Motion of Flexible Fibers in a Newtonian Flow

Part 2: Evolution of the Configuration of a Single Fiber in a Simple Shear Flow

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Abstract

The objective of this study is to well understand the interaction between the configuration of flexible fibers and flow kinematics in fiber suspension flows. In this framework, the evolution of the configuration of a single fiber in a simple shear flow at low Reynolds number was numerically analyzed on the basis of an 'immersed boundary method' developed by Peskin. In the computations fibers with force-free configuration of being straight were used. While deformation of a fiber is fundamentally three-dimensional, stretching/compression and bending deformations in the plane of flow were simply assumed.

In Part 2 of the series of papers, the effects of fluid viscosity and shear rate on the evolution of the configuration of a single fiber and flow kinematics were described. The behavior of a fiber becomes more complex as the fluid viscosity increases. Furthermore, it was confirmed that a flow-induced bending parameter, which is a function of fiber length, bending stiffness, fluid viscosity and shear rate, is a useful dimensionless parameter for classification of rotation of an elastic fiber, such as rigid rotation, springy rotation, S-turn, snake turn and complex rotation.

Keywords: configuration of flexible fiber, evolution of flow kinematics, immersed boundary method, effect of fluid viscosity and shear rate, simple shear flow.

1. Introduction

The characterization of motion of fibers in a variety of flows is of particular interest in current textile and FRP processing technology. It is fundamentally important in these processes to control motion of fibers in air or polymer liquid flows because the configuration and orientation of fibers in products strongly affect the quality and productivity. The addition of short fibers to solvents can drastically change the flow kinematics even at very low concentrations, and in recent years the importance of coupling flow field with fiber configuration and/or orientation has been recognized.

The objective of this study is to well understand the interaction of change in the configuration of flexible fibers with flow kinematics around the fibers in a Newtonian flow. In this framework, the evolution of the configuration of a single fiber in a simple shear flow has been numerically calculated using an 'immersed boundary method' developed by Peskin[1]. For the computations the elastic and bending stiffness of a fiber, fluid viscosity and shear rate were varied. While deformation of a fiber is fundamentally three-

dimensional, we have restricted ourselves to stretching/compression and bending deformations in the plane of flow.

In Part 1[2] of the series of papers, the governing equations and the numerical scheme were described in detail. Furthermore, we described the effect of the elastic stiffness on the extensional strain of a fiber and the effect of the bending stiffness on the evolutions of fiber configuration and flow kinematics for force-free fiber configuration of being straight: (1) a recirculating flow occurred around a fiber and the fiber rotated in the recirculating flow, (2) a fiber with large bending stiffness could keep an almost straight configuration during rotation, while a fiber was deformed from somewhat straight configuration into an arc for small bending stiffness, (3) the flow regime affected by the presence of fiber became narrower as the bending stiffness decreased.

In the present paper, we will concentrate on how fluid viscosity and shear rate affect the evolutions of the configuration of a single fiber and flow kinematics in a simple shear flow at very low Reynolds number. Both great potential of the immersed boundary method for future

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applications in many other aspects of fiber motion and its limitations will also be demonstrated.

2. Computations of motion of a single fiber in a simple shear flow

The governing equations for flow field are the mass and momentum conservation equations, and the external force exerted on the fluid element arises only from stretching/compression and bending deformations of a fiber in the plane of flow. Furthermore, planar deformation of the fiber is generated by the movement of a segment on the fiber at the fluid velocity. Then, the evolutions of the configuration of the fiber and flow kinematics are solved simultaneously.

Computation conditions were as follows: a single elastic fiber with $L_f = 1.0\text{ mm}$ in force-free length was initially placed in the middle of a simple shear flow domain $L_y = 3.0\text{ mm}$ wide and $L_x = 9.0\text{ mm}$ long. The flow domain and flow direction are shown in Fig.1. The force-free configuration of the fiber was straight. The initial location and configuration of the fiber are illustrated in Fig.2: the initial location of the endpoint, X_0 as shown in Fig.3, of the fiber is $x_0 = 4.5\text{ mm}$, $y_0 = 1.5\text{ mm}$ and the initial curvature and inclination angle are $5.0 \times 10^{-1}\text{ mm}^{-1}$ and -2.80 rad , respectively. An inclination angle ϕ is defined as follows: the vector pointing from X_0 to X_{N_f} makes ϕ with the x direction (see Fig.3). Where ϕ is measured anti-clockwise. In the computations, the governing equations of flow were solved using the MAC method (Marker-and-Cell method) and the grid sizes were 0.1 mm in both the flow and width directions. On the other hand, the fiber position was discretized as a set of 21 moving points. Furthermore, the evolutions of fiber configuration and flow kinematics were calculated at the time step of $\Delta t = 5.0 \times 10^{-6}\text{ s}$. The fluid density was $\rho = 1.0 \times 10^{-6}\text{ kg/mm}^3$. The Reynolds number is defined by $\text{Re} = \rho U L_y / 4\mu$, where U denotes the velocity of the top and bottom walls of a channel and μ is the fluid viscosity.

2.1 Effect of fluid viscosity on the evolutions of fiber configuration and flow kinematics

Computations were carried out for the fluid viscosity $\mu = 1.0 \times 10^{-5}$, 1.0×10^{-4} and $2.0 \times 10^{-4}\text{ kg/mm} \cdot \text{s}$ (0.01 , 0.1 and $0.2\text{ Pa} \cdot \text{s}$), the bending stiffness $\sigma_b = 7.5 \times 10^{-5}$ and $6.25 \times 10^{-3}\text{ kg} \cdot \text{mm}^3/\text{s}^2$ to examine the effect of fluid viscosity on the evolutions of fiber configuration and flow kinematics. The elastic stiffness and the shear rate were constant, $\sigma_s = 1.0\text{ kg/mm} \cdot \text{s}^2$ and $G = 10\text{ s}^{-1}$, respectively.

Figures 4–6 show the evolutions of fiber configuration

Fig.1 Computational domain, simple shear flow, and fiber configuration.

Fig.2 Initial position and configuration of a fiber used in the computations. The coordinate of the endpoint of the fiber is $x = 4.5\text{ mm}$, $y = 1.5\text{ mm}$, the curvature is $5.0 \times 10^{-1}\text{ mm}^{-1}$ and the inclination angle is -2.80 rad .

Fig.3 Definition of the inclination angle of a fiber, ϕ .

and flow kinematics up to $t = 2.0\text{ s}$ for $\sigma_b = 7.5 \times 10^{-5}\text{ kg} \cdot \text{mm}^3/\text{s}^2$, and Figs.7–9 for $\sigma_b = 6.25 \times 10^{-3}\text{ kg} \cdot \text{mm}^3/\text{s}^2$. The prediction for $\sigma_b = 6.25 \times 10^{-3}\text{ kg} \cdot \text{mm}^3/\text{s}^2$ and $\mu = 1.0 \times 10^{-4}\text{ kg/mm} \cdot \text{s}$ (Fig.8) was already illustrated in Fig.10 in Part 1. Experiments by Mason and co-workers [3, 4] identified a wide range of fiber behaviors, which they separated into distinct orbit classes based on the flexibility

Fig.4 Evolutions of the streamlines and fiber configuration for $\sigma_b = 7.5 \times 10^{-5} \text{ kg} \cdot \text{mm}^3/\text{s}^2$, $\mu = 2.0 \times 10^{-4} \text{ kg}/\text{mm} \cdot \text{s}$, $\sigma_s = 1.0 \text{ kg}/\text{mm} \cdot \text{s}^2$ and $G = 10 \text{ s}^{-1}$.