PAPER

Competitive Strategies for Evacuating from an Unknown Affected Area

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SUMMARY This article presents efficient strategies for evacuating from an unknown affected area in a plane. Evacuation is the process of movement away from a threat or hazard such as natural disasters. Consider that one or $n(n \geq 3)$ agents are lost in an unknown convex region P . The agents know neither the boundary information of P nor their positions. We seek competitive strategies that can evacuate the agent from P as quickly as possible. The performance of the strategy is measured by a competitive ratio of the evacuation path over the shortest path. We give a 13.812-competitive spiral strategy for one agent, and prove that it is optimal among all monotone and periodic strategies by showing a matching lower bound. Also, we give a new competitive strategy EES for $n(n \geq 3)$ agents and adjust it to be more efficient with the analysis of its performance.

key words: evacuation problem, computational geometry, path planning, convex region, competitive ratio

1. Introduction

Because emergencies have happened frequently in recent years, the evacuation problem has received much attention in computational geometry and robotics. Research on evacuation focuses on how to evacuate efficiently from the affected area.

The evacuation problem has been extensively studied. Skutella [1] has a survey on the network flows in evacuation. The monograph by Hamacher and Tjandra [2] surveys the state of the art on the mathematical modeling of evacuation problems. Chen and Zhan [3] present and analyze competitive strategies for evacuating in three different networks. Lu et al. [4] and Shekhar et al. [5] consider evacuation route-planning algorithms with capacity constraints. Min et al. [6] and Gupta et al. [7] consider the cost of evacuation strategy as time spent instead of path length.

Previous research mainly focuses on details of evacuation such as flow and other constraints to analyze the strategy under complete information. But in an actual situation, some information may be unavailable. In a recent work, Xu and Qin [8] consider a new situation in which the agents

have no boundary information of the affected area in advance. Xu and Qin [8] give a $3/\cos(\pi/n)$ -competitive strategy and a lower bound of 3 on the competitive ratio for $n(n \geq 3)$ groups to evacuate from a convex region in Euclidean plane. Note that the groups can communicate with each other during the evacuation.

In this paper, we denote the affected area by a convex polygon P . Assume that one or $n(n \geq 3)$ agents are lost in P . The agents know neither the boundary information of P nor their positions. There is heavy smoke in P such that the agents can recognize the boundary of P only when they hit it. We seek competitive strategies for the agent to leave from P as quickly as possible.

The paper is organized as follows. In Sect. 2, we describe the details of the evacuation problem. In Sect. 3, we propose a 13.812-competitive strategy for one agent, and prove that it is optimal among all monotone and periodic strategies by showing a matching lower bound. In Sect. 4, we give a new competitive strategy EES for $n(n \geq 3)$ agents. Then we analyze the performance of EES and adjust it to be more efficient. Some concluding remarks are made in Sect. 5.

2. Preliminaries

In this paper, we consider two cases of the evacuation problem: only one agent or $n(n \geq 3)$ agents are lost in P .

In the former case, the agent starts at the origin s in P and moves to leave from P . Let Π be the evacuation path along which the agent moves. Assume that the agent first hits the boundary of P at a point t . The cost of the strategy is defined by the length of Π from s to t and denoted by $|\Pi|$.

In the latter case, the agents start at the same origin s in P and choose different routes to leave from P with the same unit speed. The agents can not stay at a point for a while during the evacuation. When one agent reaches the boundary of P , it can inform other agents the position of the point that has been reached. Successful evacuation requires that all the agents get out of P . The cost of the strategy is defined by the length of the evacuation path along which the last agent walks and denoted by $|\Pi|$.

The performance of the competitive strategy is usually measured by a competitive ratio which was introduced by Sleator and Tarjan [9]. That is, compare the cost of the competitive strategy to the cost of the optimal strategy under full information. For any instance, if there are constants C and A , such that $\Pi \leq C \cdot \Pi_{opt} + A$ holds, the competitive strat-

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egy is called C – competitive with a competitive ratio of C . Since then, competitive analysis has been used in many other papers [10]–[12].

For the above two cases, let Π_{opt} denote the shortest path from the origin s to the boundary of P . The additive constant A is used for initial situations. For example, for any $\epsilon > 0$ there are infinitely many convex polygons with $|\Pi_{opt}| = \epsilon$. Therefore, the relative detour of the competitive strategy will be arbitrarily large when ϵ is very small. In this paper, we assume that $|\Pi_{opt}|$ is at least 1. Thus, we can omit the additive constant A , and consider the competitive ratio C by $|\Pi|/|\Pi_{opt}|$. We would like to find a strategy that guarantees a competitive ratio not greater than C for all possible convex polygons, and C should be as small as possible.

