Adaptive Subscale Entropy Based Quantification of EEG

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SUMMARY This letter presents a new entropy measure for electroencephalograms (EEGs), which reflects the underlying dynamics of EEG over multiple time scales. The motivation behind this study is that neurological signals such as EEG possess distinct dynamics over different spectral modes. To deal with the nonlinear and nonstationary nature of EEG, the recently developed empirical mode decomposition (EMD) is incorporated, allowing an EEG to be decomposed into its inherent spectral components, referred to as intrinsic mode functions (IMFs). By calculating Shannon entropy of IMFs in a time-dependent manner and summing them over adaptive multiple scales, the result is an adaptive subscale entropy measure of EEG. Simulation and experimental results show that the proposed entropy properly reveals the dynamical changes over multiple scales.

Key words: EEG, adaptive subscale entropy, empirical mode decomposition, intrinsic mode function

1. Introduction

An electroencephalogram (EEG) is a good indicator of the electric activity of the brain, and its waveform contains useful information about the states of the brain. Recently, EEG has been studied in relation to functional brain mechanisms as a potential tool for the identification of several brain disorders [1]. However, visual inspection cannot discern the subtle information embedded in EEG. Thus, the need for objective measures gives rise to the development of a quantitative EEG measure to uncover neurological states [2]. Until recently, quantitative EEG measures based on entropy have shown to be promising for monitoring and detecting brain rhythm [3],[4]. Among those, Shannon entropy [5] has been widely used due to its simplicity. However, EEG signals are representative of the interacting mechanisms between numerous neurons across multiple temporal and spatial scales, resulting in dynamical changes in both time and frequency associated with the states of the brain [1]. Thus, Shannon entropy which is calculated using gross EEG signals may not properly reveal the underlying dynamics of EEG over multiple scales.

Recently, to extract the scale-dependent oscillatory components of EEG, the empirical mode decomposition (EMD) has been introduced [6]. It decomposes a time-series into different spectral components inherent in the time-series, namely intrinsic mode functions (IMFs), in a data-driven sense. Due to the potentials of the EMD, it has been increasingly used to analyze nonstationary physiological signals [7]. More recently, intrinsic mode entropy (IMEn) [8] has been developed, which is obtained by calculation of sample entropy of the accumulated sum of IMFs. However, IMEn lays emphasis on fine scales of the underlying time series. Moreover, the use of the sample entropy in IMEn requires a large computational cost [9].

In this letter, by computing the Shannon entropy of each IMF in a manner of a time-dependent scheme and summing over adaptive multiple scales, a simple and scale-dependent quantitative measure of the EEG recording is presented, termed an adaptive subscale entropy.

Through simulation and experimental studies, the proposed adaptive subscale entropy has shown to be effective in terms of sensitivity for reflecting the dynamical changes over scales.

2. Adaptive Subscale Entropy

2.1 Empirical Mode Decomposition of EEG

Recently, Huang et al. [6] have developed a data-driven decomposition method, thus being suited for nonlinear and nonstationary time series. In an iterative manner, termed a sifting process, EMD extracts the highest frequency oscillation (finest temporal scale) from the underlying time series, being considered as an IMF. The remaining part after the extraction contains lower frequency oscillatory components. The resulting IMFs represent the oscillatory patterns at different scale. This gives rise to the following major feature of EMD: EMD results in basis functions which are derived from the time-series in self-originated way, whereas other conventional methods such as Fourier and wavelet analyses rely on the use of pre-defined basis functions.

Let s(i) denote the raw sampled EEG signal. Then, the EMD consists of the following steps:

1. Identify all the local maxima and minima of s(i).
2. Interpolate between local maxima and minima respectively, getting an upper envelope \( e_u(i) \) and a lower envelope \( e_l(i) \).
3. Compute the mean between \( e_u(i) \) and \( e_l(i) \), i.e., \( \mu(i) = [e_u(i) + e_l(i)]/2 \).
4. Subtract the mean from the original signal \( d(i) = s(i) - \mu(i) \).
5. Repeat steps 1–4 until \( d(i) \) satisfies the above two criteria to be an IMF. If \( d(i) \) satisfies conditions, it becomes
the first IMF that contains the finest temporal scale in the signal. Also denote as $d_1(i)$.  
6. Compute the residue $r_1(i) = s(i) - d_1(i)$. 
7. Iterate through steps 1–6 with $s(i)$ instead of $s(i)$ until the residue satisfies some stopping criterion [6].

