic FC WF-net, and the necessary and sufficient condition on the problem.

3.1 Problem

The growth of business involves (i) extending the existing workflow. The extended workflow must (ii) inherit the behavior of the existing workflow, and further (iii) be logically correct. We model the existing workflow and the extended behavior of the existing workflow, and further (i) extend the existing workflow, (ii) inherit the behavior of the existing workflow, and further (iii) be logically correct.

Van der Aalst[4] has shown that many actual workflows can be modeled as FC WF-nets. The Workflow Management Coalition[10] (WfMC for short), an international standardization organization on workflows, has identified four routing constructions: sequential, parallel, selective, and iterative. Acyclic FC WF-net can model workflows composed of the former three routing constructions. Therefore, various extension of actual workflows would be modeled as acyclic FC WF-nets. Thus we limit our analysis to acyclic FC WF-nets and give the formal definition as follows.

**Definition 5** (protocol inheritance preserving soundizability problem of acyclic FC WF-nets): Let $N_X$ be a sound acyclic FC WF-net and $N_Y$ a non-sound acyclic FC WF-net such that (i) $N_X$ is a subnet of $N_Y$; and (ii) $N_Y$ is a subclass of $N_X$ under protocol inheritance. Is there a sound acyclic FC WF-net $N_Z$ that satisfies the following? (i) $N_Y$ is a subnet of $N_Z$; and (ii) $N_Z$ is a subclass of $N_X$ under protocol inheritance. □

Let us consider two instances of the problem. In fact, those instances have different answers: The first is yes, but the second is no.

The first instance is shown in Fig. 1. $N_2$ is not sound because $N_2$ violates Condition (ii) of Property 1, i.e. $N_2$ has a PT-handle $p_1^t p_2^t p_3^t p_4^t p_5^t$ which has no TP-bridge from the handle to the circuit. Can we modify $N_2$ as a sound acyclic FC WF-net which is a subclass of $N_1$ under protocol inheritance? The answer is yes. We have only to add a new TP-bridge from the handle to the circuit. The obtained WF-net $N_3$ is shown in Fig. 3 (a). $N_3$ is sound because each PT-handle in $N_3$ has a TP-bridge from the handle to the circuit. Moreover, $N_3$ is a subclass of $N_1$ under protocol inheritance because $N_3$ satisfies the conditions of Property 2.

The second instance is shown in Fig. 3. $N_4$ is not sound because $N_4$ violates Condition (i) of Property 1, i.e. there is a TP-handle $p_1^t p_2^t p_4^t p_5^t$. Can we modify $N_4$ as a sound acyclic FC WF-net which is a subclass of $N_2$ under protocol inheritance? The answer is no. This is because the TP-handle will never be removed.

We deduce from the analysis results that TP-handle plays a core role in the protocol inheritance preserving soundizability problem of acyclic FC WF-nets.

3.2 Necessary and Sufficient Condition

Based on our deduction, we divide the protocol inheritance preserving soundizability problem of acyclic FC WF-nets into two cases based on the existence of TP-handle. Let us
first consider the case with TP-handles.

**Lemma 1:** Let $N_X$ be a sound acyclic FC WF-net and $N_Y$ a non-sound acyclic FC WF-net such that (i) $N_X$ is a subnet of $N_Y$; and (ii) $N_Y$ is a subclass of $N_X$ under protocol inheritance. If $N_Y$ has a TP-handle $h$, there is no sound acyclic FC WF-net $N_Z$ such that (i) $N_Z$ is a subnet of $N_Y$; and (ii) $N_Z$ is a subclass of $N_X$ under protocol inheritance.

Proof: Assume $N_Z$ exists. Since $N_Y$ is a subnet of $N_Z$, $h$ still exists in $N_Z$. This implies $N_Z$ is not sound from Condition (i) of Property 1. It is inconsistent with the assumption. Q.E.D.

This lemma means a necessary condition on the protocol inheritance preserving soundizability problem of acyclic FC WF-nets.

Next, let us consider the case with no TP-handle. Any non-sound acyclic FC WF-net satisfying protocol inheritance has a structural property as follows.

**Property 3:** Let $N_X$ be a sound acyclic FC WF-net and $N_Y$ a non-sound acyclic FC WF-net such that (i) $N_X$ is a subnet of $N_Y$; and (ii) $N_Y$ is a subclass of $N_X$ under protocol inheritance. $N_Y$ equals ‘$N_X$ with 0 or more TT-handles with length 2’ with PP-handles and/or PP-bridges, where the PP-handles and/or PP-bridges are not included in ‘$N_X$ with 0 or more TT-handles with length 2’ except for their terminal nodes.

