The Rheology of Woven and Knitted Fabrics*

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SYNOPSIS

Energy minimisation techniques are particularly powerful when used for the study of relaxed fabric structures and elastic deformation characteristics, allowing direct comparisons to be made in terms of normalised dimensionless parameters between different woven and knitted fabric constructions.

The energy analysis is based on the fundamental principle that elastic structures always assume a configuration of minimum strain energy, regardless of the deformation applied. The resultant minimization of total yarn-strain energy within the fabric (consisting of yarn-bending, torsion, lateral compression, and longitudinal extension) is treated as an optimal-control problem and is subject to certain constraints acting within the fabric.

It is shown how the mechanistic model of relaxed woven fabrics as analysed by force method in 1937 by F.T. Peirce may be derived from energy equations for values of weave crimp ranging from low to moderate. For higher values of weave crimp, the introduction of an extra constraint on the curved yarn results in the geometrical model also proposed originally by Peirce and widely used in the textile literature.

The energy analysis has been applied to the uniaxial and biaxial tensile deformation of fabrics by including the possibility of yarn extension in the theory. Fabric load-extension curves and yarn-decrimping curves are computed and compared with experimental results for fabrics in both the grey and finished states.

The computed curves are discussed in terms of the following parameters expressed in dimensionless or normalized form: the applied tension per thread and the interaction force per crossing thread, the relative fabric extension (fabric extension/initial weave crimp), the initial relative fabric tensile modulus, the fabric Poisson's ratio, and the initial tensile modulus of the crimped set yarn unravelled from the fabric.

The pure-bending behaviour of the plain-weave structure is also evaluated as a generalization of the tensile deformation. The ratio of fabric-bending rigidity to yarn-bending rigidity is computed for a range of fabric structures of different values of weave crimp and degree of set by the introduction of inequality constraints on the yarn curvature, a concept borrowed from optimal-control theory. The theoretical results are compared with the pure-bending behaviour of woven fabrics in both the grey and commercially finished states.

The implications of the work reported in this paper for future theoretical and experimental studies of the structure and mechanical properties of fabrics and their objective specification are discussed.

1. Energy Minimization Techniques Applied to Fabric Mechanics

In Part 1 of this series of papers, it was shown that the analysis of the mechanics of woven and knitted fabrics by force methods requires some assumptions about the nature of yarn interlacing within the fabric to develop a system of internal forces (and couples) exerted by one yarn on the interlacing yarn within the fabric structure. In energy methods, however, it is merely necessary to evaluate the total strain energy (a scalar quantity) acting within the fabric, i.e. the sum of the strain energy terms due to yarn bending, torsion, lateral compression and longitudinal extension; this step merely


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requires knowledge about the yarn mechanical properties but not about the magnitude and direction of the force vectors acting within the fabric. In an energy method of analysis, it is important to take into account all forms of energy stored within the fabric structure including potential energy terms arising from any external forces or couples applied to the fabric. For conditions of equilibrium, the total energy is minimised by using some form of optimisation technique subject to the action of constraints imposed by the conditions of yarn or loop interlocking; this procedure applies whether the fabric under consideration is relaxed (free from external forces) or whether it is deformed in some way, provided that the minimisation procedure is carried out on the total energy stored within the fabric.

In recent years, some authors have applied energy techniques to calculate the stress-strain behaviour of a fabric by initially assuming one of the geometrical models for the relaxed or undeformed unit cell of the structure; the strain energy required to deform the geometrical models is then minimised from which a stress-strain curve can be predicted. This type of approach, however, still includes the inherent limitations of the initially assumed geometrical model.

