PAPER

Fingerprint Verification and Identification Based on Local Geometric Invariants Constructed from Minutiae Points and Augmented with Global Directional Filterbank Features

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SUMMARY This paper addresses the problems of fingerprint identification and verification when a query fingerprint is taken under conditions that differ from those under which the fingerprint of the same person stored in a database was constructed. This occurs when using a different fingerprint scanner with a different pressure, resulting in a fingerprint impression that is smeared and distorted in accordance with a geometric transformation (e.g., affine or even non-linear). Minutiae points on a query fingerprint are matched and aligned to those on one of the fingerprints in the database, using a set of absolute invariants constructed from the shape and/or size of minutiae triangles depending on the assumed map. Once the best candidate match is declared and the corresponding minutiae points are flagged, the query fingerprint image is warped against the candidate fingerprint image in accordance with the estimated warping map. An identification/verification cost function using a combination of distance map and global directional filterbank (DFB) features is then utilized to verify and identify a query fingerprint against candidate fingerprint(s). Performance of the algorithm yields an area of 0.99967 (perfect classification is a value of 1) under the receiver operating characteristic (ROC) curve based on a database consisting of a total of 1680 fingerprint images captured from 240 fingers. The average probability of error was found to be 0.713%. Our algorithm also yields the smallest false non-match rate (FNMR) for a comparable false match rate (FMR) when compared to the well-known technique of DFB features and triangulation-based matching integrated with modeling non-linear deformation. This work represents an advance in resolving the fingerprint identification problem beyond the state-of-the-art approaches in both performance and robustness.

key words: affine invariant, directional filterbank, fingerprint matching, fingerprint verification, different fingerprint scanner characteristics

1. Introduction

A biometric system uses signature points of measurable uniqueness derived from either physical or behavior characteristics possessed by an individual person to characterize and determine his/her identity. A number of biometric technologies have been developed based on diverse biometric cues, such as DNA [1], ear [2], face [3], fingerprint [4], gait [5], hand and finger geometry [6], iris [7], keystroke [8], odor [9], palm print [10], hand writing and signature [11], and voice [12]. These cues showed their usefulness in various applications, ranging from Internet access and computer

system security, secure electronic access, passport control, banking, mobile phone, credit card fraud protection, secure access to building, health and social services, parenthood lineage determination, antiterrorist, and forensics.

Fingerprinting is the reproduction of a fingertip epidermis produced when a finger is pressed against a smooth surface, and it is the most widely used biometric feature due to its uniqueness and immutability. The most evident structural characteristics of a fingerprint are patterns of interleaved ridges and valleys. In a fingerprint image, ridges appear dark whereas valleys appear light. Fingerprints are used in person verification and identification. In verification, the identity of a person is authenticated by comparing the captured fingerprint against his/her own fingerprint template stored in the system. The one-to-one comparison of the verification system either rejects or accepts the submitted claim. The goal of fingerprint identification, on the other hand, is to recognize an individual by matching a query fingerprint against the entire template database. The fingerprint identification system performs a many-to-one comparison.

Fingerprint matching can be classified into 3 main categories: 1) correlation-based matching, 2) minutiae-based matching, and 3) ridge feature-based matching. Correlation-based fingerprint matching is the simplest and earliest method. In the matching scheme, two fingerprint images, the template and the query fingerprint, are aligned, and the cross correlation function between the two images is computed [13], [14]. The performance of this technique relies significantly on the alignment accuracy, which is invariably sensitive to transformation changes. One of the early solutions to this problem, which was designed to only deal with rigid transformations, was to iteratively rotate and shift the query image until the maximum of the cross correlation function is attained. The iterative correlation-based method suffers from two problems. First, the performance of such method is significantly degraded where a transformation other than a rigid transformation is involved. Second, the method is computationally intensive. To bypass these problems, many researchers proposed computing the correlation locally instead of globally, and different criteria were proposed to select the local region [15]–[17].

Since fingerprints are graphical patterns of ridges and valleys on the finger tips [18], a number of research works have used these features, called minutiae (ridge endings and bifurcations), for fingerprint verification and identification.

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Fingerprint matching based on minutiae algorithm [19]–[21] has been generally addressed as a point pattern matching problem [19], [22], whose goal is to find a corresponding pair of points under on the query and the corresponding template in the presence of geometric transformations. Many approaches have been proposed for minutiae matching, some of which include relaxation [23], algebraic and operational research solutions [24], [25], tree pruning [26], energy minimization [27], and Hough transform [28], [29]. For a good overview of these algorithms, readers are referred to the handbook [18].

Many of the minutiae-based fingerprint matching algorithms construct Delaunay triangle tessellation from the minutiae set and extract various features derived from the triangles [30]–[32], which are used to find corresponding triangles on the query and template fingerprints. From the corresponding triangular triplets, estimates of the transformation parameters are obtained and used to align the two fingerprint images. Although the minutiae-based fingerprint matching is considered the most efficient method with highest matching capability, the performance is limited when the fingerprint image suffers from non-linear deformation. To cope with this problem, many researchers attempted to model the non-linear distortion based on the local inter-ridge distance [33] and the spatial distribution on minutiae points [32], [34]. The estimated deformable model is then used to warp the template to the query coordinates in accordance with the non-linear transformation. One of the major challenges is how to establish correspondences between two sets of minutiae points without prior knowledge and without setting up an expensive iterative matching/estimation process that toggles between the two fingerprints assuming a match, estimates the parameters, and then iterates until a cost function is minimized. Chen *et al.* [32] proposed fuzzy features match based on a features set of local triangles to match the triangles between two fingerprints. Ratha *et al.* [35] established correspondences between two sets of minutiae based on local neighborhood structures. Wahab *et al.* [36] exploited local structure features to seek correspondences. The disadvantage of modeling the non-linear distortion based on the spatial distribution of minutiae is the sparse distribution of minutiae points across the fingerprint image. The problem worsens in the case of occlusion or missing data, in which not all minutiae points appear in both images. Ross *et al.* [34] proposed using corresponding ridge curves to estimate the non-linear distortion. In their approach, the minutiae correspondences between the template and the query images were first established. Ridge curves associated with the minutiae were found, and bifurcation points and uniform sampling points on the ridge curves were then used to estimate the non-linear transformation parameters. Establishing correspondences based on ridge curves is superior to that based on minutiae for two reasons. First, unlike minutiae, there exist a large number of points on ridge curves. Second, ridge curves are distributed over the entire image region including those regions where minutiae points were undetected. One major drawback as-

sociated with ridge curves, however, is that they are only preserved under affine transformations and some of the non-linear mappings. This means that ridge curves will no longer appear as ridge curves when the fingerprint is subjected to some non-linear mappings.

Recently, many research studies proposed using features different from minutiae for fingerprint matching. One such feature is ridge pattern (shape and frequency) [33], [34], [37], [38]. While the minutiae are local features, ridge flow provides a topological pattern as well as global information. One example of ridge-pattern features is the orientation field [39] and the associated singular points where ridge flow exhibits the distinctive regions. These features, however, are very sensitive to noise, and ways of handling the drawback have been reported [40]. Jain *et al.* [38] proposed different features based on a set of Gabor filters, i.e., Gabor filterbank, which is a well-known technique to capture information in the specific bandpass channels. These Gabor filterbank features are then combined with minutiae derived features to capture both the global information of ridges and valleys in the fingerprint as well as the local characteristics. This has been referred to as hybrid matching which combines local minutia feature with global filterbank features. The hybrid fingerprint matching proposed by Jain *et al.*, however, requires core point detection which is used for initial alignment of the fingerprint image. Ross *et al.* [41] proposed hybrid fingerprint matching that does not require core point detection. This proposed hybrid matching technique combines minutiae-based matching (local information) with ridge feature map (global information). The ridge feature map is an eight-dimensional feature map. The feature map in each direction represents the variance of the pixel intensities in each tessellated cell across all filtered images. Benhammadi *et al.* [42] suggested hybrid fingerprint matching by constructing a circular tessellation of eight-directional Gabor-filtered images surrounding each minutia point. The eight directions of the Gabor filterbank are varied according to the minutiae orientation in the circular filtered zone. Although the technique does not require core point detection, the proposed feature is invariant only to rigid transformation.

