LETTER

Robust Subband Adaptive Filtering against Impulsive Noise

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SUMMARY In this letter, a new subband adaptive filter (SAF) which is robust against impulsive noise in system identification is presented. To address the vulnerability of adaptive filters based on the L_2 -norm optimization criterion to impulsive noise, the robust SAF (R-SAF) comes from the L_1 -norm optimization criterion with a constraint on the energy of the weight update. Minimizing L_1 -norm of the *a posteriori* error in each subband with a constraint on minimum disturbance gives rise to robustness against impulsive noise and the capable convergence performance. Simulation results clearly demonstrate that the proposal, R-SAF, outperforms the classical adaptive filtering algorithms when impulsive noise as well as background noise exist.

key words: subband adaptive filter, L_1 -norm, system identification, robustness, impulsive interference

1. Introduction

Adaptive filtering has been of considerable interest in various signal processing fields such as channel equalization, system identification, acoustic echo cancellation. Among adaptive filtering algorithms, the normalized least mean square (NLMS) algorithm has been preferred due to its simplicity of implementation [1]. However, NLMS suffers from degraded convergence if the input signal is correlated. To overcome this limitation, adaptive filtering in the subband has recently been developed, referred to as the subband adaptive filters (SAFs) [2]–[4]. The remarkable feature of the SAF is that it allocates an input signal and a desired response into almost mutually exclusive subbands. By carrying out a pre-whitening on the correlated input signal, the SAF achieves the improved convergence rate over the fullband based LMS-type filters. This property stems from the fact that the eigenvalue spread for the subband signals is normally less than the eigenvalue spread for the original signal when the original signal is correlated [4]. Recently, through a multiple-constraints optimization problem based on the principle of minimal disturbance, the normalized SAF (NSAF) has been developed that offers superior convergence rates over the NLMS, while it has nearly the same computational burden as NLMS [3],[4].

Consider the case that impulsive noise is present in a system identification scenario such as echo cancellation.

Since impulsive noise with non-Gaussian distribution leads to more perturbation rather than Gaussian noise, the convergence behavior of an adaptive filter based on L_2 -norm optimization is heavily impaired. To address this issue, several adaptive filtering algorithms which utilize lower order norms have been developed [5]–[7]. Among them, the L_1 -norm optimization yields robustness as well as simple implementation [7]. However, the poor convergence of the adaptive filters based on the L_1 -norm optimization in the case of correlated input signals remains as a major drawback [8]. More recently, an affine projection algorithm based on a L_1 -norm optimization has been introduced [8].

Here, to tackle the robustness and convergence behavior issues in subband framework, a new robust SAF (R-SAF) which comes from the L_1 -norm optimization criterion is presented. Formulating the L_1 -norm of the *a posteriori* error in each subband with a constraint of the energy of the weight update as the cost function and minimizing this cost function, resulting in robust and capable updates of the R-SAF. This letter deals with the issue of robustness against impulsive interference with non-Gaussian distribution. Furthermore, the novelty of this work lies in the improved convergence compared to conventional L_1 -norm based adaptive filters. The author has presented a subband adaptive filter with L_1 -norm optimization for sparse system identification [9]. Compared to previous work, this letter addresses the robustness issue of the filter in the presence of impulsive noise. Through various simulations, the resulting R-SAF has proven its superior robustness and convergence performance over the classical NLMS and the normalized sign algorithm (NSA) [6] in cases when impulsive noise exists.

2. Robust SAF (R-SAF)

Consider a desired signal $d(n)$ that arise from the system identification model

$$d(n) = \mathbf{u}(n)\mathbf{w}^\circ + v(n), \quad (1)$$

where $\mathbf{w}^\circ$ is a column vector for the impulse response of an unknown system that we wish to estimate, $v(n)$ accounts for measurement noise with zero mean and variance σ_v^2 and $\mathbf{u}(n)$ denotes the $1 \times M$ input vector,

$$\mathbf{u}(n) = [u(n) \ u(n-1) \ \cdots \ u(n-M+1)]. \quad (2)$$

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Fig. 1 Structure of the SAF.