In 1977, De Jong and Postle published an energy analysis of both woven and knitted fabrics without resorting to any initial geometrical assumptions about the nature of yarn interlacing in the weave crimp or in the knitted loop. By assuming linear elasticity for the yarn in bending, torsion and longitudinal extension, a series of equations was established on a three-dimensional coordinate system to describe in the most general way the shape of the interlacing yarn axis within the fabric; the strain energy required to deform the geometrical models is then minimised from which a stress-strain curve can be predicted. A yarn “contact potential” or lateral compression function was assumed to act between interlacing yarns within the fabric, the value of this potential rising quite steeply as two adjacent yarn axes approach each other in space at a region of yarn interlacing. It is thus possible to evaluate the total energy acting within the fabric structure by summing all the strain energy terms and the potential energy terms due to the forces and couples arising from yarn interlacing. By employing a computerised optimisation technique (based on optimal-control theory), the shape of the interlacing yarns and the associated force and couple distribution acting within the fabric can be systematically manipulated in order to produce a condition of minimum stored energy subject to the constraints applied by yarn or loop interlocking within the fabric structure.

Because the energy optimisation technique is based on the fundamental principle that elastic structures always assume a configuration of minimum strain energy regardless of the deformation applied, it is possible by varying the constraints applied to the interlacing yarn within the fabric to generate the more limited models derived by previous workers. For example, it has been shown how the mechanism model of the relaxed plain-weave structure proposed by Peirce may be derived from the energy equations for values of weave crimp ranging from low to moderate levels; for higher values of weave crimp, the introduction of an extra constraint to restrict the weave crimp to the shape governed by the cross-section of the interlacing yarn results in the geometrical model proposed originally by Peirce. The model of the relaxed plain-knitted fabric assumed by Hepworth and Leaf can also be generated from the energy equations by making the extra assumption of a relatively incompressible yarn. Further, it can be shown that the nature of loop interlocking in the knitted structure is dynamic (rather than static) depending on the ratio of yarn lateral compressibility to yarn flexibility and the fabric tightness of construction. Energy minimisation techniques allow a detailed study to be made of the changes in the nature of loop interlocking as these parameters vary.

The energy minimisation principle has also been applied to the study of tensile properties (both uniaxial and biaxial), bending and shear behaviour for woven fabrics and to the study of tensile properties of knitted fabrics: bending and shear properties of both warp- and weft-knitted structures are presently under investigation. From the stress-strain curves so obtained, the magnitude of fabric tensile modulus, bending and shear rigidities are evaluated. The fabric properties are quoted in terms of normalised dimensionless parameters, e.g. \( \frac{YL^2}{B} \) where \( Y \) is the fabric tensile modulus, \( L \) is the normalised curvilinear crimp or loop length (i.e. the curvilinear length of yarn associated with a single interlacing within the fabric), and \( B \) is the yarn bending rigidity. Fabric stress-strain curves can therefore be plotted in a normalised form whereby direct comparisons.
are possible between the mechanical properties of various woven and knitted materials.

The energy equations can also be used to generate the normalised decrimping curves for initially crimped yarns which have been unravelled from either woven or knitted materials. These decrimping curves relate directly to the behaviour of a single end or pick for a woven fabric, or a single course for a knitted fabric, in the absence of the lateral pressure and constraints applied by the interlacing yarns within the fabric. Detailed analysis of these decrimping curves is important for the elucidation of the mechanisms of fabric deformation. In the following sections of this paper, it will be shown how the energy minimisation technique can be applied to the analysis of the relaxed plain weave structure, tensile properties, bending properties and decrimping curves for woven fabric, and to the calculation and comparison of dimensionless or normalised parameters for woven and knitted fabrics.

2. Analysis of the Relaxed Plain Weave Structure by Energy Minimisation Techniques

For application to the plain-weave structure (where the crimp is two-dimensional in shape), the general three-dimensional energy equations have been reduced to two-dimensional form and the boundary conditions evaluated. The weave-crimp shape, which has the requisite interlacing characteristics for the plain-weave construction, was fed into the computer, whereupon the shape was manipulated systematically to arrive at the minimum-energy or stable weave-crimp shape for a relaxed fabric (i.e., one that is not subjected to any externally applied forces or couples). From this equilibrium crimp shape, it is possible to evaluate the internal forces acting within the fabric itself as well as the relations between the various woven-fabric constructional parameters, e.g., thread-spacings, \( \rho \), crimp heights, \( h \), weave-crimp angles, \( \theta \), and the curvilinear yarn length, \( L \), in both warp and weft directions (see Fig. 1 of Part 1).

