Static Analysis of Finite Hydrodynamic Journal Bearing in Turbulent Regime with Non-Newtonian Lubricant

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The aim of the present paper is to investigate and predict the static performance of finite journal bearing in turbulent flow condition considering non-Newtonian lubrication. The Navier-Stokes equation has been modified considering turbulent as well as non-Newtonian lubrication and is solved for steady state parameters. Ng and Pan’s linear turbulence model is utilised in the analysis. The momentum and continuity equations in cylindrical coordinates representing the flow field in the clearance space of a finite circular journal bearing using a Newtonian lubricant were solved by the finite element method. The non-Newtonian lubrication effect is introduced by modifying the viscosity term using the cubic shear stress law in each iteration. In this paper, the effect of turbulence and non-Newtonian lubrication has been determined by solving the modified Navier-Stokes equations obtained by the application of linear theory proposed by Ng and Pan and using the cubic shear stress law model simultaneously. The modified Navier-Stokes equations, in cylindrical coordinates, have been solved by finite element method using Galerkin’s technique and a suitable iteration procedure. Steady-state performance characteristics of a finite circular journal bearing have been analysed in terms of Sommerfeld number, load carrying capacity, friction coefficient variable, temperature rise parameter and total flow at various eccentricities for different Reynolds numbers up to 13300 and various values of non-linear factor of cubic shear stress law of non-Newtonian model. Computed results have been compared with the published results obtained by linearized theory of Ng and Pan and cubic shear stress law of non-Newtonian lubrication. The results obtained revealed better performance in turbulent regime as compared to laminar condition for cubic shear stress non-Newtonian model.

Keywords: finite journal bearing, static characteristics, finite element method, turbulence, non-Newtonian lubricant

1. Introduction

Large capacity turbo-machinery employs large journal bearings to support heavy weight of rotor loads at higher r. p. m. These journal bearings are operated in turbulent flow regime due to higher surface velocities. In the present literature, the value of Reynolds numbers varies from 10000 and more have been reported by the different investigators [4,5,11,12] for the turbulence analysis of journal bearing. With the latest trends of turbo-machinery, larger value of Reynolds numbers may be faced in the future generation application of journal bearing. So, in the analysis and design of journal bearing, the effect of turbulence is very important to predict the behavior and performance of these bearings.

Use of additives controls the lubricant performance to meet various requirements of high speed machinery. Polymer thickened lubricants either behaves as a dilatent or pseudo plastic fluid depends upon the properties of particular lubricant. The viscosity of these lubricants with additives is not constant and they follow non-linear relation between shear stress and shear strain rates. This relationship can often be represented by cubic shear stress law of non-Newtonian models.

In future designs of journal bearing, higher values of Reynolds number and use of various process fluid lubricants are expected for industrial machinery of large diameters and higher operational speeds.

Constantinescu [1,2] has analyzes theoretically the three-dimensional motion in the lubricant layer by using Prandtl’s mixing length theory. Using the Prandtl’s mixing length hypothesis, exact and approximate solutions has been obtained for the velocity distribution into the lubricant layer. The result has also pointed out the pressure gradient and the Reynolds number influence.
on the velocity distributions, as well as the differences with respect to the laminar flow. The results have compared with the experimental data obtained by Smith and Fuller and the good qualitative agreement has been pointed out. Ng [3] has critically examined the theory of turbulent fluid film lubrication proposed by Constantinescu’s approach of Prandtl’s mixing length concept. Author has also proposed the best choice of the numerical constants in Reichardt’s formula as 

\[ k = 0.4, \delta^* = 10.7 \].