Proof: We show the contraposition. Assume that $N_Y$ equals ‘$N_X$ with 0 or more TT-handles with length 2’ with a PT/TP/TT-handle or PT/TP/TT-bridge $\rho$. Note that $\rho$ is not included in ‘$N_X$ with 0 or more TT-handles with length 2’ except for their terminal nodes. If $\rho$ is a PT/TP-handle or PT/TP-bridge, since $\sigma_{t_i(t)}$ makes a new source place or new sink place, $N_Y$ is not a subclass of $N_X$ under protocol inheritance (Refer to Lemmas 1 and 2 of Ref. [6]). If $\rho$ is a TT-handle or TT-bridge with length 4 or more, for the similar reason $N_Y$ is not a subclass of $N_X$ under protocol inheritance (Refer to Lemma 5 of Ref. [6]). If $\rho$ is a TT-handle with length 2, it would be included in ‘$N_X$ with 0 or more TT-handles with length 2’, so this case does not occur. If $\rho$ is a TT-bridge with length 2, since a new causality occurs between its terminal nodes, $N_Y$ is not a subclass of $N_X$ under protocol inheritance (Refer to Lemma 4 of Ref. [6]). Thus this property holds. Q.E.D.

This property means that all the transitions newly added to $N_X$ are contained in the PP-handles and/or PP-bridges. We illustrate this property with an instance shown in Fig. 4. $N_X$ is an acyclic FC WF-net obtained by extending $N_Y$, and is a subclass of $N_Z$ under protocol inheritance. $N_Y$, however, is not sound, because $N_Y$ has a PT-handle $p_1^5p_2^5p_3^5p_4^5$ which has no TP-bridge from the handle to the circuit, i.e. the PT-handle violates Condition (ii) of Property 1. From Property 3, we can obtain that $N_Y$ equals $N_Z$ with a PP-handle $p_1^5p_2^5p_3^5p_4^5$ and a BB-bridge $p_1^5p_2^5p_3^5p_4^5$. Note that all the transitions newly added to $N_Z$, i.e. $t_1^5$, $t_2^5$ and $t_3^5$, are contained in the PP-handle and/or PP-bridge.

From Condition (ii) of Property 1, $N_Y$ must have a PT-handle which has no TP-bridge from the handle to the circuit. We say, a PT-handle is wrong if the handle has no TP-bridge from the handle to the circuit. Otherwise the PT-handle is said to be right. From Property 3, $N_Y$ equals ‘$N_X$ with 0 or more TT-handles with length 2’ with PP-handles and/or PP-bridges. ‘$N_X$ with 0 or more TT-handles with length 2’ has no wrong PT-handle, because it is sound (Condition (ii) of Property 2). This means that a wrong PT-handle is caused by the PP-handles and/or PP-bridges added to ‘$N_X$ with 0 or more TT-handles with length 2’. In other words, the wrong PT-handle must include such a PP-handle or PP-bridge. Let us consider the instance shown in Fig. 4. $N_X$ has a wrong PT-handle $p_1^5p_2^5p_3^5p_4^5p_5^5p_6^5$ which has 0 or more TT-bridges, and a PP-bridge $p_1^5p_2^5p_3^5p_4^5$. The PT-handle includes the PP-bridge.

Intuitively, for each of the wrong PT-handles, if we add an arc as a PT-bridge from the handle to the circuit, we can make $N_Y$ sound. If the TP-bridge starts from a transition added to $N_X$, it can be removed with the removal of the transition by encapsulation operator, so the obtained net is a subclass of $N_X$ under protocol inheritance.

**Lemma 2:** Let $N_X$ be a sound acyclic FC WF-net and $N_Y$ a non-sound acyclic FC WF-net such that (i) $N_X$ is a subnet of $N_Y$; and (ii) $N_Y$ is a subclass of $N_X$ under protocol inheritance. If $N_Y$ has no TP-handle, there is a sound acyclic FC WF-net $N_Z$ such that (i) $N_Y$ is a subclass of $N_Z$; and (ii) $N_Z$ is a subclass of $N_X$ under protocol inheritance.

Proof: We construct a WF-net $N$ by extending $N_Y$, and prove $N = N_Z$ by showing the following: (1) $N$ is acyclic FC; (2) $N$ is sound; and (3) $N$ is a subclass of $N_X$ under protocol inheritance.