2.1 Structure of SAF

Figure 1 shows the structure of the well known NSAF, where the desired signal $d(n)$ and output signal $y(n)$ are partitioned into N subbands by the analysis filters $H_0(z), H_1(z), \dots, H_{N-1}(z)$. The resulting subband signals, $d_i(n)$ and $y_i(n)$ for $i = 0, 1, \dots, N-1$, are critically decimated to a lower sampling rate commensurate with their bandwidth. Here, the variable n to index the original sequences, and k to index the decimated sequences are used for all signals. Then, the decimated filter output signal at each subband is defined as $y_i(k) = \mathbf{u}_i(k)\mathbf{w}(k)$, where $\mathbf{u}_i(k)$ is $1 \times M$ row vector such that

$$\mathbf{u}_i(k) = [u_i(kN), u_i(kN-1), \dots, u_i(kN-M+1)]$$

and $\mathbf{w}(k) = [w_0(k), w_1(k), \dots, w_{M-1}(k)]^T$ denotes an estimate for $\mathbf{w}^\circ$ with length M . Then, the *a priori* and *a posteriori* errors at each subband $\{e_{a,i}(k), e_{p,i}(k)\}$ are defined by $e_{a,i}(k) = d_i(k) - \mathbf{u}_i(k)\mathbf{w}(k)$ and $e_{p,i}(k) = d_i(k) - \mathbf{u}_i(k)\mathbf{w}(k+1)$, where $d_i(k) = d_i(kN)$ is the decimated desired signal at each subband for $i = 0, 1, \dots, N-1$. Then, the *a posteriori* error vector $\mathbf{e}_p(k)$ can be formulated as

$$\mathbf{e}_p(k) = [e_{p,0}(k), e_{p,1}(k), \dots, e_{p,N-1}(k)]^T$$

In the presence of impulsive noise, the desired signal $d(n)$ is corrupted, in turn, the error signal becomes sparse. When $d(n)$ goes through a subband filter bank, the prewhitening process spreads the impulsive outliers to non-contaminated subbands [10].

2.2 Derivation of the R-SAF

Here, the proposed optimization criterion is formulated as follows: Minimizing the L_1 -norm of the *a posteriori* error vector

$$\min_{\mathbf{w}(k+1)} \|\mathbf{e}_p(k)\|_1 \quad (3)$$

subject to a constraint on the energy of the filter update, which is given by

$$\|\mathbf{w}(k+1) - \mathbf{w}(k)\|_2^2 \leq \mu^2, \quad (4)$$

where μ^2 denotes a control parameter which prevents the weight update from the abrupt change. The constraint (4) plays a role in restricting the effect of the perturbation, which is based on the principle of minimum disturbance. In this regard, the parameter, μ , needs to be small. Then, the constrained optimization criterion with the Lagrange multiplier is given as follows:

$$J(k) = \|\mathbf{e}_p(k)\|_1 + \lambda(k)[\|\mathbf{w}(k+1) - \mathbf{w}(k)\|_2^2 - \mu^2], \quad (5)$$

where $\lambda(k)$ is a Lagrange multiplier. Since the L_1 -norm is a non-differentiable convex function, a subgradient analysis [11] is incorporated to find subgradients which exist at some point. Taking the gradient of (5) with respect to the weight vector $\mathbf{w}(k+1)$, it leads to

$$\begin{aligned} \nabla_{\mathbf{w}(k+1)} J(k) &= \nabla_{\mathbf{w}(k+1)}^S \|\mathbf{e}_p(k)\|_1 + 2\lambda(k)[\mathbf{w}(k+1) - \mathbf{w}(k)]^T \\ &= [\nabla \mathbf{e}_p(k)]^T \text{sgn}(\mathbf{e}_p(k)) + 2\lambda(k)[\mathbf{w}(k+1) - \mathbf{w}(k)]^T \\ &= - \sum_{i=0}^{N-1} \text{sgn}[e_{p,i}(k)] \mathbf{u}_i(k) + 2\lambda(k)[\mathbf{w}(k+1) - \mathbf{w}(k)]^T, \end{aligned} \quad (6)$$

where $\nabla_{\mathbf{w}(k+1)}^S f(\cdot)$ denotes a subgradient vector of a function $f(\cdot)$ with respect to $\mathbf{w}(k+1)$ and $\text{sgn}(\cdot)$ represents the signum function. Setting (6) equal to zero, the following is obtained