The energy analysis has been extended by Knoll to include the effect of continuous contact between crossing threads in a relaxed or tensioned fabric by the introduction of an additional inequality constraint in a manner similar to that employed in considering the pure-bending properties of woven fabrics. By the application of the energy equations derived for woven fabrics, Knoll was able to show that the relaxed-fabric parameters for levels of weave crimp from low to moderate (up to about 11% for a balanced square weave) agree with those predicted by the Peirce mechanistic model shown in Fig. 1 (b) of Part 1, but, for greater values of weave crimp as structural jamming is approached, the fabric parameters predicted by the energy equations move away from those predicted by the purely mechanistic model of Peirce towards Peirce's geometrical model shown in Fig. 1 (a) of Part 1. This transition from the purely mechanistic to the purely geometrical model when the maximum yarn curvature is constrained by the diameter of the crossing yarn, as shown in Fig. 1 (a) of Part 1, was described in Peirce's original paper, but, as Knoll points out, this upper limit to the yarn curvature was not considered by subsequent workers in the field before the development of the general energy equations.

To illustrate how the energy equations can be used to derive both of Peirce's models, the values of weave-crimp angle, \( \theta \), are plotted in Fig. 1 against \( h/L \) (the ratio of crimp height to curvilinear yarn length). These values were obtained by Knoll, using the energy equations derived by De Jong and Postle for the plain-weave structure, by assuming various values for the internal reaction force, \( Q \), under conditions of zero external tension (\( P=0 \)). Two curves were derived from the energy equations, one for the balanced square-weave construction (\( L_2/L_1=1 \)) and one for a non-square weave, where \( L_2/L_1=1.5 \). Both curves are
compared with the corresponding curves derived from Peirce's mechanistic and geometrical models.

In order to facilitate comparison with Peirce's models, the curves shown in Fig. 1, derived from the energy equations, were based on the assumption of incompressible yarns of circular cross-section, but the analysis can be readily extended to include compressed yarn with flattened lenticular cross-sectional shapes.

On considering the curves for the square-weave structure shown in Fig. 1, it is clear that, for values of \( \theta \leq 35^\circ \) (corresponding to crimp values less than 11% approximately), the Peirce mechanistic model based on single-point contact between warp and weft is applicable. As \( \theta \) increases towards \( 35^\circ \), i.e., as the weave crimp approaches 11%, the warp- and weft-yarn systems make contact over a larger area and the curve derived from the energy equations approaches the limiting case of Peirce's geometrical model (as structural jamming is approached). Complete jamming of the structure occurs at the maximum or limiting value of \( h/L = 0.478 \); the reaction force \( Q \) between the warp and weft threads (not shown) rises steeply as jamming is approached.

For the non-square weave (where \( L_2/L_1 = 1.5 \)), the condition of continuous yarn contact is reached at lower values of crimp and \( h/L \) than for the square weave.

Similar curves may be obtained for other crimp ratios (different values of \( L_2/L_1 \)) and for pairs of fabric parameters other than \( \theta \) and \( h/L \).

It may be concluded that the basic energy optimization formulation of woven-fabric mechanics can be used to derive the elastic mechanistic model of the relaxed plain-weave structure \( (P = 0) \) for values of the weave crimp ranging from low to moderate. However, the introduction of an inequality constraint, limiting the maximum value of the yarn curvature to that governed by the shape of the crossing thread, means that the energy equations produce results corresponding to Peirce's purely geometrical model as the weave crimp increases and the structure becomes very tight and approaches the jamming condition. The actual value of the weave crimp at which jamming occurs is very dependent on the ratio of curvilinear yarn lengths in warp and weft, \( L_2/L_1 \), and on the actual yarn cross-sectional shape.

It is clear from this work that the two models of the relaxed plain-weave structure proposed by Peirce are not mutually exclusive as has often been implied but rather are both applicable at different levels of the weave crimp. The basic energy formulation of woven fabrics has thus provided a unified basis for the detailed study of the mechanical properties of woven fabrics.

### 3. Analysis of the Tensile Properties and Yarn Decrimping Curves for Woven Fabrics

The energy equations have been used to investigate the plain-weave structure in biaxial tension \( (P = 0) \) (uniaxial tension merely being considered as the special case where the tension in one of the principal directions of the fabric is zero). An important mechanism of woven-fabric extension is considered in the theory, namely the possibility of yarn extension.

