PAPER

Image Quality Assessment by Quantifying Discrepancies of Multifractal Spectrums

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SUMMARY  Visual quality evaluation is crucially important for various video and image processing systems. Traditionally, subjective image quality assessment (IQA) given by the judgments of people can be perfectly consistent with human visual system (HVS). However, subjective IQA metrics are cumbersome and easily affected by experimental environment. These problems further limit its applications of evaluating massive pictures. Therefore, objective IQA metrics are desired which can be incorporated into machines and automatically evaluate image quality. Effective objective IQA methods should predict accurate quality in accord with the subjective evaluation. Motivated by observations that HVS is highly adapted to extract irregularity information of textures in a scene, we introduce multifractal formalism into an image quality assessment scheme in this paper. Based on multifractal analysis, statistical complexity features of nature images are extracted robustly. Then a novel framework for image quality assessment is further proposed by quantifying the discrepancies between multifractal spectrums of images. A total of 982 images are used to validate the proposed algorithm, including five type of distortions: JPEG2000 compression, JPEG compression, white noise, Gaussian blur, and Fast Fading. Experimental results demonstrate that the proposed metric is highly effective for evaluating perceived image quality and it outperforms many state-of-the-art methods.

key words: image quality assessment, multifractal spectrums, statistical complexity feature, human visual system

1. Introduction

Digital images always become distorted during acquisition, processing, compression, storage, transmission, and displaying, which will eventually lead to loss of visual quality [1]. Therefore, an efficient scheme to dynamically monitor image quality and judge distortion degree is becoming essential in numerous image applications. Recently, research has been focused on how to develop objective image quality assessment (IQA) methods that can automatically predict perceived image quality [2]. Hitherto, IQA metrics are categorized as full-reference (FR), reduced-reference (RR) and no-reference (NR) methods. In FR metrics, a complete reference image is needed [3], [4]. RR methods are used where only partial information of reference images is available [5], [6], nor no information of the reference images is accessible in NR scheme [7], [8]. This paper focuses on the FR metrics.

Various FR IQA metrics have been proposed in literature. In early works, the most widely used approaches are mean squared error (MSE) and peak of signal-to-noise ratio (PSNR) [9] for the ease of mathematical calculation and clear physical meaning. However, they have been criticized for not correlating well with perceived quality [13]. In recent years, the most popular FR metrics is structural similarity index (SSIM), which is based on the assumption that HVS is highly adapted for extracting structural information from natural scene [14]. Various research works have shown that SSIM could significantly improve the correlation results to the human subjective evaluation [15], [16]. Extensive perceptual image quality assessment approaches were proposed based on SSIM soon after [17]–[19]. However, SSIM-based methods are less efficient when used to evaluate blurred images. Later on, a contourlet transform based algorithm (MSDD) [20] was put forward that could effectively measure the image quality across various distortion types. More recently, both an information fidelity criterion (IFC) [1] and a visual information fidelity (VIF) metrics [3] were proposed based on natural scene statistics as well as HVS models. Compared with previous methods, IFC and VIF achieved much better adaptability and accuracy to different types of image distortion. Nevertheless, the mathematical calculation expense for IFC and VIF is very high, which may further limit their wide applications. In addition, a variety of IQA methods were put forward based on NSS, especially in NR field, which could efficiently evaluate a certain type of image distortion [10]–[12]. From the development of IQA methods, it is obvious that both HVS and NSS characteristics are crucially important in visual perception.

To satisfy both accuracy and efficiency of IQA metrics, novel image quality assessment metrics based on multifractal theory is proposed in this paper. Multifractal analysis is put forward to extract the statistical complexity information of images which is in accordance with HVS. Therefore multifractal analysis is a powerful tool to extract significant image features for image understanding. Fractal geometry provides a mathematical model for many complex objects found in nature. Many researchers have applied the fractal theory to several fields of scientific research, such as biology [22], [23], physics [24], [25], and medicine [26], [27]. Pentland noticed that the fractal model of image can be used to acquire shape information and to distinguish smooth or rough textured regions, which is in accordance with human perception [28]. Beyond fractal theory, multifractal analysis characterizes how globally irregular a scene is. Multifractal spectrum concentrates on describing the fluctuations along

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space of the local regularity of an object. Image distortions can change the multifractal spectra of original image as well as perceived image quality. In this context, the goal of the present contribution is to evaluate visual quality by quantifying the discrepancies of multifractal spectrums of reference images and that of the distorted images.