In this paper, we introduce a novel hybrid fingerprint matching that uses combined geometric invariants constructed from local convex hull minutiae-based triangles, augmented with the directional filterbank (DFB) features in order to overcome large variations of fingerprint problems, especially non-linear distortion and partial overlap. There are two main contributions of our research work to fingerprint verification and identification. For the first contribution, we develop a technique for fingerprint alignment between the query and reference fingerprint in the presence of affine and some non-linear transformations. We propose to construct the minutiae triangulation by connecting the neighboring minutiae vertices on the local convex hull and then extracting the affine invariant features from the local convex hull triangles. These affine invariant features sorted in a meaningful fashion are then used to establish correspon-

dences between two sets of minutiae to be aligned. Once the query and reference fingerprint are aligned, a cost function is then computed for further identification and/or verification. Establishing correspondences using invariants not only allows for a fast non-iterative procedure for alignment but also makes the correspondences robust to noisy or missing data as these invariants are based on the local triangles constructed from the minutiae points. Triangles that are distorted by noise or have no counterpart on the query are discarded. We rely only on “strong” matches that are reliable and present (i.e., where the error metric between the local absolute invariants is below a set threshold). The minimum number of such matches is largely dictated by the number of parameters in the global warping transformation that need to be estimated.

For the second contribution, we introduce novel Directional Filterbank (DFB) features without requiring reference point detection on the fingerprint images, combined with the conventional distance map [43] features to serve as an identification/verification cost function. The proposed DFB features derive from applying the eight-directional filter bank on the image of the extracted triangle filled with the binarized ridge curve (binary texture). Imposing DFB features in the hybrid fingerprint matching not only exploits the topological information of the local triangles constructed from the minutiae points but also the ridge texture information within. We have compared our hybrid matching algorithm to mainstream algorithms such as the DFB features based algorithm [38], and the triangulation-based matching integrated with modeling non-linear deformation using Radial Basis Function (RBF) [53] algorithm for a comparable FMR. The results are promising: the smallest FNMR (one half to one third the recorded rates of the state of the art systems) can be obtained.

This paper is organized as follows: Sect. 2 is related to triangulation of minutiae set. Section 3 describes the minutiae-based matching. Section 4 introduces the directional filterbank (DFB) derived features. Section 5 explains the identification and verification rules that combine distance map error and DFB error. Section 6 shows experimental results of the proposed algorithm. The conclusion and discussion are given in Sect. 7.

2. Triangulation of Minutiae Set

Given a set of minutiae points on the query fingerprint and another on one of the templates, we want to consider a model that is preserved under the existing (rigid, affine or non-linear which is piecewise affine) map. For a set of m minutiae points, we can form a set of up to $\binom{m}{3}$ triangles by considering all possible triangles formed by taking three non-collinear minutiae points at a time. Under a rigid map, the lengths of the sides, the angles, and the area of each triangle are absolute invariants and are preserved under a rigid transformation, i.e., the corresponding triangles after the rigid transformations will have the same invariant values, whereas under an affine map only the area is a relative

invariant, which can be made into an absolute invariant by considering ratios of pairs of triangles $\left(\frac{\binom{m}{3}}{2}\right)$. Finally, under a non-linear map, only a subset of triangles will exhibit such rigid and/or affine invariance. If we want to consider a smaller subset to align the fingerprints before and after the transformation, we can consider a Delaunay tessellation [30] in the rigid transformation case, which is of order m instead of $\binom{m}{3}$. In the affine transformation, however, the Delaunay tessellation is not preserved, i.e., we might end up with a tessellation of non-corresponding triangles. In such a case, we can consider triangles constructed from the convex hull set [44]–[46] obtained from the local minutiae set, which is of order less than $\binom{m}{3}$. For the non-linear case, however, neither the Delaunay triangulation nor the convex hull is preserved. In that case, we have to consider the entire set and use a combination of rigid and affine absolute invariants when either of the invariants is locally encountered in some of the triangles.

3. Minutiae-Based Matching

Our fingerprint matching is based on minutiae registration where sets of triangles constructed from the minutiae points are obtained on the query as well as the candidate fingerprint. The correspondence between two sets of triangles is then established based on certain criteria depending upon class of transformation, including rigid transformation, affine transformation, and non-linear transformation which is piecewise locally rigid and/or affine. Once sufficient correspondences are established using the matching method described below, the transformation parameters are determined, the images are aligned, and the distance map error between the candidate fingerprint image and the undistorted image is then computed and later used in fingerprint identification and/or verification (see Sect. 5 for details). Figure 1 depicts the registration process. Actually, the convex hull triangulation can be applied in all cases of transformations. Nevertheless, to study the invariant features under the rigid transformation, the Delaunay triangulation of m triangles, where m is the total number of minutiae, is considered since the computational complexity is less.

3.1 Matching in the Presence of Rigid Transformation

The rigid transformation case corresponds to the case in which the same scanner is used for the query as for the candidate fingerprints in the database, but when the orientation and position of the query are different from the candidate's. In that case, the triangle tessellation considered is a Delaunay triangulation, which is preserved under a rigid transformation [30], processed on the extracted minutiae points. For each triangle, its shape (side lengths and angles) and area are preserved and hence can be used for establishing correspondence.

The matching criterion between each pair of triangles, saying triangles i^{th} and j^{th} , is according to

Fig. 1 Registration process.

$$\% \varepsilon_i^2 = \frac{\|\mathbf{F}_j - \mathbf{F}_i\|^2}{\|\mathbf{F}_i\|^2} \times 100 < \xi \quad (1)$$

where $\mathbf{F}_i = [d_{i1}, d_{i2}, d_{i3}, \alpha_{i1}, \alpha_{i2}, \alpha_{i3}, A_i]$ is the feature vector of the i^{th} triangle on the template, with d_{i1}, d_{i2} and d_{i3} being the lengths of the sides of the i^{th} triangle, α_{i1}, α_{i2} and α_{i3} are the angles of the i^{th} triangle, and A_i is the area of the i^{th} triangle. $\mathbf{F}_j$ is the feature vector of the j^{th} triangle on the query. ξ is a threshold value. Finally, the match on the longest string (N) of triangles that yields the minimum average error of $\frac{\sum_i \% \varepsilon_i^2}{N}$ is declared.

3.2 Matching in the Presence of Affine Transformation

3.2.1 Convex Hull Triangulation

As Delaunay triangulation is not invariant under affine transformation, the triangle tessellation in this case is based on the convex hull [44]–[46]. Due to the properties of convex hull [44], [45] including affine invariance and local controllability, the affine invariants can be constructed from the convex hull to deal with the affine transformation and/or the partial occlusion. One possible set of affine invariants may be constructed from the areas of the polygons formed by connecting the neighboring vertices on the convex hulls.

Fig. 2 Process of local convex hull triangulation surrounding a central minutia: (a) Triangles formed by the central minutia point and any other two successive minutiae points (some triangles are shown); (b) Smallest N areas surrounding the central minutia point; (c) Each minutiae set consisting of the nearest neighbor minutiae points derived from vertices of the smallest areas, and the central minutia; (d) Local convex hull of the minutiae set; (e) Local convex hull triangles surrounding the central minutia.