The fabric load-extension curves and yarn-decrimping curves for the plain-weave construction were computed for a realistic range of input parameters: the yarn-bending rigidity \( (B) \), the yarn-extension modulus \( (Y) \), the weave crimp \( (c) \), the degree of set \( (\phi) \), and the curvilinear lengths of yarn in the two principal directions of the fabric \( (L_1, L_2) \). Typical curves are shown in Fig. 2 where it can be seen that the load-extension
curves are non-linear.

The computed results are discussed in terms of the following dimensionless parameters: the applied (dimensionless) tension per thread \((PL^2/B)\), the relative extension, \(\varepsilon_r (= \varepsilon/c)\), where \(\varepsilon\) is the actual fabric or yarn decrimping extension), the initial relative modulus \(E_{rs}\) in the range \(0 < PL^2/B < 2\), Poisson’s ratio of the fabrics, and the relative extension of the fabric (yarn) \(\varepsilon_{rs}\) at \(PL^2/B=8\).

Computed yarn-decrimping curves are shown in Fig. 3 for various values of \(YL^2/B\) and two yarn-crimp levels, \(c=5.5\%\) and 16.6%. It can be seen that, for values of \(YL^2/B > 2000\), the yarn extensibility or the crimp value of the yarn makes little difference to the computed curves plotted as the force \((PL^2/B)\) against the relative extension, \(\varepsilon_r = \varepsilon/c\), where \(c\) is now the retained yarn crimp. This fact justifies the approach of expressing all the load-extension curves in this manner. Experimental values of initial relative modulus, \(E_{rs}\), and relative extension, \(\varepsilon_{rs}\), at \(PL^2/B=8\) compare very well with the theoretical values obtained from the decrimping curves of Fig. 3 for both wool fabrics and also cotton plain-weave fabrics.


The energy analysis of woven-fabric mechanics can be extended by considering the pure-bending behaviour of the plain-weave structure as a generalization of the tensile deformation.

A non-square plain-weave fabric under biaxial tension or pure bending represents an example where more than one yarn is considered in the structural repeat unit of the fabric. Yarn extension and set are included in this example. The continuously distributed forces acting between the yarns within the fabric are replaced in pure bending by the point forces \(Q\). The woven fabric structural repeat unit in bending, as represented by its yarn axes, is shown in Fig. 4, with the forces \(P\), \(Q\) acting at ends \(A\) and \(B\) of the yarn.

In Fig. 4, \(P\) and \(Q\) are the axial and transverse yarn forces, respectively. During bending, \(Q\) may be considered to remain constant. If point \(A\) \((s=0)\) of the bent yarn shown in full is fixed, any variation in yarn shape from its equilibrium position will cause forces \(P\), \(Q\) and couple \(M_B\) at point \(B\) \((s=1)\) to move and do work. The minimum of the energy functional \(U\) is then found only over those variations of the yarn shape which give rise to pure fabric bending.

It is required to investigate whether the approximation provided by the point forces is realistic in fabric bending. For this, constraints are imposed on the yarn curvature which have the effect of preventing the control or curvature, \(m\) to increase beyond a certain limit. This limit may be due to the continuous contact between the crossing yarns in a repeat, and the curvature limit might be some function of the interyarn distance. Optimal control theory shows that when constraints are placed on the control variable, the equilibrium equations still define the equilibrium solution, except where the constraint would be violated, in which case the control variable equals the constraint. Consequently, geometric constraints have now been introduced which do not affect the basic minimisation process so
that the internal forces are still part of the solution. The results show that the interyarn forces, and dimensions of the structural repeat are only slightly affected when the crimp of the yarns is low (less than 10% crimp). The constraints increase the bending rigidity of the fabric. Typical computed yarn shapes are shown in Fig. 5.