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \frac{1}{2\lambda(k)} \sum_{i=0}^{N-1} \mathbf{u}_i^T(k) \text{sgn}[e_{p,i}(k)]. \quad (7)$$

Here, according to the Karush-Kuhn-Tucker (KKT) conditions [11], the complementary slackness conditions holds

$$\lambda(k)(\|\mathbf{w}(k+1) - \mathbf{w}(k)\|_2^2 - \mu^2) = 0. \quad (8)$$

Substituting (7) into (8), it results in

$$\frac{1}{2\lambda(k)} = \frac{\mu}{\sqrt{\sum_{i=0}^{N-1} \text{sgn}[e_{p,i}(k)] \mathbf{u}_i(k) \cdot \sum_{i=0}^{N-1} \mathbf{u}_i^T(k) \text{sgn}[e_{p,i}(k)]}}. \quad (9)$$

Assuming that the *a priori* error $e_{a,i}(k)$ approximates the *a posteriori* error $e_{p,i}(k)$ and substituting (9) into (7), the proposed R-SAF updates the weight as follows:

$$\begin{aligned} \mathbf{w}(k+1) &= \mathbf{w}(k) + \\ &\mu \frac{\sum_{i=0}^{N-1} \mathbf{u}_i^T(k) \text{sgn}[e_{a,i}(k)]}{\sqrt{\sum_{i=0}^{N-1} \text{sgn}[e_{a,i}(k)] \mathbf{u}_i(k) \cdot \sum_{i=0}^{N-1} \mathbf{u}_i^T(k) \text{sgn}[e_{a,i}(k)] + \delta}}, \end{aligned} \quad (10)$$

where μ plays a role in controlling the convergence as a step-size parameter and δ is a regularization parameter to avoid the situation in which the input signal $\mathbf{u}_i$ gets close to zero.

3. Simulation Results

To validate the performance of the proposed R-SAF, computer simulations are carried out in a system identification scenario in which the unknown channel is randomly generated. The length of the unknown system is $M = 128$ in experiments. The adaptive filter and the unknown system are assumed to have the same number of taps. The input signals $u(n)$ are obtained by filtering a white, zero-mean, Gaussian random sequence through a first-order system $G(z) = 1/(1-0.9z^{-1})$. The measurement noise $v(n)$ with white Gaussian distribution is added to the system output $y(n)$ such that the signal-to-noise ratio (SNR) is 30 dB, where the SNR is defined as $\text{SNR} = 10 \log_{10}(E[y^2(i)]/E[v^2(i)])$, where $y(n) = \mathbf{u}(n)\mathbf{w}^\circ$. Impulsive noise $z(n)$ is added to the system output $y(n)$ with the signal-to-interference ratio (SIR) of -30 , -10 , and 10 dB. This impulsive noise is modeled by a Bernoulli-Gaussian (BG) distribution [12], which is obtained as the product of a Bernoulli distribution and a Gaussian one, i.e., $z(n) = \omega(n)\eta(n)$, where $\omega(n)$ is a Bernoulli process with a probability mass function given by $P(\omega) = 1 - p$ for $\omega = 0$ and $P(\omega) = p$ for $\omega = 1$. The average power of the BG distribution is $p \cdot \sigma_\eta^2$. When keeping the average power constant, the smaller p value, the spikier the BG interference. In echo cancellation, the Bernoulli-Gaussian distribution with $p = 0.001$ were used for modeling double-talk which is considered as an impulsive noise [8], [13]. With this in mind, $p = 0.001$ is used in the simulations except Fig. 5 where various p values are considered. In addition, $\eta(n)$ is additive white Gaussian noise with zero mean and variance σ_η^2 . In order to compare the convergence performance, the instantaneous normalized square deviation (SD), where $\text{Normalized SD} = \|\mathbf{w}^\circ - \mathbf{w}(k)\|^2 / \|\mathbf{w}^\circ\|^2$, is taken. The cosine-modulated filter banks [4] with the subband numbers of $N = 1, 2, 4, 8$, and 16 are used in the simulations. The prototype filter of length $L = 32$ and $\delta = 0.01$ are used.