However, as our purposed method is based on the alignment of query fingerprint image against the template image, establishing correspondence between two sets of vertices is needed. Thus, we propose constructing the triangles by connecting the neighboring triplet of vertices on the local convex hull and extracting the affine invariant features from the local convex hull triangles. The areas of triangles are sorted in a conformal order. As the area is a relative invariant, to establish correspondence, the sequence of absolute invariants is required by taking the ratio of the particular area in the list with its consecutive area (in the list). The convex hull triangulation can be summarized as the following steps.

1) At each minutia, now called a central minutia point, the triangles are formed by the central minutia point and any other two successive minutiae points on the fingerprint as shown in Fig. 2 (a). The areas of triangles are computed to find the nearest neighbor minutiae.

2) To remove the problem of noisy data interference, we impose an additional constraint on the range of area and the triangle inequality to determine the smallest N areas. To avoid the collinear triangles and the triangle having too small acute angle, based on the triangle inequality, the triangles having the sum of the lengths of any two sides greater than the length of the remaining side by the optimized value of 20% are considered. Based on our experiment, the optimized area threshold of 200 is set. Only triangles whose computed areas are in the determined range are considered. Figure 2 (b) shows the smallest N areas and the nearest neighbor minutiae points derived from their vertices are shown in Fig. 2 (c).

3) For each minutiae set consisting of the nearest neighbor minutiae points and the central minutia point, the convex hull of minutiae set is formed based on the algorithm developed by Bykat [47] as shown in Fig. 2 (d). This convex hull is defined as a local convex hull. As the nearest neigh-

bor points surrounding the considered minutia are derived based on the area, they are hence relative affine invariant as well as the convex hull being applied on them.

4) All possible triangles are constructed from the vertices of local convex hull. The local convex hull triangles surrounding the central minutia point are shown in Fig. 2(e).

5) Local convex hull triangulation is applied on all minutiae points. For a set of n minutiae points contributed to the local convex hull surrounding the central minutia, all possible triangles of $\binom{n}{3}$ are then derived, hence yielding the total $m \cdot \binom{n}{3}$ triangles, where m is the total number of minutiae. For the set of local convex hull triangles surrounding a central minutia, the local convex hull triangles are ordered according to an increasing order of computed area prior to the computation of absolute affine invariant features. The absolute affine invariant will then be further used in the correspondence-establishing process.

As long as the central minutiae points (before and after the transformation) are corresponding points (i.e., are affine transformations of one another), the local convex hulls constructed by considering the nearest neighbor minutiae points surrounding the corresponding central minutiae points (before and after the transformation) will be affine transforms of one another. Since the area is affine invariant, the triangle area constructed from corresponding minutiae triplet will also be affine invariant and can be used for establishing correspondence in the case of affine transformation.

3.2.2 Establishing Correspondences of Convex Hull Triangles

In the case where a shear (in a given direction) on the query sample is applied, an affine map rather than a rigid map between the query and the template takes place. When that happens, the triangle side lengths and angles are no longer preserved. The areas of the corresponding triangles, however, become relative invariant which the two corresponding areas are related to each other through the determinant of the linear transformation matrix T in the affine map transform (T, b) , where b is the translation vector. If the area of triangles of the sequence on the template are $A(k)$, the corresponding area $A_a(k)$ of the sequence of triangles on the query under affine transformation are related to those of the template in accordance with the following relative invariant

$$A_a(k) = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} A(k), \quad k = 1, 2, \dots, n \quad (2)$$

where $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ is the determinant of the affine transformation matrix and n is the number of corresponding areas. The a_{ij} is the element of the matrix T of the affine transformation (T, b) . As the linear transformation matrix is unknown, absolute affine invariants are constructed out of the area relative invariants by taking the ratio of two triangles to cancel out the dependence of the area relative invariant on the determinant of affine transformation matrix.

To facilitate the process of establishing correspondences between two set of vertices, i.e., the minutiae, the triangles surrounding each central minutia are sorted in an increasing order of areas. Sorting the triangles in conformal order also provides the specificity of each group. By taking the ratio of the consecutive elements in the sequence, the sets of absolute invariants in Eqs. (3) and (4) are obtained

$$I(k) = \frac{A(k)}{A((k+1) \bmod m)}, \quad k = 1, 2, \dots, m \quad (3)$$

$$I_a(k) = \frac{A_a(k)}{A_a((k+1) \bmod n)}, \quad k = 1, 2, \dots, n \quad (4)$$

where m is the number of triangles on the template and n is the number of triangles on the query. In the case of noise free measurement, the absolute invariant of the query equals that of the template. Each of $I_a(k)$ will have a counterpart $I(k)$ with $I_a(k) = I(k)$. These counterparts are easily determined through a circular shift involving n comparison, where n is the number of invariants of the query. In the presence of noise and partial occlusion, only a subset of absolute invariant between the query and that of the template can be declared as matching provided that the error percentage difference between matched absolute invariants does not exceed a threshold value. We allow a small error percentage difference between corresponding invariants to achieve only small difference between the areas of triangles. This may reduce the length of the matched triangle sequence. The lower the error percentage is, the stricter the matching. Experimentally, an error percentage of 5% was applied. We adopt a run length method to decide on the correspondences between the two ordered sets of triangles. For every starting point on the sequence, the run length method computes a sequence of consecutive invariants satisfying the criterion

$$\%e_i^2 = \frac{(I_a(j) - I(i))^2}{I^2(i)} \times 100 < \xi \quad (5)$$

where $I(i)$ is the absolute invariant of the i^{th} triangle on the template and $I_a(j)$ is the absolute invariant of the j^{th} triangle on the query. We declare the match on the longest string (M) of triangles that yields minimum average error of $\frac{\sum_i \%e_i^2}{M}$. Once correspondences are found, the vertices on the matched triangles are used to estimate the affine transformation. The algorithm for matching triangles in query and template fingerprints under an affine map is shown in Fig. 3.

3.3 Matching Beyond the Affine Map

Non-linear distortions that cannot be described by a global affine map occur when a non-uniform pressure of the finger is exerted against the sensor. The most evident effect of this kind of distortion is local compression/stretching of the fingerprint ridges and valleys. Establishing local correspondences in the presence of non-linear distortions uses the shape information introduced in Sect. 3.1 (if a local rigid map seems like the one under play) and/or the affine invariants introduced in Sect. 3.2 (if a local affine map seems like

Fig. 3 Matching in the case of affine transformation: (a) Set of triangles on fingerprint minutiae of the template; (b) Sort of (a) in an increasing order of areas; (c) area of (b); (d) Ratio of area in (c); (e) Set of triangles on fingerprint minutiae on the query; (f) Sort of (e) in an increasing order of areas; (g) area of (f); (h) Ratio of area in (f). Matching triangles between (d) and (e) are shown in box.

the one under play) or a combination of both through an appropriate weighting function of both shape characteristics. Remember that we are only in need of just a few of such local matches from which you will be able to estimate the global non-linear transformation parameters.

3.3.1 Establishing Correspondences Using Combined Features

In the case in which fingerprint images are suffered from non-linear transformation, we proposed using combined features derived from enhanced minugia feature and absolute affine invariant feature. The enhanced minugia feature is constructed based on the type of minutiae, the number of triangles, the list of triangles surrounding the considered minugia point, and the ratio of consecutive ordered triangles. Only two central minugia types are considered: ridge ending and bifurcation minugia. Matching is then performed by maximizing the similarity between the minugia feature vectors. Similarity measure between minugia feature vectors is not only based on matching the shape and size of the triangle as in the case of rigid transformation (Sect. 3.1) but also on matching the absolute invariant derived from the ratio of consecutive area as in the case of affine transformation (Sect. 3.2).