First of all, we give how to construct $N$ by extending $N_Y$. $N_Y$ has wrong PT-handles. Let $n$ be the number of
the wrong PT-handles. Let $h_i (i = 1, 2, \cdots, n)$ denote each wrong PT-handle, and $c_i$ the circuit of $h_i$ (See Fig. 5). For PT-handle $h_i$, let $p_{i}(o)$ and $t_{i}(o)$ denote respectively the start node and the end node, and $s_{i}(o)$ denote the input place of $t_{i}(o)$ which appears in $c_i$. Let $\rho_i$ be a PP-handle or PP-bridge added to $'N_X$ with 0 or more TT-handles with length 2’ such that $\rho_i$ is a part of $h_i$. For PP-handle or PP-bridge $\rho_i$, let $d_{i}(o)$ and $r_{i}(o)$ denote respectively the start node and the end node, and $u_{i}(o)$ denote the input transition of $r_{i}(o)$ which appears in $\rho_i$. We construct $N = (P, T, A, \ell)$ as follows: $P=p_6$, $T=T_4$, $A=A_5\cup\{(u_{i}(o), s_{i}(o))|i=1, 2, \cdots, n\}$, $\ell=\ell_4$. $N$ is obviously an extended WF-net of $N_Y$.

Firstly, we prove (1). The extended part, $\{(u_{i}(o), s_{i}(o))|i=1, 2, \cdots, n\}$, consists of arcs from transition $u_{i}(o)$ to place $s_{i}(o)$. Therefore the free-choiceness is preserved. There is no path from $s_{i}(o)$ to the nodes of $\rho_i$ because such a path makes $N$ non-FC. Therefore the part does not produce any circuit, i.e. $N$ is acyclic.

Secondly, we prove (2). Arc $\{(u_{i}(o), s_{i}(o))|i=1, 2, \cdots, n\}$ forms a TP-bridge from $h_i$ to $c_i$. Note that $\overline{N_Y}$ originally has a TP-bridge from $c_i$ to $h_i$. Without the TP-bridge, $\delta(Y_{T\rightarrow T_4})$ would make a new source place, i.e. $N_Y$ does not become a subclass of $N_X$ under protocol inheritance. $\overline{N_Y}$ has no TP-handle. This means that the TP-bridge does not form any TP-handles. From the symmetry of structure, arc $\{(u_{i}(o), s_{i}(o))|i=1, 2, \cdots, n\}$ also forms no TP-handle. Therefore $N$ is sound.

Finally, we prove (3). Since $u_{i}(o)$ is a transition in PP-handle or PP-bridge $\rho_i$, we can remove arc $\{(u_{i}(o), s_{i}(o))$ because $u_{i}(o)$ can be removed by encapsulator operation. $N$ is a subclass of $N_Y$ under protocol inheritance.

Summarizing the above results, we have $N = N_Z$. Thus this lemma holds.

Q.E.D.

This lemma means a sufficient condition on the protocol inheritance preserving soundizability problem of acyclic FC WF-nets.

Let us consider again the instance shown in Fig. 4. We can say that $N_6$ is soundizable because $N_6$ has no TP-handle. $\overline{N_6}$ has a wrong PT-handle $p_1^5p_2^5p_3^5p_4^5p_5^5p_6^5p_7^5p_8^5p_9^5$. The end node of the PT-handle is $t_2^5$. The input place of $t_2^5$ appears in its circuit is $p_2^5$. The PT-handle includes a PP-bridge $p_1^5p_2^5p_3^5p_4^5p_5^5p_7^5p_8^5p_9^5$. The end node of the PP-bridge is $p_2^5$. The input transition of $p_2^5$ which appears in the PP-bridge is $t_2^5$. An arc $(t_2^5, p_2^5)$ forms a TP-handle from the PT-handle to the circuit. Adding the arc to $N_6$, we can obtain WF-net $N_7$ shown in Fig. 6. $N_7$ is an sound acyclic FC WF-net, and is a subclass of $N_Y$ under protocol inheritance.

From Lemmas 1 and 2, we can immediately obtain the following necessary and sufficient condition on the protocol inheritance preserving soundizability problem of acyclic FC WF-nets.

**Theorem 1:** Let $N_5$ be a sound acyclic FC WF-net and $N_Y$ a non-sound acyclic FC WF-net such that (i) $N_X$ is a subnet of $N_Y$; and (ii) $N_Y$ is a subclass of $N_X$ under protocol inheritance. If $N_Y$ has no TP-handle, there is a sound acyclic FC WF-net $N_Z$ such that (i) $N_Y$ is a subnet of $N_Z$; and (ii) $N_Z$ is a subclass of $N_X$ under protocol inheritance.

4. Polynomial Time Procedure and Examples

4.1 Polynomial Time Procedure

Based on Theorem 1, we construct a procedure for solving the protocol inheritance preserving soundizability problem of acyclic FC WF-nets.

\textbf{<Decision of Soundizability for Acyclic FC WF-nets>}

\textbf{Input:} Sound acyclic FC WF-net $N_X$ and non-sound acyclic FC WF-net $N_Y$ such that (i) $N_X$ is a subnet of $N_Y$; and (ii) $N_Y$ is a subclass of $N_X$ under protocol inheritance.