Through the sifting process, the raw EEG signal $s(i)$ is decomposed as follows:

$$s(i) = \sum_{k=1}^{K} d_k(i) + r_k(i),$$

where $K$ is the number of all extracted intrinsic mode functions, $d_k(i)$ is the $k$th IMF, and $r_k(i)$ is the final residue. The last residue $r_k(i)$ can be considered as the last IMF, and thus Eq. (1) can be rewritten as $s(i) = \sum_{k=1}^{K+1} d_k(i)$.

### 2.2 Measuring Adaptive Subscale Entropy

Next, the distribution of the time-varying individual oscillatory components obtained in Eq. (1), i.e., $d_k(i)$, are utilized to evaluate the adaptive subscale entropy. To deal with continuously acquired signals, EEG recording is divided into a time-varying oscillatory components in EEG. Let assume $s(i)$ of EEG in the sliding window of the raw EEG signal are defined by:

$$s_n(i) = \{s(i); i = 1+n\Delta, \ldots, w+n\Delta\},$$

where $n = 0, 1, \ldots, \lceil (N - w + 1)/\Delta \rceil$ and $[x]$ denotes the integer part of $x$.

Then, EMD is incorporated to utilize the underlying time-varying oscillatory components in EEG. Let assume EEG is decomposed by a sifting process, yielding totally $K$ IMFs and one residual which is considered as $K + 1$ IMF. A set of IMFs, EMD[$s_n(i)$], is obtained from EEG in the sliding window $s_n(i)$:

$$EMD[s_n(i)] = [d_n^1, d_n^2, \ldots, d_n^{K+1}],$$

where $d_n^k = [d_k(i); i = 1+n\Delta, \ldots, w+n\Delta]$ for $k = 1, \ldots, K+1$ denote the $k$th IMF in the $n$th sliding window.

In order to compute the probability distributions of the IMFs, $d_n^k$ for $k = 1, \ldots, K+1$ are partitioned into $M$ disjoint intervals $[l_m, m = 1, \ldots, M]$ spanning the range between the minimum and maximum values of IMF with $l_1 = \min(d_n^1)$ and $l_M = \max(d_n^{K+1})$, where $l_1 < l_2 < \ldots < l_M$, which is as follows:

$$EMD[s_n(i)] = [d_n^1, d_n^2, \ldots, d_n^{K+1}] = \bigsqcup_{m=1}^{M} I_m,$$

Then, $p^k_m(m)$ is the probability that the IMF belongs to the interval $I_m$ in $k$th IMF $d_n^k$. It is computed as a ratio of number of samples of $d_n^k$ within $I_m$ and the total sample number of $d_n^k$.

By sliding the window $w$, an adaptive subscale entropy of Shannon framework in the $k$th scale is defined as

$$ASE^k(n) = -\sum_{m=1}^{M} p^k_m(m) \log(p^k_m(m)),$$

where $k = 1, \ldots, K + 1$, $0 \leq p^k_m(m) \leq 1$, and $\sum_{m=1}^{M} p^k_m(m) = 1$. Finally, the adaptive subscale entropies in each scale are summed over all scales, leading to the adaptive subscale entropy

$$ASE(n) = \sum_{k=1}^{K+1} ASE^k(n).$$

### 3. Results

#### 3.1 Simulation

To verify the capability of the proposed adaptive subscale entropy, a synthesized signal consisting of Gaussian distribution and multiple sinusoidal components is used, which is shown in Fig. 1 (a). For the first 4 sec, the synthetic signal has Gaussian distribution. Following periods of the synthetic signal has different number of sinusoids in time-dependent manner as follows: From 4 to 8 sec, it begins with a single sinusoid of 1 Hz, followed by the addition of one more sinusoid with 5 Hz after 4 sec. From 12 to 16 sec, it consists of three sinusoids whose frequencies are 1, 5, and 10 Hz. During following 4 sec, it consists of four sinusoids whose frequencies are 1, 5, 10, and 20 Hz. During the last 4 sec, five sinusoids with 1, 5, 10, 20, and 40 Hz are included. From the perspective of entropy, it is expected that the more the number of sinusoidal components, the higher value of entropy. Figure 1 (b) depicts the results of the Shannon entropy, the IMEn and the adaptive subscale entropy.

For the IMEn, $m = 2$ and $r = 0.2$ are used as recommended in [8]. In the figure, the Shannon entropy is almost constant and the IMEn is not capable of characterizing the underlying components regardless of the distribution and the number of sinusoidal components of the signal. Comparing with those methods, the adaptive subscale entropy has higher value in accordance with the increase of sinusoidal components and is discriminative with Gaussian distribution.