Table 1 shows the ratio of the fabric-bending rigidity for two values of fabric crimp (c = 5.5% and 16.6%, and \( L_2/L_1 = 1 \)) and for three different values of set (\( \phi = 0.0, \phi = 0.70, \phi = 0.99 \)). Table 1 was computed by first calculating the solution of the fabric free from external couples or tensions and then computing the solution with two different values of MB (MB \( L/B = 0.3, 0.6 \)), which gave varying (dimensionless) fabric curvatures of up to \( L/p = 0.7 \), where \( p \) is the radius of curvature. The ratio of the fabric-bending rigidity, \( B_r \) per thread, to the yarn bending rigidity, \( B \), was very nearly equal at these two values of \( M_s \), which showed that, if yarns behave linearly in bending, the fabric-bending rigidity, \( B_r \), is also constant. Only the mean values are given in Table 1. If there is no curvature constraint on the yarn, i.e., if the yarns are free to bend around each other subject to point forces between them, and the yarn is also completely set (\( \phi = 0.99 \)), then the ratio \( B_r/B = 1/(1+c) \), a result that is in agreement with the literature. When \( \phi = 0.0, 0.70 \), however, \( B_r/B > 1 \) even when there is no constraint imposed on the yarn curvature. With increasing constraint for either of the two yarn crimps, the ratio \( B_r/B \) increases sharply, the ratio \( B_r/B \) being always greater for smaller degrees of set, \( \phi \). Consequently, the interyarn forces have a very significant effect on the fabric-bending rigidity.

5. Comparison of Woven and Knitted Fabrics in terms of Normalised Parameters

It is instructive to compare the properties of woven and knitted structures in terms of dimensionless quantities. Table 2 shows such a comparison and includes values calculated from energy minimisation techniques. It is noteworthy that the interyarn forces for a woven fabric with average crimp (5.5%) are comparable with the knitted-fabric interyarn forces expressed in terms of modular length, \( L \) (the yarn length associated with a single interlacing in each structure), and the yarn bending rigidity, \( B \). Because the modular

<table>
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<th>Curvature Constraint*</th>
<th>( \phi = 0 )</th>
<th>( \phi = 0.70 )</th>
<th>( \phi = 0.99 )</th>
<th>Crimp</th>
<th>Percentage Contact†</th>
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<td>Grey</td>
<td>Set Finished</td>
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<td>1.14</td>
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</tr>
<tr>
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<td>1.36</td>
<td>16.06</td>
<td>60</td>
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</tbody>
</table>

*For inequality constraints on the curvature or control variable, e.g., \( m_s < \) constraint, the first of Equations applies unless the constraint is violated, in which case the solution of \( m_s \) at point \( s \) is equal to the constraint.
†Percentage of yarn length during which curvature constraint was active.

*The parameter \( L \) is equal to the half-weave repeat unit for the plain-weave structure and the quarter-loop repeat for the plain- and 1x1 rib-knitted structures.
yarn length, \( L \), is normally much lower for woven than knitted fabrics, the actual forces are much higher, which gives rise to considerable yarn-flattening in woven fabrics.

In terms of minimum interyarn distances, the woven fabric is the tightest construction, for it incorporates the greatest \( d_{\text{min}} / L \) within its structure. When the bending energies (per unit length) are compared, the woven fabric has slightly smaller bending energy than either the plain or the 1x1 rib weft-knitted fabric. Again, because the modular lengths of the woven fabric are much smaller (\( L=0.05 \) cm) than those of the knitted fabrics (\( L=0.25 \) cm), the actual bending energy is much larger. The total energy as measured by the Hamiltonian \( H/L^2/B \) defined in the general energy theory is comparable for each of the fabric structures.

The normalised yarn-decrimping curves shown in Fig. 6 for crimp levels ranging from 6% (woven) to 164% (knitted) are almost independent of crimp level after dividing the real extension, \( \varepsilon \), by the crimp, \( c \). The normalised load-extension curves of the plain-weave structure were compared in the same manner, e.g., relative slopes \( E_r=(PL^2/B)/(\varepsilon/c) \) were calculated. In Table 2, the initial relative slopes \( E_{rn} \) (for \( 0<PL^2/B<2 \)) for the woven structure are compared with the relative slopes \( E_{rn} \) for the decrimping curves of the yarn unravelled from the fabric (for little or no yarn extension). It may be noted that the presence of the crossing threads in the structure approximately doubles the initial relative slope. If an extra constraint is considered for the woven fabric of 16.6% crimp, this effect of the crossing threads on the initial slope is much greater.