For each minugia, the feature vector set ($\mathbf{V}_i$) associated with the minugia m_i is constructed based on the local convex hull triangles surrounding the minugia as introduced in Sect. 3.2.1. The minugia feature vector set is defined as

Fig. 4 Matching beyond affine transformation: (a) List of local convex hull triangles surrounding the central minugia including type of minutiae of the template; (b) List of local convex hull triangles surrounding the central minugia including type of minutiae of the test; (c) Matching triangles between minugia number 1 of the template and number 2 of the test. The matching triangles are shown in the box. The number in the triangle designates the related absolute invariant.

$\mathbf{V}_i = \{\mathbf{F}_{ic}\}$, where $\mathbf{F}_{ic}$ is the feature vector $\mathbf{F}_c$ of the c^{th} convex hull triangle surrounding the minugia m_i . $\mathbf{F}_c$ is defined as $\mathbf{F}_c = [d_{c1}, d_{c2}, d_{c3}, \alpha_{c1}, \alpha_{c2}, \alpha_{c3}, A_c, I_c]$, where d_{c1}, d_{c2} and d_{c3} are the lengths of the sides of the c^{th} triangle, α_{c1}, α_{c2} and α_{c3} are the angles of the c^{th} triangle, A_c is the area of the c^{th} triangle, and I_c is the area ratio of the c^{th} convex hull triangle to its consecutive ordered triangle as presented in Sect. 3.2.2. List of local convex hull triangles surrounding the central minugia including types of minutiae is shown in Fig. 4. The algorithm is summarized as follows:

1) At each reference minugia, either ridge ending or bifurcation, apply the local convex hull triangulation explained in Sect. 3.2.1 to the reference minugia point to derive the list of local convex hull triangles surrounding the central minugia on the template. Construct the minugia feature vector set ($\mathbf{V}_i$) from the feature vectors ($\mathbf{F}_{ic}$) of all $\binom{n}{3}$ local convex hull triangles. Figure 4(a) shows the list of local convex hull triangles associated with the reference minugia, the area ratio of consecutive ordered triangles in the list, and the type of the reference minugia of the template.

2) Construct the minugia feature vector sets for all minugia on the query fingerprint. Figure 4(b) shows samples of the list of local convex hull triangles surrounding the central minugia on the query, the area ratio of consecutive

ordered triangles in the list, and the type of minutiae for the query.

3) Given a list of triangles surrounding one minutia on the query fingerprint, search for the corresponding triangles in the list surrounding the corresponding minutia in the template that maximizes the similarity between the minutiae feature vector on the query and the template. The similarity measure is given by $S = 1/E$ where E is defined as

$$E = \alpha \frac{\sum_i \% \varepsilon_i^2}{N} + (1 - \alpha) \frac{\sum_i \% e_i^2}{M} \quad (6)$$

where α is the weighting factor, $0 \leq \alpha \leq 1$. We chose α in accordance with the fraction $\frac{N}{N+M}$. The $\% \varepsilon_i^2$ and $\% e_i^2$ are defined in Eqs. (1) and (5), respectively, and N and M are the number of matching triangles in the longest rigid and affine matching triangle strings, respectively. From Eq. (6), the resultant E is close to zero when the matching triangles are under a local rigid transformation or affine transformation. However, when the matching triangles are under the non-linear transformation (polynomial one and partly affine one), the resultant E would be higher. Therefore, we declare a match when E does not exceed a threshold value of 5%.

4) The corresponding minutiae are then used to estimate the non-linear transformation. Once the transformation parameters are determined, the query fingerprint is then aligned with the template.

3.3.2 Estimating Non-Linear Transformation

After establishing the correspondences using the combined features, the corresponding minutiae points of the template and the query are applied in Eq. (9) to compute the transformation matrix T which is consequently used to align the query fingerprint with the template.

When there is an absence of non-linear transformation, an affine transformation between the template and the query is modeled as

$$F = G \cdot T \quad (7)$$

where F and G are the matrices of matched minutiae between the query and the template, respectively, which are

$$F = \begin{bmatrix} x'_1 & y'_1 \\ x'_2 & y'_2 \\ \vdots & \vdots \\ x'_n & y'_n \end{bmatrix}$$

$$G = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots \\ x_n & y_n & 1 \end{bmatrix}$$

where n is the number of matched minutiae between the two fingerprints, and T is the affine transformation matrix

$$T = \begin{bmatrix} m_{00} & m_{01} \\ m_{10} & m_{11} \\ m_{20} & m_{21} \end{bmatrix}.$$

Since there are only six parameters of affine transformation to be estimated, at least 3 pairs of matched minutiae points are needed.

For the map beyond the affine transformation, we opt for an explicit polynomial map. To avoid the instability problem associated with the explicit (and implicit) polynomial, e.g., unboundedness of the polynomial away from the data, a polynomial transformation of degree 3 is considered. A cubic polynomial transformation for n matched minutiae points is defined as

$$F = G \cdot T \quad (8)$$

where F and G are the matrices of matched minutiae between the query and the template, respectively, which are

$$F = \begin{bmatrix} x'_1 & y'_1 \\ x'_2 & y'_2 \\ \vdots & \vdots \\ x'_n & y'_n \end{bmatrix}$$

$$G = \begin{bmatrix} x_1^3 & y_1^3 & x_1^2 y_1 & x_1 y_1^2 & x_1^2 & y_1^2 & x_1 y_1 & x_1 & y_1 & 1 \\ x_2^3 & y_2^3 & x_2^2 y_2 & x_2 y_2^2 & x_2^2 & y_2^2 & x_2 y_2 & x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n^3 & y_n^3 & x_n^2 y_n & x_n y_n^2 & x_n^2 & y_n^2 & x_n y_n & x_n & y_n & 1 \end{bmatrix},$$

and T is the non-linear transformation matrix

$$T = \begin{bmatrix} m_{00} & m_{01} \\ m_{10} & m_{11} \\ \vdots & \vdots \\ m_{90} & m_{91} \end{bmatrix}.$$

The transformation matrix T can be recovered from a pair of 10 matched landmarks, or estimated using least square error (LSE) sense if the number of matched landmarks is greater than 10. The LSE solution of T is given as

$$T = (G^T G)^{-1} (G^T F). \quad (9)$$

In general, the cubic polynomial map can be used in all cases. When the polynomial map is used and the fingerprint image is subjected to only linear transformation, the estimated parameters from the first row to the seventh row of transformation matrix in Eq. (9) will be close to zero.

4. Directional Filterbank (DFB) Derived Features

This section introduces the computing of Directional Filterbank (DFB) features, which will be combined with the average distance map error to serve as the identification and verification cost function. DFB was originally introduced by Bamberger and Smith [48], which is effective in processing images with directional information in such applications as object and character analysis, texture classification, denoising, segmentation, enhancement, remote sensing, and data analysis [49]. There are two reasons that DFB is very

suitable for enhancing fingerprint identification and classification: 1) fingerprint image consists of only a background and foreground (ridges and valleys); 2) fingerprint image is directional in nature.

The idea of using filterbank-based features for fingerprint matching was proposed by Jain *et al.* [38]. Their approach is based on extracting a reference point, mostly a singular point, and on tessellating a region of interest (ROI) around that reference point. The ROI is then filtered into eight different directions using a bank of Gabor filters. The eight feature vectors are derived by averaging the absolute deviation from the mean of gray level in individual section. Unlike their approach, our DFB appearance method does not use a reference point extraction (generally not a straightforward task), but uses the minutiae triangle tessellation. In addition, our approach is (or made) invariant to geometric transformations through the un-warping method described in Sect. 5 using the shape and size invariants.