This work is divided into five sections, including this introduction. In Sect. 2, the box-counting method for image multifractal spectrums is introduced to extract statistical complexity features. A novel IQA metric based on multifractal properties is proposed in Sect. 3. Section 4 demonstrates the performance comparison of the proposed method and some related state-of-the-art metrics. Finally, Sect. 5 draws up a conclusion.

2. Fractal Dimension and Multifractal Spectra

Digital images are generally highly structured and the complexity of texture is crucially important to human perception. A good image quality metrics should be able to extract the key features. Fractal theory is a powerful tool to analysis digital images and to capture the complexity information of an object. Texture irregularity and surface complexity is fundamental to nature scene, such as the edge and contour information of images. Quantifying the surface morphology is difficult since it is not easy to describe the surface mathematically. However, this becomes possible since Mandelbrot first proposed the concept of fractal [29]. Furthermore, these properties are associated to the visual perception of an image. In other words, fractal geometry establish a bridge between mathematical model of a digital image and the subjective visual distinction. Therefore, fractal is becoming a powerful and efficient tool to characterize and describe digital images.

The most widely used fractal measure is the fractal dimension, which aims to measure the complexity of irregular objects through the following expression:

$$FD = \lim_{r \to 0} \frac{\ln(N(r))}{1/r}$$  \hspace{1cm} (1)

where $N(r)$ is some kind of complexity measure and $r$ is the scale which the measure is taken. The fractal dimension can represent the irregular degree of an image, for a larger fractal dimension refer to more complex and rough textures in the image.

Although fractal dimension is an efficient tool to describe all the richness of a complex image, some research has noticed that images of different content may have the same fractal dimension. In order to extract more fractal information of images, we focus on multifractal which can be seen as an extension of fractal. The multifractal analysis is based on the definition of multifractal spectra. Multifractal spectrums describe the evolution of the probability distribution of fractal structures. Box-counting method was defined by Russel et al. [30], which is the most frequently used method to calculate multifractal spectrums for the ease of computational complexity and empirical estimation. The box-counting based multifractal analysis for image is introduced as follows.

We start with a digital image with $M \times M$ pixels. This image is mapped onto a three-dimensional spatial surface with $(x, y)$ coordinating the image dimension and $z$ representing intensity of pixel in that point. As illustrated in Fig. 1 (c), which describes the three-dimension surface of the gray-level selective patch in the original image. Then the $(x, y)$ plane of the image is divided recursively into four equal boxes and each step constitute a decomposition scale. At each scale, deposition probability of box $(i, j)$ is computed by:

$$P_{ij}(\epsilon) = \frac{h_{ij}}{\sum h_{ij}}$$  \hspace{1cm} (2)

where $h_{ij}$ is the sum of all pixels intensity contained in box$(i, j)$ and $\epsilon$ is scale measure computed by the ratio of box size to the whole image size. After the calculation of $P_{ij}$, the partition function $\chi_q(\epsilon)$ is introduced and it is expressed as a power law of $\epsilon$ an exponent $\tau(q)$:

$$\chi_q(\epsilon) = \sum P_{ij}(\epsilon)^q = \epsilon^{\tau(q)}$$  \hspace{1cm} (3)

where $q$ is the moment order. This power law constitute
by performing a Legendre transformation as follows:

\[ \chi_q(h) = \frac{\partial}{\partial q} \ln \langle x^q \rangle(h) \]


Multifractal analysis of selective patch image “Monarch”. (a)-(e): color image, gray-level imaged, computation of logscale diagram \( \ln \chi_q(e) \) vs. \( \ln(e) \), \( \tau(q) \) vs. \( q \) and multifractal spectra \( D(h) \).

Fig. 2  Multifractal analysis of selective patch image “Monarch”. (a)-(e): color image, gray-level imaged, computation of logscale diagram \( \ln \chi_q(e) \) vs. \( \ln(e) \), \( \tau(q) \) vs. \( q \) and multifractal spectra \( D(h) \).

the fundamental relation connecting the concept of fractal. The exponent \( \tau(q) \) can be obtained from the slope of \( \ln \chi_q(e) - \ln e \). The multifractal spectra \( D(h) \) can be obtained by performing a Legendre transformation as follows:

\[ h = \frac{d[\tau(q)]}{d(q)}, D(h) = hq - \tau(q) \]

where \( h \) is defined to be the singularity probabilities and \( D(h) \) be the fractal dimension of \( h \) subset. The procedure for multifractal spectra calculation of selective patch in image “Monarch” is illustrated in Fig. 2. The color image Fig. 2 (a) is first transformed into gray-scale image Fig. 2 (b). Then we decompose the selective patch into small boxes of different scale and compute the deposition probability of every boxes in each decomposition level to get partition function as showed in Fig. 2 (c). The exponent \( \tau(q) \) is calculated by partition function which is represented in Fig. 2 (d). At last, through Legendre transformation, we finally get the multifractal spectra as depicted in Fig. 2 (e).