\textbf{Output:} Is there a sound acyclic FC WF-net $N_Z$ such that (i) $N_Y$ is a subnet of $N_Z$; and (ii) $N_Z$ is a subclass of $N_X$ under protocol inheritance.

1° Construct a flow network $D_{N_Y}=(V_{N_Y}, E_{N_Y})$ whose every edge has capacity 1, where $V_{N_Y}=P_Y\cup T_Y$, $E_{N_Y}=A_Y$.

2° For each vertex pair $(\nu_i, \nu_j)\in(V_{N_Y}\times V_{N_Y})$, if

- $\nu_i$ corresponds to a transition $\nu_i$ of $N_Y$ s.t. $|\nu_i|\geq 2$;
- $\nu_j$ corresponds to a place $\nu_j$ of $N_Y$ s.t. $|\nu_j|\geq 2$; and
- The maximum value of flow between $\nu_i$ and $\nu_j$ exceeds 1,

then output no and stop.

3° Output yes and stop.
Property 4: Algorithm \( \ll Decision \) of Soundizability for Acyclic FC WF-nets outputs yes if \( N_Y \) is soundizable, i.e. \( N_Y \) has no TP-handle. \( \square \)

Proof: The max-flow min-cut theorem states that if every edge of a flow network has capacity 1, the number of the disjoint paths from a vertex to another vertex is equal to the maximum value of flow between those vertices [11].

The “if” part: Since \( N_Y \) has no TP-handle, the number of the disjoint paths from any transition \( t \) to any place \( p \) is at most one. The maximum value of flow between the vertices corresponding to \( t \) and \( p \) does not exceed 1. Therefore the procedure outputs yes.

The “only-if” part: If \( N_Y \) has a TP-handle, the number of the disjoint paths from the start transition to the end place of the TP-handle is two or more. The maximum value of flow between the corresponding vertices exceeds 1. Therefore the procedure outputs no. \( \quad \Box \)

Property 5: The following problem can be solved in polynomial time: Given a sound acyclic FC WF-net \( N_X \) and a non-sound acyclic FC WF-net \( N_Y \) such that (i) \( N_X \) is a subnet of \( N_Y \); and (ii) \( N_Y \) is a subclass of \( N_X \) under protocol inheritance, to decide whether there is a sound acyclic FC WF-net \( N_Z \) such that (i) \( N_Y \) is a subnet of \( N_Z \); and (ii) \( N_Z \) is a subclass of \( N_X \) under protocol inheritance. \( \square \)

Proof: Algorithm \( \ll Decision \) of Soundizability for Acyclic FC WF-nets can run in polynomial time, because Step 1 takes time \( O(|P_Y| + |T_Y| + |A_Y|) \) and Step 2 takes time \( O(|T_Y||P_Y||(P_Y + |T_Y|)^3) \). Note that the computation of the maximum flow takes time \( O(|P_Y| + |T_Y|)^3) \). \( \Box \)

4.2 Examples

We illustrate the proposed procedure with the instances shown in Figs. 1 and 3. Let us first consider the instance shown in Fig. 1. In Step 1, we construct a flow network \( D_{N_1} \) shown in Fig. 7. In Step 2, we compute the maximum values of flow from \( v_{11} \) to \( v_{12} \), from \( v_{11} \) to \( v_{13} \), and from \( v_{11} \) to \( v_{14} \). Since the maximum values are all 1, our procedure outputs yes and stops. In fact, there exists a sound acyclic FC WF-net \( N_1 \).

Next, let us consider the instance shown in Fig. 3. In Step 1, we construct a flow network \( D_{N_3} \) shown in Fig. 8. In Step 2, we obtain the maximum value of flow from \( v_{11} \) to \( v_{13} \) as 2. Therefore our procedure outputs no and stops.

5. Conclusion

In this paper, we have tackled the protocol inheritance preserving soundizability problem of acyclic FC WF-nets. We have first given the formal definition of the problem. Next we have proposed a necessary and sufficient condition on the problem. The condition is the existence of TP-handle. Based on this condition, we have also constructed a polynomial time procedure to solve the problem. The proposed procedure enables us to check it efficiently. It would contribute to strengthen the organization’s competitive power in business environment that is changing rapidly.

This paper is the first step for soundization of WF-nets. The next step is to give how to soundize soundizable nets. A WF-net obtained by soundization is not unique in general, so it is desirable to have a minimal one. As a future work, we first propose a measure of quality of soundized nets, e.g. net size. Then considering the measure, we are going to construct a procedure of soundization.

Acknowledgment

This work was supported by JSPS KAKENHI Grant Number 23560532.

References


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