The yarn-decrimping curves for the weft-knitted fabrics have a similar shape and an initial slope of the same order as the decrimping curves of the yarns removed from the woven fabric, provided that they are expressed in terms of relative extensions (\( \varepsilon_r=\varepsilon/c \)). The corresponding relative slopes of the plain-knitted structure are shown to be considerably higher than the woven-fabric values. Thus, in the plain-knitted structure, there is considerable yarn interference preventing yarn-decrimping. In the wale direction, \( E_{rw} \) is lower than the value \( E_{rn} \) for the course direction because considerable yarn slippage occurs.

The Poisson's ratios, \( \nu \), for both the plain-weave and plain-knitted structures are also compared. Despite the greater "necking" of the knitted structures during extension, their Poisson's ratio is lower, because their real
extension is far higher than that of the woven fabrics.

Because of this surprising similarity in fabric properties expressed in terms of normalised dimensionless parameters, experimental studies on fabric mechanical properties should be carried out, not only under constant-load or constant-extension conditions, but also by stressing the fabric to a constant dimensionless tension value, $PL^2/B$. In this way, the effects of tension on different yarn-loop and crimp structures could be adequately compared. The possibility of developing a measure of the real tightness of construction of a fabric based on normalised dimensionless parameters has also been explored by studying the relationship between the yarn compression behaviour and the distributed yarn forces acting within the fabric structure. Typical distributed forces between the interlocking yarns in a plain-knitted fabric at equilibrium are shown in Fig. 7, where the 1 and 2 directions correspond to the course and wale-wise directions respectively, and the 3 direction is perpendicular to the fabric plane.

Jamming forces between courses and wales can be found simultaneously. Analogous results have been obtained for the 1 x 1 rib fabric.

It is important to note that no assumptions have been made in this analysis about the nature of yarn contact, or the geometry of the loop shape. The only assumption made concerning the interyarn forces is that, as a result of the assumed yarn compression potential function depending only on the interyarn distances, the interyarn forces must pass through the yarn axis and apply therefore no twisting couple to the yarns. A result of the analysis is that the contact between interlocking yarns is dynamic, depending on the yarn compression characteristics, and on the applied fabric tension. The effect of different yarn compression properties shows that yarn compression does not give the fabric more significant extensibility (the reduction of interyarn distance being small on extension). The major mechanism that governs the ability of a fabric to extend is the freedom of yarn movement within the structure. Compressible yarns need to be knitted more tightly and therefore have greater effective contact along the length of yarn in the repeat, thereby reducing fabric extensibility.

6. Inelastic Fabric Deformation, Set and Recovery

The major limitation of energy minimisation techniques as far as knitted and woven fabrics are concerned, is that they may only strictly be applied to the analysis of completely elastic structures. When an inelastic structure (such as a fabric) is deformed, some energy is dissipated and accordingly the principles of energy conservation cannot be applied. Energy is dissipated during fabric deformation as a result of viscoelastic losses of the fibres as well as inter-fibre frictional losses. The inelastic nature of fabric deformation was studied by Olofsson who attempted to include both sources of inelastic behaviour in his general analytical work. In the paper describing a geometrical-mechanical model of the plain-weave structure, Olofsson was able to derive tensile stress-strain curves by the recognition that the interlaced yarns are at least partially set into their crimped shape. The concept of a “form factor” or degree of set was further developed by De Jong and Postle in their energy minimisation analysis in order to account for the cohesive set developed by wool yarns in fabrics. The degree of set can be estimated experimentally by comparing the projected length of a crimped yarn released from a fabric to the projected length of the same yarn when confined within the fabric structure (i.e. the thread or loop spacing). This setting factor becomes an important variable in fabric mechanics and allows grey-state fabrics (for which the degree of set is approximately 70%) to be compared with commercially finished fabrics (for which the degree of set is generally greater than 95%).