After the minutiae-based matching and fingerprint alignment, the computation of the DFB features can be performed as the following: Given two matched triangles formed by two sets of 3 corresponding minutiae points, the fingerprint images capsulated by the corresponding query and candidate triangles are filtered each into eight direc-

tions using the DFB, resulting into eight triangle patches. Each is filled with the binarized ridge curve pattern of the corresponding eight-directional filtered image as shown in Fig. 5(a) for the query image. Square boxes are then bounded on the eight-directional triangle patches of interest to derive the eight-directional filtered sub-images shown in Fig. 5(b). Each sub-image is then divided into n vertical stripes, saying 20, where for each stripe we compute the energy in each stripe, normalize them into gray level and input the computed gray level into the corresponding stripe. Dividing the sub-image into vertical stripes can represent the local ridge orientation including the number of ridges along that orientation. The large number of binarized ridge curves corresponding to the filter direction results in more bright level in the DFB feature vector as shown in Fig. 5(c). The eight feature vectors are then concatenated to derive a primary DFB feature vector shown in Fig. 5(d). Similar feature vectors will be obtained from the corresponding minutiae triangles on the candidate fingerprint. We compute the primary DFB feature for each triangle and re-order the primary DFB features in an increasing order of area of all considered triangles to derive the secondary DFB features.

5. Verification and Identification of a Query Fingerprint

In this section, we describe the fingerprint verification and identification based on the minutiae-based triangles matching described in Sect. 3.3, augmented by the DFB appearance embedded in these triangles as explained in Sect. 4. An overall fingerprint matching algorithm is shown in Fig. 6. To start the fingerprint verification and identification process, the best warping transformation between the candidate and the query fingerprints is estimated based on the matched triangles. The query fingerprint is then mapped into the candidate coordinate system in accordance with that estimated map. For all minutiae points on the query, we find the closest minutiae points on the candidate using a distance map [43] and look up tables constructed beforehand for each candi-

Fig. 5 Process of computing DFB feature: (a) Directional filtered images; (b) Square boxes bounded on the eight-directional triangle patches of interest; (c) DFB feature vector of the eight-directional filtered sub-images; (d) Primary DFB feature vector.

Fig. 6 Fingerprint matching algorithm.

date fingerprint in the database. These constitute the corresponding minutiae points of the query for a given candidate. An average geometric distance map error function is then computed. The average distance map error function E_{MAP} is defined as

$$E_{MAP} = \sqrt{\frac{\sum_{i=1}^n (m_c(i) - m_q(i))^2}{n}} \quad (10)$$

where $m_q(i)$ is the i^{th} minutia coordinate on the transformed query fingerprint, $m_c(i)$ is the i^{th} minutia coordinate on the candidate closest to that on the query fingerprint, and n is the number of minutiae of the transformed query fingerprint. This is performed for every other possible candidate fingerprint. In an identification problem, the query is identified as the candidate that results in the smallest average geometric distance map error function. In a verification problem, the identity of the query fingerprint is verified if the average geometric distance map error function is below a set threshold.

The identification and/or verification could also be augmented with an appearance-based matching process for which interior features (DFB features) of a triangle are compared to those of the matching triangle, where the interior features are derived from the eight-directional filterbank appearance images. The DFB error function E_{DFB} is defined as

$$E_{DFB} = \frac{\sum_i \sum_j (S_c(i, j) - S_q(i, j))^2}{\sum_i \sum_j S_c^2(i, j)} \times 100 \quad (11)$$

where $S_c(i, j)$ is the DFB feature (Fig. 5 (d)) at the j^{th} stripe of the i^{th} corresponding triangle on the candidate fingerprint and $S_q(i, j)$ is the DFB feature at the j^{th} stripe of the i^{th} corresponding triangle on the warped query fingerprint. From Eq. (11), $\sum_i \sum_j (S_c(i, j) - S_q(i, j))^2$ is the summation of the errors of all corresponding DFB features on the query and their corresponding ones on the candidate and $\sum_i \sum_j S_c^2(i, j)$ is the summation of the corresponding DFB features on the candidate.

With the augmentation of the error metric based on the DFB features, the overall verification and identification rule combining the average geometric distance map error (E_{MAP}) with DFB error (E_{DFB}) is

$$\tilde{E} = \beta E_{DFB} + (1 - \beta) E_{MAP} \quad (12)$$

where $\tilde{E}$ is the combined error and β is factor between 0 and 1. The β takes a value equal to, greater or less than 0.5, depending on which error factor is favored.

6. Experimental Results

The experiments are divided into four parts. The first is a robustness evaluation of the proposed affine invariant features. The second shows the registration of two fingerprints in the presence of affine transformation, non-linear distortion, and partial occlusion. The third reports the results of fingerprint verification of our proposed algorithm on the database of fingerprints. The computational aspect of the proposed method

Table 1 Error and the number of matched triangles in each string resulting from the robustness evaluation.

Long string no.	Number of matched triangles	Average error (%)
1	15	0.10
2	19	0.13
3	22	0.18
4	23	0.17

Fig. 7 Long strings of matched triangles of (a) the fingerprint with low pressure and (b) the fingerprint with high compression against sensor surface. Numbers correspond to long string numbers in Table 1.

is shown in the fourth.

6.1 Robustness Evaluation of Affine Invariant Features

To evaluate robustness of the proposed absolute affine invariant features in Sect. 3.2.2 against the non-linear distortion, the matching algorithm presented in Sect. 3.3.1, which the list of triangles surrounding each central minutia is constructed, was applied to find the overall long strings of matched triangles between the two fingerprints of the same individual. One is a fingerprint with low pressure and the other is a fingerprint with high compression against sensor surface, the latter of which is served as non-linear distortion. Error of the absolute affine invariant as the area ratio of consecutive triangles was calculated according to Eq. (6) by setting α equal to 0. The error and the number of matched triangles in each string are shown in Table 1. An overall average error of the proposed absolute affine invariant features is less than 0.2%. The results of long strings of the matched triangles between the two fingerprints are shown in Fig. 7. From the results of robustness evaluation, the proposed absolute affine invariant features are robust and stable against the non-linear distortion.

6.2 Fingerprint Registration Based on Minutiae-Based Matching

Fingerprints of the same individual's thumb were scanned with an L SCAN 100R fingerprint scanner. A resolution of the scanner is 500 pixels per inch (ppi) under different transformations. Prior to the computation of the triangulation, pre-processing was performed using Gaussian blurring, Gabor filtering, thinning, and minutiae points detection. Only two types of minutiae points are considered:

Fig. 8 Two fingerprints (a) before and (b) after alignment in the presence of affine transformation. Template fingerprint and matched triangles of the template are presented in blue. Query fingerprint and matched triangles of the query are presented in red.

Table 2 Average distance map and alignment errors of the two fingerprints in the presence of affine transformation.

	Before alignment	After alignment
Average distance map (inch)	0.084	0.015
Alignment errors on average (% of the size of the finger)	14.872	2.575

ridge ending and bifurcation points. By using the minutiae-based matching algorithm described in Sect. 3, the matched triangles of the two fingerprints were found and the vertices of their corresponding triangles were then used to estimate the transformation parameters. The matching results and the alignment are represented under different transformations, namely affine, non-linear, and partial occlusion.

6.2.1 Matching Results in the Presence of Affine Transformation

In the presence of affine transformation, the matched triangles are found based on the absolute affine invariants constructed from the minutiae-based local convex hull triangulation described in Sect. 3.2. The alignment of the two fingerprints is shown in Fig. 8. The matched triangles on the template and the query fingerprints are shown in Fig. 8 (a).