3. Evacuation Strategy for One Agent

In this section, we consider the competitive strategy for one agent. The performance of the strategy is measured by the competitive ratio

$$C = \frac{|\Pi|}{|\Pi_{opt}|}.$$

We first present a spiral strategy. We then prove that this strategy has a competitive ratio of 13.812. Finally, we prove that this strategy is optimal among all monotone and periodic strategies by showing a matching lower bound.

3.1 A Competitive Strategy

To evacuate from P efficiently, we consider a logarithmic spiral (see Fig. 1). Let a and b denote two constant positive real values, and thus the spiral can be given in polar coordinates by

$$r(\varphi) = ae^{b\varphi}, -\infty < \varphi < +\infty.$$

An important property of a logarithmic spiral is that every ray, R , emanating from the spiral's origin s intersects the spiral at the same tangent angle, α [13]. And α satisfies

$$b = \cot \alpha.$$

Fig. 1 The spiral and the worst case

To get the upper bound of the competitive ratio, we consider the worst case. Figure 1 shows an instance of the worst case. Assume that the agent walks along the spiral and first hits the boundary of P at point t . At this time, we choose the smallest $|\Pi_{opt}|$ to maximize the competitive ratio C . Let L denote a tangent to the spiral passing through t and w denote the tangent point. Let q be the point on L such that sq is perpendicular to L . We regard L as an edge of P . Note that this edge does not actually intersect the spiral, but the agent slightly miss this edge at w . For any possible boundary information of P , it is easy to see that no one can achieve a $|\Pi_{opt}|$ smaller than $|sq|$. From now on, we consider $|\Pi_{opt}|$ as $|sq|$ in this instance.

We set angle $\beta = \angle tsw$ (see Fig. 1). Lemma 1 shows that β depends only on parameter b of the spiral.

Lemma 1. The angle β is given by the equation

$$\sin \alpha \csc(\alpha + \beta) = e^{b(2\pi - \beta)}.$$

Proof. In polar coordinates, the equation of a line passing through (r_0, φ_0) and perpendicular to the line $\varphi = \varphi_0$ is

$$r(\varphi) = r_0 \sec(\varphi - \varphi_0).$$

In the worst case, segment sq is perpendicular to L . Thus, $|sq| = |sw| \sin \alpha = ae^{b\varphi_w} \sin \alpha$. And $\varphi_w + 2\pi - \varphi_q = \pi/2 - \alpha$ holds (see Fig. 1). The equation of tangent L is given by

$$\begin{aligned} r(\varphi) &= r_q \sec(\varphi - \varphi_q) = r_q \sec(\varphi + \pi/2 - \alpha - \varphi_w - 2\pi) \\ &= r_q \csc(\alpha + \varphi_w - \varphi). \end{aligned}$$

The point t is both on L and the spiral, thus,

$$r(\varphi_t) = ae^{b\varphi_w} \sin \alpha \csc(\alpha + \varphi_w - \varphi_t) = ae^{b\varphi_t}.$$

Further, $\varphi_t = \varphi_w + 2\pi - \beta$ holds (see Fig. 1). Thus, we have

$$\begin{aligned} ae^{b\varphi_w} \sin \alpha \csc(\alpha + \beta) &= ae^{b(\varphi_w + 2\pi - \beta)} \\ \iff \sin \alpha \csc(\alpha + \beta) &= e^{b(2\pi - \beta)}. \end{aligned}$$

Note that β depends only on parameter b of the spiral. $\square$

Theorem 1. The optimal spiral strategy for one agent to evacuate from a convex region achieves a competitive ratio of 13.812.