![Fig. 1](image-url)
3.2 Experimental Study on EEG Following Brain Injury

Next, this study investigates EEG signals from rats subject to hypoxic-ischemic brain injury due to cardiac arrest. The experimental model of brain injury by cardiac arrest has been approved by Animal Care and Use Committee of the Johns Hopkins Medical Institutions [11]. Nine adult male Wistar rats (300±25 g) were used. Anesthesia was induced with 4% halothane in 50%:50% nitrous oxide:oxygen. A 10 min of baseline EEG was recorded including 5 min washout period to ensure that halothane did not influence the EEG. Subsequently, 7 min asphyxia was induced by stopping and disconnecting the ventilator and clamping the tracheal tube. More details on the experiment are described in [11].

Two channels of EEG using subdermal needle electrodes in right and left parietal areas of rats were recorded continuously before the insult, during insult, and through 3 h of recovery. The EEG signals were digitalized with DATAQ acquisition package (DATAQ Instruments INC., Akron, OH). Sampling frequency of 250 Hz and 12-bit A/D conversion were used. All rats underwent neurological testing at 72 h from the beginning of recovery. Neurological deficit score (NDS) was used as the measure for comprehensive neurological outcome of rats. Since NDS is evaluated quantitatively, which ranges from 0 (worst) to 80 (best), it serves as an appropriate tool for relating entropy measures to neurological outcome.

Figures 2 (a)–2 (c) show the EEG recordings of the experiment which were recorded at different stage as follows: at 5 min (Fig. 2 (a)), 50 min (Fig. 2 (b)), and 180 min (Fig. 2 (c)) from the start of experiment, respectively. In the figures, the EEG recordings at different stage show distinct waveforms in both amplitude and frequency. To reveal the inherent oscillations of EEGs, the five IMFs of each EEG recording are presented in Figs. 2 (d)–2 (f), respectively. The first IMF, here, denoted as $d_1$, has highest frequency and frequency component in the IMFs decreases along with the next IMFs.

For calculating the adaptive subscale entropy, the following parameters were used: sliding temporal window length of $w = 10$ sec, sliding interval of $\Delta = 10$ sec, and $M = 20$. For the IMEn, $m = 2$ and $r = 0.2$ are used. Figures 3 (a)–3 (c) show the statistical results (mean ± standard deviation) of the Shannon entropy, the IMEn, and the adaptive subscale entropy for three rats with different NDS values (NDS = 74, 59, 50), which implies distinct neurological outcome of rats. To demonstrate the entire trend, entropies for each rat were averaged over selected intervals of recovery phase. To test difference between rats per each recovery period, hypothesis testing (t-test) was carried out. The asterisk denotes t-test with $p < 0.001$.

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In Fig. 3 (a), the
Shannon entropy values during 30-60 min and 60-120 min are not clearly separable for different rats with NDS=74 and 59. In addition, Fig. 3 (b) shows that the IMEn is not able to capture the difference between animals with NDS = 59 and 50. On the other hand, the adaptive subscale entropies shown in Fig. 3 (c) are consistently separable for the three animals with different neurological deficit scores. This result indicates that the higher neurological score, the higher entropy value at the end of the four hour recovery period.

To assess with a larger sample, the Shannon entropy, the IMEn, and the adaptive subscale entropy of nine rats including the previous three rats were calculated. The comprehensive time average of entropies of recovery period (30-240 min from the start of experiment) was considered. To analyze the capability of entropies as a predictor of neurological outcome, Pearson correlation coefficient (r) and hypothesis testing using p-value were evaluated between NDS and entropies over the recovery period. From Figs. 4 (a)–4 (c), Pearson correlation coefficients between the adaptive subscale entropy and NDS were more significant than the Shannon entropy and the IMEn. Additional hypothesis testing using a Student-t distribution (n = 9) also reveals that the adaptive subscale entropy is more closely correlated to NDS than its counterparts.

4. Conclusion

A new quantitative measure of EEG which takes into account the dynamic changes over multiple time scales was presented. Utilizing a data-driven (adaptive) decomposition tool, i.e., EMD, makes possible to capture locally changing features from fine to coarse scales of EEG. Following evaluation of entropy using probability distribution of IMFs at each subscale, it leads to an effective quantitative measure of both spectral and temporal changes in EEGs. Finally, this study lays the foundation for applying this novel approach to clinical studies of EEG signals recorded during comparable episodes of brain injury.

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