3. Image Quality Assessment Based on Multifractal Spectrum

Various distortions can change the multifractal spectra of original images, so quality of images can be measured by quantifying the discrepancies between multifractal spectrums of reference image and distorted image. With the purpose to evaluate the image quality of different distortion degree, we proposed our image quality assessment method based on multifractal theory MF-IQA. The procedures of our proposed image quality metrics are the following.

First, divide reference and distorted images into small patches of \( 64 \times 64 \) pixels, this is mainly for the capture of fine complexity information and ease of mathematical computation. If the size of images are not a multiple of 64, the images are resized using nearest-neighbor interpolation before the division. Second, calculate the multifractal spectrums of each patch by means of box-counting method. Third, quantify the multifractal spectrums differences globally by multifractal mean weighted spectrums distance between each corresponding patches of reference image and the distorted image. The mean weighted spectrums distance of all the moment order \( q \) is defined as:

\[ L = \frac{\sum_{q=-N}^{N} w_q \sqrt{(D_{ref}(h) - D_{dis}(h))^2 + (h_{ref} - h_{dis})^2}}}{2N + 1} \]

where \( N \) is the number of moment order \( q \) which is used to gain partition function. \( N \) should be neither too small to get the accurate multifractal spectra of an image nor too large to avoid overflow during calculation. Take these factors into consideration, \( N \) is set to 60.

Based on the definition of multifractal spectra, the left side of spectra describes the subset with a larger deposition probability in different scales. While the right hand of spectra describes the subset with a smaller deposition probability. In other words, the right side of spectra describes the rough and irregular texture of an image, such as the edges and contours of objects. And the left side of spectra represents the smooth and regular part of an image like the surface. Because HVS is more sensitive to the irregular and erratic part of images, we divided all the moment order into 6 groups by its value and let the correlation coefficient between each group and the subjective evaluation score as weights \( w_j \).

Finally, we get the predicted image quality by calculation of the mean \( L \) of all patches:

\[ Q = \frac{\sum_{j=1}^{m} L_j}{m} \]

where \( m \) is the total number of patches that the test images are divided into. Figure 3 illustrates the calculation procedure of mean weighted spectrums distance.

As illustrated in Fig. 4, (a) is the reference image, (b)-(d) are different distortion types of “bikes” with DMOS. DMOS is degradation mean opinion score of subjective image quality experiment, where a smaller DMOS refer to a better perceptual image quality. As depicted in Fig. 5, the black curve is the multifractal spectra of selective patch in the reference image (a), and the blue triangle, red square and
Fig. 3 Procedure mean weighted spectrums distance.

Fig. 4 Different distortion levels of an image “bikes” from LIVE. (a) is the reference image, (b) is JPEG2000 distortion of “bikes”, DMOS = 38.4690, (c) is Gauss blur distortion of “bikes”, DMOS = 55.8753, (d) is Fastfading distortion of “bikes”, DMOS = 74.0254.

Fig. 5 Multifractal spectrums of different distortion for “bikes”.

green circle spectrums correspond to the selective patches in distorted images (b)-(d) respectively. We can easily find the multifractal spectra in blue triangle is the most similar to the multifractal of the reference image in black, which indicates image (b) has the best image quality. Image (d) has lost much detail information, so the width of multifractal spectra of selective patch in image (d) is much narrow. It is clear that the multifractal spectra of selective patch in image (d) is the most different from that of the reference image, which suggests image (d) has the worst image quality. These are all in accordance with DMOS obtained by psychometric tests.