Proof. The length of Π on the spiral from s to t is given by

$$|\Pi| = \frac{\sqrt{1+b^2}}{b} ae^{b\varphi_t},$$

(for details see [13]). As $b = \cot \alpha$ holds, we have

$$\frac{\sqrt{1+b^2}}{b} = \frac{\sqrt{1+\cot^2 \alpha}}{\cot \alpha} = \frac{\sqrt{\sin^2 \alpha + \cos^2 \alpha}}{\cos \alpha} = \sec \alpha.$$

The length of Π_{opt} from s to q is given by

Fig. 2 A bundle of m rays and the representation of strategy S

$$|\Pi_{opt}| = |sw| \sin \alpha = ae^{b\varphi_w} \sin \alpha.$$

Thus, the competitive ratio of the spiral strategy is given by

$$\begin{aligned} C &= \frac{|\Pi|}{|\Pi_{opt}|} = \frac{ae^{b\varphi_i} \sec \alpha}{ae^{b\varphi_w} \sin \alpha} \\ &= \frac{ae^{b(\varphi_w + 2\pi - \beta)} \sec \alpha}{ae^{b\varphi_w} \sin \alpha} \\ &= e^{b(2\pi - \beta)} \sec \alpha \csc \alpha. \end{aligned}$$

With Lemma 1, we could minimize the competitive ratio $e^{b(2\pi - \beta)} \sec \alpha \csc \alpha$ as a function of b . It numerically yields a minimum value of 13.811... for $b = 0.212...$ $\square$

3.2 Lower Bound

To get a lower bound of the competitive ratio, we consider a discrete version of this problem. For an integer m and integers $j = 0, \dots, m-1$, let R_j be the ray $\varphi = 2j\pi/m$ (see Fig. 2). The goal of the agent is to search the region boundary intersecting with these m rays.

We present a search strategy, S , which is described as follows: the agent reaches a ray R_j at distance y_i from the origin in the i th step. Note that $y_i \leq y_{i+1}$ for integers $i = 1, 2, \dots, m, \dots$. Then, within length

$$\sqrt{y_{i+1}^2 - 2y_i y_{i+1} \cos \frac{2\pi}{m} + y_i^2}.$$

(by the law of cosine) it moves to the next ray at distance y_{i+1} . Assume that the agent visits the m rays in periodic order and the distance y_i increases in each step (see Fig. 2). We say the path of a strategy is periodic if it can be defined in polar coordinates $(\varphi, r(\varphi))$ with increasing angle $\varphi \in [0, +\infty)$, and $r(\varphi) < r(2\pi + \varphi)$ holds. And the path is monotone if $r(\varphi_1) < r(\varphi_2)$ holds for any $\varphi_1 < \varphi_2$. Thus, if m goes to infinity, any monotone and periodic evacuation strategy for one agent can be described in this way.

Note that $|\Pi_{opt}|$ is at least 1. Therefore, we set $y_1 = 1$ for initializing the first visits on the m rays.

Assume that the agent reaches the boundary of P when it visits the ray l at distance y_k and the ray l was visited at distance y_n the second last time (see Fig. 3). Thus, $k = n + m$.

Fig. 3 The worst case of strategy S

Let segment $y_k y_{n+w}$ denote a tangent to the path from s to y_k and y_{n+w} is the tangent point (n, k and w are positive integer constants). We regard segment $y_k y_{n+w}$ as an edge of P . Note that this edge does not actually intersect the searching path, but the agent slightly misses this edge at point y_{n+w} .

It is easy to see that the worst case can be achieved when $|\Pi_{opt}|$ is as small as possible. Consider all possible boundary information of P , the smallest $|\Pi_{opt}|$ is in the convex polygon P' enclosed by a sequence of segments $y_i y_{i+1}$ ($n+w \leq i \leq k-1$) and segment $y_k y_{n+w}$ (see Fig. 3).

P' can be seen as a composition of a sequence of triangles $\Delta s y_i y_{i+1}$ ($n+w \leq i \leq k-1$) and $\Delta s y_k y_{n+w}$ (see Fig. 3). Let $|d_i|$ and $|d_k|$ denote the shortest distance from s to segment $y_i y_{i+1}$ ($n+w \leq i \leq k-1$) and segment $y_k y_{n+w}$, respectively.

Lemma 2. Let $\Delta s y_i y_{i+1}$ and $\Delta s y_{i+1} y_{i+2}$ be two adjacent triangles in the triangle set $\Delta s y_i y_{i+1}$ ($n+w \leq i \leq k-1$). Thus, $|d_i|$ is not greater than $|d_{i+1}|$.