The inclusion of a parameter for the degree of set in fabric mechanical analyses still does not allow any predictions or conclusions to be made about the recovery behaviour of the fabric from an imposed stress or
strain. This is a very serious deficiency as fabric recovery behaviour is often the limiting factor when considering such practical fabric performance characteristics as wrinkling behaviour, crease retention properties, dimensional stability and shape retention properties. In order to study the recovery behaviour of fabrics in detail, it is necessary to develop methods of analysis that allow the inclusion of energy losses due to fiber viscoelastic behaviour and inter-fiber friction. These inelastic mechanisms of fabric deformation and recovery and their relationship to the objective specification of fabric mechanical behaviour in biaxial tension, shear and bending from the basis of Part 3 of this series of papers. The basic mechanical properties of fabrics are also related in Part 3 to their practical performance characteristics, such as wrinkling, drape, handle and tailorability.

7. Conclusion

Because of the inherent complexity of yarn interlacing in fabrics, particularly for knitted structures, energy minimisation techniques are very powerful for the study of fabrics in either their relaxed or deformed state. When the fabric mechanical properties or stress-strain curves, including decrimping curves for yarns unravelled from fabrics, are quoted in terms of normalised dimensionless parameters, it is possible to make direct comparisons between different woven and knitted fabric constructions.

The energy analysis, based on the fundamental principle that elastic structures always assume a configuration of minimum strain energy, has been formulated for fabric mechanics with the aid of optimal-control theory.

It has been shown how the two models of the plain weave fabric structure originally proposed in 1937 by Peirce\(^1\), namely, the geometric model and the mechanistic model derived by force methods of analysis, may be derived from the energy analysis. Comparison of the experimental and theoretical results calculated on the basis of both curvature constraints on the yarn (geometric model) and no curvature constraints (mechanistic model) shows that fabrics during both uniaxial tension and pure-bending deformations act as a mixture of the geometric model and the mechanistic model.

Yarn-decrimping results show that yarn extensibility plays only a small part in the load-extension behaviour. Comparison of yarn- and fabric-bending rigidities shows that the yarn-bending rigidity is not appreciably increased when the yarn is in the fabric, but rather that the increase in fabric-bending rigidity is due to the imposition of yarn-curvature constraints.

The energy analysis provides a unified approach to the study of both woven and knitted fabrics and has been shown to be a very powerful tool in the interpretation of experimental results on fabrics. Work on the application of the energy analysis to other fabric structures and deformations is continuing. In Part 3 of this series of papers, the application of fabric rheological principles to the objective specification of basic fabric mechanical properties and practical performance characteristics will be discussed. This work includes the development of fabric mechanics to include inelastic phenomena such as fibre viscoelasticity and inter-fibre friction in fabrics.

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(The following is a consolidated reference list for Parts 1 and 2 of this series of papers.)
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本書は、1973年～1979年の7年間に、米国で取得された233件に及ぶ特許を体系的に集めた、解説書である。織物の加工では、古くから各種の柔軟加工剤が使われて来た。最近になって、織維製品の高付加価値化を反映して、これら加工剤の種類や用途は急激に増加している。それともに、これらの加工剤は、単に工業用としてはかりでなく、家庭用としても、我々の日常生活に入り込むに至った。新しい加工剤としては、帯電防止加工剤、防汚加工剤発泡発泡加工剤、耐摩耗加工剤、S R加工剤、など多岐にわたっている。本書では、これら広範囲に及ぶ各種加工処理剤を、7つの章に分類し、米国特許を基礎に、その製法と使用時の処法を中心にまとめている。米国特許とは、世界的主な技術は米国特許が取得されているので、現在の世界的技術水準を知るうえで興味深い。

参考までに、本書の目次を紹介する（くわしい内容については、海外織物技術文献集、1981年6月号、P.40〜42の抄録を参照された）。
1. 柔軟剤と帯電防止剤を含む洗剤の成分
2. 洗濯のすすぎの際に使われる処理剤
3. その他の洗濯用仕上加工処理剤
4. 洗濯の乾燥工程で使われる柔軟加工剤と静電防止加工剤
5. 織維製品用の仕上加工剤
6. カーペット処理剤
7. 織維用加工処理剤