Since the true correspondences of the scanned ridge points are unknown, we opt to use the distance map that displays the distance between any point of one ridge coordinate and the closest point on the other image after undoing the transformation to the second image. The average distance map before alignment is performed by aligning the centroids computed from all ridge points on the two fingerprints and then computing the distance map between sets of minutiae points on the two fingerprints. After transforming the query image to the second image, the average distance map after alignment is obtained by computing the distance map between the entire minutiae points set on the template and the transformed image. The averages of the distance errors before and after the alignment are shown in Table 2. The alignment errors on average are approximately 2.575% of the size of the finger.

Fig. 9 Two fingerprints (a) before and (b) after alignment in the presence of non-linear distortion. Template fingerprint and matched triangles of the template are presented in blue. Query fingerprint and matched triangles of the query are presented in red.

Table 3 Average distance map and alignment errors of the two fingerprints in the presence of non-linear distortion.

	Before alignment	After alignment
Average distance map (inch)	0.079	0.042
Alignment errors on average (% of the size of the finger)	9.188	4.889

6.2.2 Matching Results in the Presence of Non-linear Distortion

In case of the non-linear distortion, the subject smeared and compressed his/her finger against the sensor, resulting in the local compression/stretching in the acquired fingerprint. Fingerprint matching is performed based on the combined absolute invariant features presented in Sect. 3.3. The alignment of the two fingerprints is shown in Fig. 9. The matched triangles on the template and the query fingerprints are shown in Fig. 9 (a). The averages of the distance errors before and after the alignment are shown in Table 3. The alignment errors on average are approximately 4.889% of the size of the finger.

6.2.3 Matching Results in the Presence of Partial Occlusion

The results of matched triangles and alignment of the two fingerprints based on the combined features described in Sect. 3.3 are presented in Fig. 10. The averages of the distance errors before and after the alignment are shown in Table 4. The alignment errors on average are approximately 1.346% of the size of the finger.

From the results of fingerprint alignment, our proposed fingerprint matching can be used to find the corresponding minutiae and align any two fingerprints in the case of the affine transformation, the non-linear transformation, the partial occlusion, and in the presence of noise. This makes our system portable and applicable under varying conditions and acquisition systems that differ from those under which the fingerprint database is constructed.

6.3 Fingerprint Verification Based on Combined Features

A fingerprint database was created from a total of 40 stu-

Fig. 10 Two fingerprints (a) before and (b) after alignment in the presence of partial occlusion, based on the combined features described in Sect. 3.3. Template fingerprint and matched triangles of the template are presented in blue. Query fingerprint and matched triangles of the query are presented in red.

Table 4 Average distance map and alignment errors of the two fingerprints in the case of partial occlusion.

	Before alignment	After alignment
Average distance map (inch)	0.048	0.010
Alignment errors on average (% of the size of the finger)	6.337	1.346

dents in the Electrical Engineering Department at King Mongkut's Institute of Technology Ladkrabang and in the Biomedical Engineering program at Srinakharinwirot University, Thailand. The subjects were mainly 20 to 30 years of age, approximately 60% of whom were women. For each subject, 7 impression scans per finger from a total of 6 fingers (thumb, forefinger, and middle finger on both hands) were taken. The first impression was acquired as a template fingerprint with the finger placed orthogonal to the surface sensor in an upright position. The other impressions were taken by applying low pressure, partial occlusion, lateral roll, and rotation with high compression against the sensor surface. Examples of these impressions of each finger are shown in Fig. 11. The rotated fingers deviated up to ± 45 degrees from the vertical upright position of the scanner were acquired. During the image acquisition, the sensor plate was not systematically cleaned. Among these fingerprint images, there are some images obtained from dry skin or sweaty skin, causing some noise on fingerprint images.

Our database was hence comprised of a total of 1680 ($40 \times 6 \times 7$) fingerprint images captured from the 240 fingers, with each scanned image being 620×620 pixels in size. This database was used to evaluate the performance of our fingerprint matching algorithm. The number of genuine matches is 5040 matches (21 of possible pairs of fingerprints coming from the same finger $\times$ 6 fingers $\times$ 40 individuals = 5040) and the number of imposter matches is 28680 matches ($((240 \times 239)/2)$ of possible pairs of template fingerprints coming from the different fingers). For fingerprint verification, the minutiae-based matching based on the combined absolute invariant features described in Sect. 3.3 was applied. If the matched triangles between the two fingerprints can be found, the query fingerprint will be mapped into the template coordinate system according to the transformation estimated from the corresponding minutiae. The

Fig. 11 Fingerprint impressions of (a) an upright position with normal pressure, (b) low pressure, (c) partial occlusion, (d) and (e) lateral roll in 2 different directions, and (f) and (g) rotation in two different directions with high compression against the sensor surface.

combined error ($\tilde{E}$) presented in Sect. 5 was then calculated from the geometric distance map error (E_{MAP}) augmented with the DFB error (E_{DFB}), both of which were weighed by 0.5. Finally, the combined error was inverted to a matching score ($\tilde{S}$) as

$$\tilde{S} = 1 - \frac{\tilde{E}}{100} \quad (13)$$

where $\tilde{S}$ is a matching score and $\tilde{E}$ is the combined error defined in Eq. (12). Matching scores resulting from the genuine matches and the imposter matches were collected to plot their histogram distributions.

As a result, histogram distributions of matching scores for the genuine matching pairs and for the imposter matching pairs are shown in Fig. 12. Based on the distributions, a receiver operating characteristic (ROC) curve is plotted and shown in Fig. 13. A zoomed-in ROC curve is shown in Fig. 14. The area under the ROC curve is 0.99967. The cut-off threshold point for the Bayesian classifier is 0.505 and the average probability of error computed based on the threshold is 0.713%.

In addition, the performance of proposed algorithm was evaluated according to the Fingerprint Verification Competition (FVC) testing procedures [50]. The False Match Rate (FMR) and False Non-Match Rate (FNMR) curves of the proposed algorithm are shown in Fig. 15. From the curves, the Equal Error Rate (EER) with 95% confi-

Fig. 12 Distribution of genuine matching scores and distribution of imposter matching scores. The x-axis represents the threshold of matching scores.

Fig. 14 Zoomed-in ROC curve in which the square mark in the circle represents the cut-off point.

Fig. 13 Receiver operating characteristic (ROC) curve in which the circle mark represents the cut-off point.

Fig. 15 False Match Rate (FMR) and False Non-Match Rate (FNMR) curves.

dence interval of our algorithm is $2.68 \pm 0.18\%$ assuming uncorrelated matching decisions [51], [52]. The threshold at the EER point is approximately 0.420. A Detection-Error Tradeoff (DET) curve plotted FMR against FNMR is also presented in Fig. 16. At the threshold of EER point, the FMR and the FNMR are 2.55% and 2.80%, respectively. A number of false non-matches are mostly found on matching of the very small partial overlap between the 2 impressions, whereas a number of false matches probably result from the miss detection of some minutiae and the spurious minutiae. The false matches, however, are mainly obtained from matching of the two individual fingerprints with different fingerprint classes. This error may be reduced by the pre-selection algorithm used in an identification system. Although there are some errors due to miss detection of minutiae and spurious minutiae, the proposed algorithm is still

used to deal with the non-linear distortion and not too small partial overlap to verify the fingerprints. Moreover, the performance of our proposed matching algorithm in comparison with that of standard minutiae-based method [22], of the well-known technique of DFB features [38], and of triangulation-based matching integrated with modeling non-linear deformation using Radial Basis Function (RBF) [53], is shown in Tables 5 and 6. From the tables, at the comparable FMR to that of the other matching algorithms, the proposed algorithm shows the lower FNMR.