Proof. In $\Delta s y_i y_{i+1}$, $|y_{i+1}| \geq |y_i|$, $\angle s y_{i+1} y_i \leq \angle s y_i y_{i+1}$. Thus, $\angle s y_{i+1} y_i \geq \frac{1}{2}(\pi - \angle y_{i+1} s y_i)$, we have

$$\begin{aligned} |d_i| &= |y_{i+1}| \sin \angle s y_{i+1} y_i \\ &\leq |y_{i+1}| \cos \frac{\angle y_{i+1} s y_i}{2}. \end{aligned}$$

In $\Delta s y_{i+1} y_{i+2}$, $|y_{i+2}| \geq |y_{i+1}|$, $\angle s y_{i+1} y_{i+2} \geq \angle s y_{i+2} y_{i+1}$. Thus, $\angle s y_{i+1} y_{i+2} \geq \frac{1}{2}(\pi - \angle y_{i+1} s y_{i+2})$, we have

$$\begin{aligned} |d_{i+1}| &= |y_{i+1}| \sin \angle s y_{i+1} y_{i+2} \\ &\geq |y_{i+1}| \cos \frac{\angle y_{i+1} s y_{i+2}}{2}. \end{aligned}$$

$\angle y_{i+1} s y_i = \angle y_{i+1} s y_{i+2}$, thus, $|d_i| \leq |d_{i+1}|$ holds. $\square$

With Lemma 2, we have $|\Pi_{opt}| = \min\{|d_{n+w}|, |d_k|\}$ in P' . In the case of $|d_k| < |d_{n+w}|$, the competitive ratio $C(S)$ is equal to $\frac{|\Pi|}{|d_k|}$. In the other case of $|d_k| \geq |d_{n+w}|$, the competitive ratio

$$C(S) = \frac{|\Pi|}{|d_{n+w}|} \geq \frac{|\Pi|}{|d_k|}.$$

Altogether, the competitive ratio $C(S)$ is bigger than

$$\sup_k \frac{y_1 + \sum_{i=1}^{k-1} \sqrt{y_{i+1}^2 - 2y_i y_{i+1} \cos \frac{2\pi}{m} + y_i^2}}{|d_k|} \quad (1)$$

In $\triangle sy_k y_{n+w}$, by the law of cosine,

$$|y_k y_{n+w}| = \sqrt{y_{n+w}^2 - 2y_k y_{n+w} \cos \frac{2w\pi}{m} + y_k^2}.$$

Applying the law of sine yields

$$\frac{|y_k y_{n+w}|}{\sin \frac{2w\pi}{m}} = \frac{|y_{n+w}|}{\sin \angle sy_k y_{n+w}}.$$

Thus,

$$|d_k| = \frac{|y_k| |y_{n+w}| \sin \frac{2w\pi}{m}}{|y_k y_{n+w}|}.$$

Theorem 2. For one agent evacuation problem, there is no monotone and periodic strategy that achieves a competitive ratio smaller than 13.812.

Proof. In the strategy S , the distance sequence y_i is monotone and periodic and fulfills some other necessary properties so that a general framework proposed by Alpern and Gal [14] can be applied. Thus, for discrete m the ratio (1) is minimized by an exponential sequence $y_i = a^i$. Simple arithmetic shows that the ratio is given by

$$f(a, w, m) = \frac{\sqrt{1 - 2a \cos(\frac{2\pi}{m}) + a^2}}{a - 1} * \frac{\sqrt{1 - 2a^{m-w} \cos(\frac{2w\pi}{m}) + a^{2(m-w)}}}{\sin(\frac{2w\pi}{m})}.$$

Thus, we can analytically find the value a_{min} and w_{min} that minimizes $f(a, w, m)$. For increasing m the corresponding value $f(a_{min}, w_{min}, m)$ converges to 13.811.... $\square$

For example, for $m = 10000$, we compute that $a_{min} = 1.000133515$, $w_{min} = 2267$ and $f(a_{min}, w_{min}, m) = 13.811....$

4. Evacuation Strategy for $n(n \geq 3)$ Agents

In this section, we consider the competitive strategies for $n(n \geq 3)$ agents. The performance of the strategy is measured by the competitive ratio

$$C = \frac{|\Pi|}{|\Pi_{opt}|},$$

where $|\Pi|$ denotes the length of the evacuation path along which the last agent walks, and $|\Pi_{opt}|$ denotes the length of the shortest path from s to the boundary of P .

We present a new evacuation strategy named EES for $n(n \geq 3)$ agents. Then we analyze the performance of EES

Fig. 4 Strategy for $n(n \geq 3)$ agents

Fig. 5 The relation between $|sq_i|$ and $|\Pi_{opt}|$

and adjust it to be more efficient.