4. Experimental Results and Discussion

We validate the proposed algorithm on the LIVE Image Quality assessment Database Release 2 made available by University of Texas at Austin [31]. The LIVE database has been widely used to test the performance of various IQA algorithms. This database contains a total of 982 distorted images and five different types of distortions: JPEG2000 compression (JP2K), JPEG compression (JPEG), white noise (WN), Gaussian blur (Gblur), and Fastfading (the transmission errors in the JPEG2000 bit stream using a fast-fading model, FF). All these distortions represent a wide variety of impairments from which images might suffer. LIVE also provide the subjective evaluation result, the degradation mean opinion scores (DMOS) for each image, which is obtained by rigorous psychometric tests. Objective scores acquired from the proposed method are transformed to the predicted subjective quality via a nonlinear regression. In our paper, we use a five-parameter logistic fitting function to obtain the objective DMOS:

\[
DMOS_{\text{objective}}(x) = \beta_1 \left( \frac{1}{2} - \frac{1}{1 + \exp(\beta_2(x - \beta_3))} \right) + \beta_4 x + \beta_5
\]
where $x$ is adopting the score obtained from the MF-IQA metric. The reason for this regression is that the objective scores can be mapped into the subjective scores despite different measures may normally have different value scopes. Figure 6 illustrates the subjective ratings of perception against predicted values for various distortion types in LIVE database. Figure 7 shows all the predicted values after nonlinear regression against subjective DMOS. Each blue point represents an objective score calculated by computer and the corresponding subjective score given by image experts. If the two scores are closer, the blue point is more likely to locate near the diagonal line in red. The following diagram indicates the predicted image quality by our proposed method is highly consistent with human perception.

To quantitatively compare our proposed algorithm with state-of-the-art algorithms, we follow the performance validation procedures released by VQEG [32]. Two performance metrics are used to evaluate performance: corre-
achieve satisfactory performance in all aspects except some images of white noise distortion. In addition, both CC and SROCC get the top or second status in almost distortion types indicates that the proposed method also accomplish better adaptability. The reason for the accurate prediction is that it uses the statistical complexity and regularity features of image which is the most important information in image understanding. The reason for unsatisfactory result in WN is mainly because the noise in WN distortion has the character of self-similarity that will disturb complexity feature extraction. MSDD deploys contour transform which enters into wavelet domain while MF is image domain, which is independent from discrete wavelet decomposition. Therefore, MF demands lower consumption of calculation expense. However, some images in LIVE include objects that do not come from nature, such as plane, boats and buildings which are manufactured by human. These objects can be described by Euclidean geometry and do not have obvious character of fractal, which might eventually influence the result of our algorithm. We believe our propose method will get superior performance on images composed of all natural entities.

The average time required to compute each of these algorithms is given in Table 3, which is calculated over the entire LIVE database. The computer configuration used for calculating is: Intel Xeon 2.66 GHz, 12 GB RAM, Linux. All these algorithms are implemented using MATLAB 2012a. From Table 3 we can see that our proposed method spends much less time than the IFC and VIF method.

5. Conclusion

In this paper, a novel image quality assessment is proposed based on multifractal analysis. The complexity of texture in images is extremely meaningful in human perception. We introduce multifractal theory to extract the global statistical complexity of images by means of box-counting method. We define the mean weighted distance between multifractal spectrums to assess image quality. The experiment upon the LIVE database shows the efficiency of our proposed method and is consistent with human visual system. The comparison result demonstrates the proposed method is competitive among the most widely used full reference image quality assessment metrics. All in all, image quality assessment through quantifying discrepancies of multifractal spectrums

Table 3 Average time required to calculate each algorithm on LIVE.

<table>
<thead>
<tr>
<th>Metrics</th>
<th>Time(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>UQI</td>
<td>0.2123</td>
</tr>
<tr>
<td>PSNR</td>
<td>0.1078</td>
</tr>
<tr>
<td>VSNR</td>
<td>3.2128</td>
</tr>
<tr>
<td>SSIM</td>
<td>1.6027</td>
</tr>
<tr>
<td>GSSIM</td>
<td>2.1137</td>
</tr>
<tr>
<td>IFC</td>
<td>8.7653</td>
</tr>
<tr>
<td>VIF</td>
<td>10.9523</td>
</tr>
<tr>
<td>MSDD</td>
<td>4.7265</td>
</tr>
<tr>
<td>Proposed</td>
<td>4.2878</td>
</tr>
</tbody>
</table>

Fig. 7 Predicted values after nonlinear regression vs. subjective score.

Table 1 CC of different state-of-the-art metrics.