Figure 2 shows the normalized SD curves of the R-SAF in cases of the different number of subbands, i.e., $N=1, 2, 4, 8$, and 16 . The step-size, $\mu = 0.01$, SIR= -30 dB, and $p = 0.001$ are used. In the figure, the higher the number of subbands, the better the convergence behavior of the R-SAF in terms of the convergence rate and the steady-state misalignment.

Figure 3 depicts the convergence performance of the R-SAF with various step-sizes for $N = 4$, where $\mu = 0.1, 0.03, 0.01, 0.003$, and 0.001 are chosen. The same values of SIR and p as those in Fig. 2 are chosen. As expected, the use of a large step-size leads to faster convergence with higher steady-state misalignment. On the contrary, a small step-size reduces the convergence rate, but achieves the lower steady-state misalignment.

Figure 4 illustrates the normalized SD curves of the NLMS, NSA, NSAF, and R-SAF in case of SIR= -30 dB,

Fig. 2 Normalized SD curves of the R-SAF with the different number of subbands, $N = 1, 2, 4, 8$, and 16 under SIR= -30 dB.

Fig. 3 Normalized SD curves of the R-SAF with the different step-sizes, $\mu = 0.1, 0.03, 0.01, 0.003$, and 0.001 under SIR= -30 dB.

Fig. 4 Normalized SD curves of the NLMS, NSA, NSAF, and R-SAF under SIR= -30 dB.

respectively. The number of subbands, $N=4$, for the NSAF and R-SAF is chosen in these simulations. The step-sizes, $\mu = 0.1$ for the NSA and $\mu = 0.003$ for the R-SAF are used. While the NLMS and the NSAF are vulnerable against impulsive noise, the NSA and the R-SAF are capable of coping with the BG interference. Moreover, the convergence

Fig. 5 Normalized SD curves of the NSA and R-SAF under different p values, $p = 0.1, 0.01$, and 0.001 .

Fig. 6 Normalized SD curves of the NLMS, NSA, NSAF, and R-SAF for a speech input signal under $\text{SIR} = -10$ dB.

behavior of the R-SAF is superior to that of the NSA.

To examine the effect of the impulsiveness of the BG interference, the convergence performances of the NSA and the R-SAF with $N = 4$ are compared in Fig. 5, where $p = 0.1, 0.01$, and 0.001 are considered. The step-sizes are same as those in Fig. 4. The SIR is set to -10 dB. It can be seen that the NSA and the R-SAF perform better in cases when the BG interference is more spiky. In addition, the R-SAF achieves highly improved convergence performance over the NSA for all cases.

Figure 6 shows the convergence capabilities of NLMS, NSA, NSAF, and R-SAF in case of a real speech signal ($\text{SIR} = -10$ dB). Other parameters were the same as those in Fig. 4. As can be seen, the R-SAF performs better than other algorithms, revealing its capability for practical echo cancellation.

Finally, the tracking capabilities of the NLMS, NSA, NSAF, the proposed robust RLS (PRRLS) [14], the robust Recursive Inverse (RRI) [15] and R-SAF to a sudden change in the system are tested. For the SAFs, a number of subbands, $N = 4$, is used. Figure 7 shows the results of suddenly multiplying the unknown system by -1 at the 10000th iteration. Same values of the step-size in Fig. 4 are used

Fig. 7 Normalized SD curves of the NLMS, NSA, NSAF, PRRLS [14], RRI [15] and R-SAF in case of a time-varying unknown system. The system is suddenly changed from $\mathbf{w}^o$ to $-\mathbf{w}^o$ at the 10000th iteration.

for algorithms. In addition, the parameters of PRRLS and RRI were used as recommended in [14] and [15]. As can be shown, the R-SAF keeps track of weight change without losing the convergence rate nor the steady-state misalignment compared to conventional algorithms.

4. Conclusion

A robust SAF derived from a L_1 -norm optimization criterion based on subband structure has been presented. The resulting R-SAF inherits robustness by utilizing a L_1 -norm optimization as well as adequate convergence performance due to its subband framework. A number of numerical simulations have shown that the R-SAF successfully addresses both vulnerability against impulsive interference and poor convergence of the sign algorithm. Notably, when the power of measurement noise increases compared to impulsive noise, the R-SAF may yield a biased solution. The power ratio between impulsive noise and non-impulsive noise for convergence of the R-SAF would be future issue.

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