6.4 Computational Aspect

The computational time for fingerprint verification/identification systems ranges in 45-125s on Intel Core2 Duo processor (1.80 GHz), 2GB of RAM. Most of the time is consumed

Fig. 16 Detection-Error Tradeoff (DET) curve plotted FMR against FNMR.

Table 5 Performance comparison of the proposed algorithm with other matching algorithms at FMR approximately 0.14%.

Matching Algorithm	False Match Rate (%)	False Non-Match Rate (%)
Standard minutiae-based method [22]	0.16	11.23
Triangulation combined with modeling non-linear deformation using RBF [53]	0.18	8.48
Proposed matching algorithm	0.14	3.27

Table 6 Performance comparison of the proposed algorithm with other matching algorithms at FMR approximately 1.09%.

Matching Algorithm	False Match Rate (%)	False Non-Match Rate (%)
Well-known DFB technique [38]	1.07	7.87
Proposed matching algorithm	1.09	2.89

in the establishing corresponding triangles, which is proportional to the number of minutiae on the query fingerprint.

7. Discussion and Conclusion

In this paper, we introduced a hybrid fingerprint-matching method that combines a geometric-based method applied to the local convex hull minutiae-based triangles with the global directional filterbank (DFB) technique for fingerprint identification. In the proposed method, the extracted minutiae points are local and hence well suited to dealing with the partial alignment problem (occlusion). To find correspondences between the minutiae points on the two fingerprint images, sets of geometric invariants were constructed based on the minutiae-based triangles. The combined features based on geometric invariance were also presented to find the corresponding minutiae in the case of non-linear distortion. After the correspondences were established, the parameters of a relevant transformation were estimated and the two images were aligned. The performance of our method

is demonstrated by the ability to register the two fingerprint images scanned under various kinds of shape transformations. The results of fingerprint alignment show that our proposed method can be used to find the corresponding minutiae and align any two fingerprints in the case of the affine transformation, the non-linear transformation, the partial occlusion, and in the presence of noise. This makes our system portable and applicable under varying conditions and acquisition systems that differ from those under which the fingerprint database is constructed.

For fingerprint verification, we also proposed the identification/verification rule that combines the geometric distance map error with the DFB error to identify/verify a query fingerprint against candidate fingerprint(s). Our performance yields an area of 0.99967 under the receiver operating characteristic (ROC) curve, based on a database consisting of a total of 1680 fingerprint images captured from the 240 fingers, with each scanned image being 620×620 pixels in size. The average probability of error is found to be 0.713%. Our algorithm is also compared to the standard minutiae-based method [22], the well-known technique of DFB features [38], and the triangulation-based matching integrated with modeling non-linear deformation using Radial Basis Function (RBF) [53] for a comparable FMR; consequently, our approach results in the smallest FNMR (one half to one third the recorded rates of the state of the art systems).

From the performance evaluation, the false match rate obtained from the matching of the two individuals with different fingerprint classes may be reduced by pre-selection algorithm or fingerprint classification when the proposed algorithm is applied in the identification system. Moreover, the proposed minutiae-based matching could be used in conjunction with a non-minutiae based matching as ridge structures to improve finding the correspondences and aligning the two fingerprints.

In conclusion, the performance of the proposed algorithm is extremely impressive, in particular in light of the fact that the fingerprint of a person (as the query) is taken under conditions that differ from those under which the fingerprint of the same person stored in a fingerprint database was constructed, especially the non-linear distortion and the partial overlap. This work represents an advance in resolving the fingerprint identification problem beyond the state-of-the-art approaches in both performance and robustness, including changes in conditions under which the fingerprint is taken.