<table>
<thead>
<tr>
<th>Metrics</th>
<th>JP2K</th>
<th>JPEG</th>
<th>WN</th>
<th>GBLUR</th>
<th>FF</th>
<th>ALL</th>
</tr>
</thead>
<tbody>
<tr>
<td>UQI</td>
<td>0.8407</td>
<td>0.8446</td>
<td>0.9361</td>
<td>0.9469</td>
<td>0.9467</td>
<td>0.8964</td>
</tr>
<tr>
<td>PSNR</td>
<td>0.8590</td>
<td>0.8420</td>
<td>0.9220</td>
<td>0.7440</td>
<td>0.8770</td>
<td>0.8772</td>
</tr>
<tr>
<td>VSNR</td>
<td>0.9530</td>
<td>0.9430</td>
<td>0.9780</td>
<td>0.9340</td>
<td>0.9020</td>
<td>0.9231</td>
</tr>
<tr>
<td>SSIM</td>
<td>0.9368</td>
<td>0.9279</td>
<td>0.9793</td>
<td>0.8741</td>
<td>0.9452</td>
<td>0.9388</td>
</tr>
<tr>
<td>GSSIM</td>
<td>0.9382</td>
<td>0.9343</td>
<td>0.9537</td>
<td>0.9076</td>
<td>0.9479</td>
<td>0.9563</td>
</tr>
<tr>
<td>IFC</td>
<td>0.9013</td>
<td>0.9024</td>
<td>0.9577</td>
<td>0.9575</td>
<td>0.9599</td>
<td>0.9257</td>
</tr>
<tr>
<td>VIF</td>
<td>0.9633</td>
<td>0.9422</td>
<td>0.9887</td>
<td>0.9737</td>
<td>0.8828</td>
<td>0.9579</td>
</tr>
<tr>
<td>MSDD</td>
<td>0.9420</td>
<td>0.9400</td>
<td>0.9840</td>
<td>0.9590</td>
<td>0.9190</td>
<td>0.8900</td>
</tr>
<tr>
<td>Proposed</td>
<td>0.9624</td>
<td>0.9660</td>
<td>0.9750</td>
<td>0.9686</td>
<td>0.9545</td>
<td>0.9511</td>
</tr>
</tbody>
</table>

Table 2 SROCC of different state-of-the-art metrics.

<table>
<thead>
<tr>
<th>Metrics</th>
<th>JP2K</th>
<th>JPEG</th>
<th>WN</th>
<th>GBLUR</th>
<th>FF</th>
<th>ALL</th>
</tr>
</thead>
<tbody>
<tr>
<td>UQI</td>
<td>0.8441</td>
<td>0.8227</td>
<td>0.9093</td>
<td>0.9390</td>
<td>0.9385</td>
<td>0.8907</td>
</tr>
<tr>
<td>PSNR</td>
<td>0.8510</td>
<td>0.8280</td>
<td>0.9380</td>
<td>0.7250</td>
<td>0.8590</td>
<td>0.8756</td>
</tr>
<tr>
<td>VSNR</td>
<td>0.9460</td>
<td>0.9080</td>
<td>0.9790</td>
<td>0.9410</td>
<td>0.9060</td>
<td>0.9274</td>
</tr>
<tr>
<td>SSIM</td>
<td>0.9317</td>
<td>0.9028</td>
<td>0.9629</td>
<td>0.8942</td>
<td>0.9411</td>
<td>0.9250</td>
</tr>
<tr>
<td>GSSIM</td>
<td>0.9326</td>
<td>0.9038</td>
<td>0.9367</td>
<td>0.9364</td>
<td>0.9451</td>
<td>0.9448</td>
</tr>
<tr>
<td>IFC</td>
<td>0.8910</td>
<td>0.8635</td>
<td>0.9380</td>
<td>0.9521</td>
<td>0.9599</td>
<td>0.9247</td>
</tr>
<tr>
<td>VIF</td>
<td>0.9563</td>
<td>0.9093</td>
<td>0.9854</td>
<td>0.9678</td>
<td>0.8662</td>
<td>0.9559</td>
</tr>
<tr>
<td>MSDD</td>
<td>0.9360</td>
<td>0.9040</td>
<td>0.9780</td>
<td>0.9580</td>
<td>0.9160</td>
<td>0.9040</td>
</tr>
<tr>
<td>Proposed</td>
<td>0.9603</td>
<td>0.9515</td>
<td>0.9656</td>
<td>0.9527</td>
<td>0.9415</td>
<td>0.9470</td>
</tr>
</tbody>
</table>
is both powerful and efficient. Further work will enhance the performance of evaluating Gblur distortion type and, on the other hand, exploit MF analysis performance on artificial images, which are mainly composed by manmade objects.

References


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