4.1 A New Competitive Strategy

For the case of $n(n \geq 3)$ agents, a good choice is to choose different routes to leave from P . We design a new competitive strategy as follows (see Fig. 4):

Equiangular Evacuation Strategy (EES)

Step 1: Divide the agents on s into n groups, i.e. $\{G_1, G_2, \dots, G_n\}$.

Step 2: For an integer n and integers $i = 0, \dots, n-1$, let r_i be the ray $\varphi = 2i\pi/n$. The n groups leave along n radials $\{r_1, r_2, \dots, r_n\}$ from s , respectively.

Step 3: Once a group G_i first arrives at a point q_i on the boundary of P . It informs other groups of the position of q_i . Other groups that do not arrive at any boundary of P stop, and then walk towards q_i until they get out of P .

Lemma 3. When group G_i first arrives at a point q_i on the boundary of P , group G_j arrives at a point q_j . The angle $\angle sq_j q_i$ is equal to $\frac{1}{2}|\pi - |j - i|| \frac{2\pi}{n}$.

Proof. The angle $\angle q_j s q_i$ is $|j - i| \frac{2\pi}{n}$ (see Fig. 4). The angle $\angle sq_j q_i$ is equal to $\frac{1}{2}(\pi - |j - i| \frac{2\pi}{n})$ if $\angle q_j s q_i \leq \pi$ (e.g. $\angle q_7 s q_8$). Or the angle $\angle sq_j q_i$ is equal to $\frac{1}{2}(|j - i| \frac{2\pi}{n} - \pi)$ if $\angle q_j s q_i > \pi$ (e.g. $\angle q_1 s q_7$). $\square$

Lemma 4. Group G_i first arrives at a point q_i on the boundary of P , $|sq_i|$ is not greater than $\frac{|\Pi_{opt}|}{\cos \frac{\pi}{n}}$.

Proof. Without loss of generality, assume that q_{opt} is a point on the boundary of P between r_i and r_{i+1} such that Π_{opt} is segment sq_{opt} (see Fig. 5). As the agents walk with unit speed during the evacuation, $|sq_i| = |sq_{i+1}|$. Further, P is a convex region, we have $|sq_{opt}| \geq |sq_i| \cos \frac{\pi}{n}$. Thus, $|sq_i|$ is not greater than $\frac{|\Pi_{opt}|}{\cos \frac{\pi}{n}}$. $\square$

Theorem 3. *The strategy EES achieves a competitive ratio of*

$$\frac{1 + 2 \sin(\lfloor \frac{n}{2} \rfloor \frac{\pi}{n})}{\cos \frac{\pi}{n}}.$$

Proof. With Lemma 3, when a group G_i first arrives at a point q_i on the boundary of P , other groups G_j can change their directions and walk towards q_i to get out of P (see Fig. 4). The cost of the strategy $|\Pi|$ is the path length of the group who last leave from P . Assume that the last group G_{last} arrives at point q_{last} when G_i arrives at q_i .

$$\begin{aligned} |\Pi| &= |sq_{last}| + |q_{last}q_i| \\ &= |sq_{last}| + 2|sq_{last}| \sin \frac{\angle q_{last}sq_i}{2} \\ &= |sq_{last}| (1 + 2 \sin(\lfloor \frac{n}{2} \rfloor \frac{\pi}{n})). \end{aligned}$$

With Lemma 4, we have

$$\begin{aligned} C &= \frac{|\Pi|}{|\Pi_{opt}|} = \frac{|sq_{last}| (1 + 2 \sin(\lfloor \frac{n}{2} \rfloor \frac{\pi}{n}))}{|\Pi_{opt}|} \\ &\leq \frac{1 + 2 \sin(\lfloor \frac{n}{2} \rfloor \frac{\pi}{n})}{\cos \frac{\pi}{n}}. \end{aligned}$$

Thus, the proof is complete. $\square$

4.2 Analysis of the Performance of EES

We analyze the performance of EES in two situations as follows, where n is an even or odd number.

Situation 1, n is an even number. With Theorem 3, we have

$$C \leq \frac{1 + 2 \sin(\frac{n}{2} \frac{\pi}{n})}{\cos \frac{\pi}{n}} = \frac{3}{\cos \frac{\pi}{n}}.$$

This is the same to the competitive ratio of strategy EDES proposed by Xu and Qin [8]. The performance of EES becomes more effective when n is increasing. It is close to be optimal when the number of groups is numerous.