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References

- [1] M. Xue, J. Li, W. Xu, Z. Lu, K.L. Wang, P.K. Ko, and M. Chan, "A self-assembly conductive device for direct DNA identification in integrated microarray based system," *Int. Electron Devices Meeting, IEDM'02*, pp.207–210, San Francisco, CA, Dec. 2002.
- [2] B. Moreno, A. Sanchez, and J.F. Velez, "On the use of outer ear images for personal identification in security applications," *Proc. IEEE 33rd Annu. 1999 Int. Carnahan Conf. Security Technology*, pp.469–476, Madrid, Oct. 1999.
- [3] Y. Gao and M.K.H. Leung, "Face recognition using line edge map," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol.24, no.6, pp.764–779, June 2002.
- [4] M.S. Alam and M. Akhteruzzaman, "Real time fingerprint identification," *Proc. IEEE Nat. Aerospace and Electronics Conf., NAECON 2000*, pp.434–440, Dayton, OH, Oct. 2000.
- [5] C.S. Lee and A. Elgammal, "Gait style and gait content: Bilinear models for gait recognition using gait re-sampling," *Proc. 6th IEEE Int. Conf. Automatic Face and Gesture Recognition*, pp.147–152, May 2004.
- [6] R. Sanchez-Reillo, C. Sanchez-Avila, and A. Gonzalez-Marcos, "Biometric identification through hand geometry measurements," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol.22, no.10, pp.1168–1171, Oct. 2000.
- [7] L.W. Liam, A. Chekima, L.C. Fan, and J.A. Dargham, "Iris recognition using self-organizing neural network," *Student Conf. Research and Development, SCORED 2002*, pp.169–172, 2002.
- [8] E. Yu and S. Cho, "GA-SVM wrapper approach for feature subset selection in keystroke dynamics identity verification," *Proc. Int. Joint Conf. Neural Networks*, vol.3, pp.2253–2257, July 2003.
- [9] T. Nakamoto and T. Morizumi, "Odor sensor using quartz-resonator array and neural-network pattern recognition," *Proc. IEEE Ultrasonics Symp.*, vol.1, pp.613–616, Chicago, IL, Oct. 1988.
- [10] L. Zhang and D. Zhang, "Characterization of palmprints by wavelet signatures via directional context modeling," *IEEE Trans. Syst. Man Cybern. B, Cybern.*, vol.34, no.3, pp.1335–1347, June 2004.
- [11] S. Palla, H. Lei, and V. Govindaraju, "Signature and lexicon pruning techniques," *9th Int. Workshop on Frontiers in Handwriting Recognition, IWFHR-9*, pp.474–478, Oct. 2004.
- [12] M.A. Bartsch and G.H. Wakefield, "Singing voice identification using spectral envelope estimation," *IEEE Trans. Speech Audio Process.*, vol.12, no.2, pp.100–109, March 2004.
- [13] E.C. Driscoll, C.O. Martin, K. Ruby, J.J. Russell, and J.G. Watson, "Method and apparatus for verifying identity using image correlation," *U.S. Patent 5,067,162*, Nov. 1991.
- [14] A. Sibbald, "Method and apparatus for fingerprint characterization and recognition using auto-correlation pattern," *U.S. Patent 5,633,947*, May 1997.
- [15] C. Beleznaï, H. Ramoser, B. Wachmann, J. Birchbauer, H. Bischof, and W. Kropatsch, "Memory efficient fingerprint verification," *Proc. Int. Conf. Image Processing, Thessaloniki, Greece*, vol.2, pp.463–466, Oct. 2001.
- [16] H. Yahagi, S. Igaki, and F. Yamagishi, "Moving-window algorithm for fast verification," *Proc. IEEE Southeastcon '90*, pp.343–348, New Orleans, LA, April 1990.
- [17] Z.M. Kovacs-Vajna, "A fingerprint verification system based on triangular matching and dynamic time warping," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol.22, no.11, pp.1266–1276, Nov. 2000.
- [18] D. Maltoni, D. Maio, A.K. Jain, and S. Prabhakar, *Handbook of Fingerprint Recognition*, Springer-Verlag, New York, 2003.
- [19] A.K. Jain, L. Hong, S. Pankanti, and R. Bolle, "An identity-authentication system using fingerprints," *Proc. IEEE*, vol.85, no.9, pp.1365–1388, Sept. 1997.
- [20] D. Maio and D. Maltoni, "Direct gray-scale minutiae detection in fingerprints," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol.19, pp.27–40, Jan. 1997.
- [21] N. Yager and A. Amin, "Fingerprint verification based on minutiae feature: A review," *Pattern Anal. Applic.*, vol.7, pp.94–113, Feb. 2004.
- [22] A.K. Jain, L. Hong, and R. Bolle, "On-line fingerprint verification," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol.19, no.4, pp.302–314, April 1997.
- [23] S. Ranade and A. Rosenfeld, "Point pattern matching by relaxation," *Pattern Recognit.*, vol.12, no.4, pp.269–275, 1980.
- [24] J. Hong and X. Tan, "A new approach to point pattern matching," *Proc. 9th Int. Conf. Pattern Recognition*, pp.82–84, Rome, Nov. 1988.
- [25] J. Sprinzak and M. Werman, "Affine point matching," *Pattern Recognit. Lett.*, vol.15, no.4, pp.337–339, April 1994.
- [26] H.S. Baird, *Model-Based Image Matching Using Locations*, MIT Press, Cambridge, MA, 1985.
- [27] J.P.P. Starink and E. Backer, "Finding point correspondences using simulated annealing," *Pattern Recognit.*, vol.28, no.2, pp.231–240, Feb. 1995.
- [28] D.H. Ballard, "Generalizing the Hough transform to detect arbitrary shapes," *Pattern Recognit.*, vol.13, no.2, pp.111–122, 1981.
- [29] G. Stockman, S. Kopstein, and S. Benett, "Matching images to models for registration and object detection via clustering," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol.PAMI-4, pp.229–241, May 1982.
- [30] G. Bebis, T. Deaconu, and M. Georgiopoulos, "Fingerprint identification using Delaunay triangulation," *Proc. 1999 Int. Conf. Information Intelligence and Systems, ICIIS'99*, pp.452–459, Bethesda, MD, 1999.
- [31] N. Liu, Y. Yin, and H. Zhang, "A fingerprint matching algorithm based on Delaunay triangulation net," *the 5th Int. Conf. Computer and Information Technology, CIT 2005*, pp.591–595, Shanghai, Sept. 2005.
- [32] X. Chen, J. Tian, X. Yang, and Y. Zhang, "An algorithm for distorted fingerprint matching based on local triangle feature set," *IEEE Trans. Inf. Forensics Security*, vol.1, no.2, pp.169–177, June 2006.
- [33] A. Senior and R. Bolle, "Improved fingerprint matching by distortion removal," *IEICE Trans. Inf. & Syst.*, vol.E84-D, no.7, pp.825–832, July 2001.
- [34] A. Ross, S.C. Dass, and A.K. Jain, "Fingerprint warping using ridge curve correspondences," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol.28, pp.19–30, Jan. 2006.
- [35] N.K. Ratha, R.M. Bolle, V.D. Pandit, and V. Vaish, "Robust fingerprint authentication using local structural similarity," *5th IEEE Workshop on Applications of Computer Vision*, pp.29–34, Palm Springs, CA, Dec. 2000.
- [36] A. Wahab, S.H. Chin, and E.C. Tan, "Novel approach to automated fingerprint recognition," *IEE Proc. Vision, Image and Signal Processing*, vol.145, no.3, pp.160–166, June 1998.
- [37] Y.L. Yin, X.B. Ning, X.M. Zhang, and Y.L. Wang, "A fingerprint matching algorithm based on ridge comparability," *Pattern Recognition and Artificial Intelligence*, vol.15, no.4, pp.502–506, 2002.
- [38] A.K. Jain, S. Prabhakar, L. Hong, and S. Pankanti, "Filterbank-based fingerprint matching," *IEEE Trans. Image Process.*, vol.9, no.5, pp.846–859, May 2000.
- [39] M. Tico and P. Kuosmanen, "Fingerprint matching using an orientation-based minutia descriptor," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol.25, no.8, pp.1009–1014, Aug. 2003.
- [40] Y. Wang, J. Hu, and D. Phillips, "A fingerprint orientation model based on 2D Fourier expansion (FOMFE) and its application to singular-point detection and fingerprint indexing," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol.29, no.4, pp.573–585, April 2007.
- [41] A. Ross, A. Jain, and J. Reisman, "A hybrid fingerprint matcher," *Pattern Recognit.*, vol.36, pp.1661–1673, July 2003.
- [42] F. Benhammadi, M.N. Amirouche, H. Hentous, K. Bey Beghdad, and M. Aissani, "Fingerprint matching from minutiae texture maps," *Pattern Recognit.*, vol.40, pp.189–197, Jan. 2007.
- [43] P.E. Danielsson, "Euclidean distance mapping," *Computer Graphics and Image Processing*, vol.14, pp.227–248, 1980.

- [44] F.P. Preparata and M.I. Shamos, *Computational Geometry: An Introduction*, Springer-Verlag, Berlin, Germany, 1985.
- [45] Z. Yang and F.S. Cohen, "Image registration and object recognition using affine invariants and convex hulls," *IEEE Trans. Image Process.*, vol.8, no.7, pp.934–946, July 1999.
- [46] D.P. Huttenlocher, "Fast affine point matching: An output-sensitive method," *Proc. CVPR' 91 IEEE Computer Society Conf. Computer Vision and Pattern Recognition*, Maui, HI, pp.263–268, June 1991.
- [47] A. Bykat, "Convex hull of a finite set of points in two dimensions," *Inf. Process. Lett.*, vol.7, no.6, pp.296–298, Oct. 1978.
- [48] R.H. Bamberger and M.J.T. Smith, "A filter bank for the directional decomposition of images: Theory and design," *IEEE Trans. Signal Process.*, vol.40, no.4, pp.882–893, April 1992.
- [49] A.E. Caldwell, H.J. Choi, A.B. Kahng, S. Mantik, M. Potkonjak, G. Qu, and J.L. Wong, "Effective iterative techniques for fingerprinting design IP," *IEEE Trans. Comput.-Aided Des. Integr. Circuits Syst.*, vol.23, no.2, pp.208–215, Feb. 2004.
- [50] D. Maio, D. Maltoni, R. Cappelli, J.L. Wayman, and A.K. Jain, "FVC 2000: Fingerprint Verification Competition," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol.24, no.3, pp.402–412, March 2000.
- [51] M.E. Schuckers, "A parametric correlation framework for the statistical evaluation and estimation of biometric-based classification performance in a single environment," *IEEE Trans. Inf. Forensics Security*, vol.4, no.2, pp.231–241, June 2009.
- [52] M. Gamassi, M. Lazzaroni, M. Misino, V. Piuri, D. Sana, and F. Scotti, "Quality assessment of biometric systems: A comprehensive perspective based on accuracy and performance measurement," *IEEE Trans. Instrum. Meas.*, vol.54, no.4, pp.1489–1496, Aug. 2005.
- [53] C. Wang and M.L. Gavrilova, "A novel topology-based matching algorithm for fingerprint recognition in the presence of elastic distortions," *Computer Science and Its Applications - ICCSA 2005, LNCS 3480*, pp.748–757, 2005.

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