Situation 2, n is an odd number. With Theorem 3, we have

$$\begin{aligned} C &\leq \frac{1 + 2 \sin(\frac{n-1}{2} \frac{\pi}{n})}{\cos \frac{\pi}{n}} = \frac{1 + 2 \sin(\frac{\pi}{2} - \frac{\pi}{2n})}{\cos \frac{\pi}{n}} \\ &= \frac{1 + 2 \cos \frac{\pi}{2n}}{\cos \frac{\pi}{n}} = \frac{1 + 2 \cos \frac{\pi}{2n}}{2 \cos^2 \frac{\pi}{2n} - 1}. \end{aligned}$$

Clearly, the performance is better than that of strategy

Table 1 Comparison of competitive ratio of EDES, EES and IEES.

number of agents	competitive ratio		
	EDES	EES	IEES
n=3	6	5.464	5.464
n=4	4.243	4.243	4.243
n=5	3.708	3.587	3.587
n=6	3.464	3.464	3.464
n=7	3.330	3.274	3.274
n=8	3.247	3.247	3.247
n=9	3.193	3.160	3.160
n=10	3.154	3.154	3.154
n=11	3.127	3.105	3.105
n=12	3.106	3.106	3.105
n=13	3.090	3.075	3.075
n=14	3.077	3.077	3.075
n=15	3.067	3.056	3.056
n=16	3.059	3.059	3.056
n=17	3.052	3.043	3.043
n=18	3.046	3.046	3.043
n=19	3.041	3.035	3.035
n=20	3.037	3.037	3.035

EDES proposed by Xu and Qin [8] (see Table 1).

Let $x = \cos \frac{\pi}{2n}$, further

$$y = \frac{1 + 2 \cos \frac{\pi}{2n}}{2 \cos^2 \frac{\pi}{2n} - 1} = \frac{1 + 2x}{2x^2 - 1}.$$

For $n \geq 3$, we have

$$\frac{dy}{dn} = \frac{dy}{dx} \frac{dx}{dn} = \frac{-(2x+1)^2 - 1}{(2x^2 - 1)^2} * \frac{\pi \sin \frac{\pi}{2n}}{2n^2} < 0.$$

Thus, $C(n)$ is a decreasing function, and the performance of EES becomes more effective when n is increasing. $\lim_{n \rightarrow +\infty} C(n) = 3$, also Xu and Qin [8] give a lower bound of 3 for $n(n \geq 3)$ groups evacuation problem. Thus, the performance of EES is close to be optimal when the number of groups is numerous.

Now, we try to combine the analysis of these two situations. Let $k(k \geq 3)$ be an odd number, then $k+1$ is an even number after k . We contrast the performance of k groups to that of $k+1$ groups. With Theorem 3, we have

$$C_k = \frac{1 + 2 \cos \frac{\pi}{2k}}{\cos \frac{\pi}{k}}, \quad C_{k+1} = \frac{3}{\cos \frac{\pi}{k+1}}.$$

Let $z = C_{k+1} - C_k$. We could compute that either $z < 0$ when $3 \leq k \leq 9$; or $z > 0$ when $11 \leq k \leq 9999$. Thus, the performance of k groups is better than that of $k+1$ groups when $11 \leq k \leq 9999$. Note that the number of agents n will not bigger than 10000 in general.

As the above analysis, we could adjust the strategy EES to be more efficient.

Improved Equiangular Evacuation Strategy (IEES)

Step 1: If the number of agents n is an even number and $n \geq 12$, divide the agents on s into $n-1$ groups; else if n is an even number and $4 \leq n \leq 10$ or n is an odd number, divide the agents on s into n groups.

Step 2 and Step 3: The same to the strategy EES.

5. Conclusion and Future Work

In this paper, we considered the problem of finding efficient strategies for evacuating from an unknown affected area. We propose and analyze competitive strategies for two cases: only one agent or $n(n \geq 3)$ agents. In the former case, we consider a logarithmic spiral strategy and prove that it has a competitive ratio of 13.812. Also, we prove that it is optimal among all monotone and periodic strategies by showing a matching lower bound. In the latter case, we design a new competitive strategy EES. Then we analyze the performance of EES and adjust it to be more efficient.

We pose two open questions for future work. There is a strong conjecture that the logarithmic spiral strategy is generally optimal for one agent. How do we obtain a general lower bound? It is also an interesting work to propose a more efficient strategy for $n(n \geq 3)$ agents.

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