Ng and Pan [4] have presented the linearised turbulent lubrication theory using the eddy diffusivity concept of Bousinesq and the “law of wall”. The turbulence was assumed isotropic and linearization was achieved by assuming the flow to be an approximately Couette flow. Orcutt and Arwas [5] have presented the steady-state and dynamic characteristics of a full circular bearing and a centrally loaded, 100 deg, arc bearing in laminar and turbulent regime adopting the linearized turbulence lubrication theory [4]. The modified Reynolds equation was solved by finite difference method to obtain numerical solution. Reynolds numbers up to 13300 were used and results were compared with measured performance of these bearings over the same range of operating parameters. They have found good correlation between the theoretical and test data for partial arc bearings; however, results for full journal bearing have showed poor comparison at high Reynolds number and large eccentricities. Booser et al. [6] has examined the behavior of large size steam turbine bearings of elliptical and pad types for speeds ranging up to 5000 rpm and Reynolds number up to 7000. Authors have found that the load on bearing is strongly related to laminar-turbulent transition speed. Hirs [7] has adopted the bulk-flow theory on the turbulent lubricant film. He has utilized experimental data to show shear stress as a function of Reynolds number. Taylor and Dowson [8] have reviewed the existing turbulent lubrication theories of Ng, Ng and Pan, Elrod and Ng and their applications to fluid film bearing design. They have also provided quantitative predictions of the various turbulent theories for the case of a plane inclined slider bearing. Soni et al. [9] has solved modified Reynolds equation by finite element method using Galerkin’s technique and has applied linearized turbulent lubrication theory of Ng and Pan. Static and dynamic performance characteristics of the noncircular (two-lobe) bearings have been studied, both for laminar and turbulent flow, in terms of load support, oil flow, fluid film stiffness coefficients, damping coefficients and critical mass for various Reynolds numbers. Singh et al. [10] has modified the classical Navier-stokes equations in cylindrical coordinates. These modified equations have been solved by finite element method using Galerkin’s technique and a suitable iteration scheme. He has adopted the nonlinear turbulent lubrication theory of Elrod and Ng to predict the performance of finite journal bearing. Performance characteristics of finite circular hydrodynamic journal bearing have been calculated at various eccentricities for Reynolds numbers up to 13300. Computed results have also been compared with the published results obtained by linearized theory of Ng and Pan. Soni et al. [11] has adopted the non-linear theory proposed by Elrod and Ng, the generalized Navier-Stokes equations has been modified and solved numerically for the flow field in the clearance space of a journal bearing. The performance characteristics of a finite circular hydrodynamic bearing have been studied for Reynolds numbers up to 13300. Shenoy and Pai [12] have applied the concept of adjustability in the design of hydrodynamic bearings to enhance the performance in laminar and turbulent regime by adopting the linearized turbulence model of Ng and Pan. Gautam and Ghosh [13] have presented a theoretical analysis to determine the leakage flow and dynamic characteristics of circumferential wave geometry annular seals using turbulent lubrication theory. Convective fluid inertia effects have been incorporated using the perturbation approaches in turbulent lubrication. They have found that circumferential wave geometry improves the performance of the annular seal at higher speeds. Bouard et al. [14] have got static and thermal performance characteristics of a four shoe tilting pad bearing by taking comparison among three turbulent models i.e. Constantinescu, Ng and Pan and Elrod and Ng turbulent model. Authors have shown that under high rotational speeds thermal effects are very important to predict the bearing performance and non-laminar flow regime effects are also taking into account. Jain et al. [15] has obtained the static and dynamic performance characteristics of hydrodynamic journal bearing by using the modified Reynolds equation based on the linearized turbulent lubrication theory of Ng and Pan [4] taking variation of viscosity into consideration. Frene et al. [16] has presented that the lubrication problem typically involves moderate or large Reynolds numbers (based on the fluid film thickness) ranging from \(2 \times 10^3\) to \(10^5\). Thus inertial forces and turbulent phenomena are expected to play a role in lubrication. Horowitz and Steidler [17] have solved the modified Reynolds equation by finite difference technique using oil viscosity as a logarithmic function of shear stress and have analyzed steady state characteristics of infinite long bearings. Tanner [18] has investigated the frictional force and fluid film reactions for a short journal bearing by solving the modified Reynolds equation for a power law fluid model. Hsu [19] has used the cubic shear stress law for the non-Newtonian lubricants and has studied the static performance characteristics of infinitely long bearings by expressing the pressure and the flow in terms of a power series. Wada and Hayashi [20,21] have modified the Reynolds equation for the cubic shear stress law and solved it by a perturbation technique to determine the static performance characteristics of finite-width bearings. Sinhasan and Goyal [22] have presented the transient
Response of a circular journal bearing with non-Newtonian lubricant. The three-dimensional Navier-Stokes equations, along with the continuity equation, have been solved for the lubricant flow field for Newtonian lubricants using the finite element method and an iterative scheme. The non-Newtonian effect is incorporated by modifying the viscosity term using the cubic shear stress law. The linear and non-linear trajectories of the journal-centre motion have been obtained by numerically integrating the equations of motion using a fourth-order Runge-Kutta method. Hayashi et al. [23] have modified Reynolds equation to obtain the pressure distribution and the load capacity of journal bearings by application of a polynomial expression for non-Newtonian characteristics of pseudo-plastic and dilatant fluid. Safar [24] have derived a modified Reynolds equation by assuming a polynomial expansion for the velocity and author has obtained the fluid film pressure distribution and load capacity for different values of the eccentricity ratio using the power law for non-Newtonian flow. Tayal et al. [25] have modified the Navier-Stokes equations for non-Newtonian fluids and the non-Newtonian effect is introduced by modifying the viscosity term using a power law. The static characteristics have been studied for a finite-width journal bearing. Tayal et al. [26] has modified the N-S equations in cylindrical coordinates for non-Newtonian fluids and the non-Newtonian effect is introduced by modifying the viscosity term using a cubic shear stress law. The static characteristics for an elliptical journal bearing have been studied. Authors have found that non-linear factor of non-Newtonian lubricant remarkably affects the performance of non-circular (two-lobe) bearing as compared with Newtonian lubricant. Sinhasan and Goyal [27] have presented, the transient response of a two-lobe journal bearing with non-Newtonian lubricant. The 3D N-S equations, along with the continuity equation in cylindrical coordinates, have been solved for the lubricant flow field for Newtonian lubricants using the finite element method and an iterative scheme. The non-Newtonian effect is incorporated by modifying the viscosity term using the cubic shear stress law. The non-linear trajectories of the journal-centre motion have been obtained for two-lobe bearing. Raghunandana and Majumdar [28] have analyzed the stability of circular journal bearing by using the non-linear time transient method. Authors have used the Dien and Elrod model of non-Newtonian lubricant in the analysis. Stability of finite journal bearing improves with non-Newtonian lubricant having higher power law index value. Jang and Chang [29] have presented the adiabatic solutions for finite width hydrodynamic misaligned journal bearing using the power law model of non-Newtonian lubricant. Viscosity of oil film was taken to be an exponential function of temperature. The adiabatic solutions showed that the load capacity was reduced very much as compared to isothermal solutions and thermal effects were found to be more noticeable at larger power index values, larger eccentricity ratios and misalignment angles. Jang and Chang [30] have presented the adiabatic solutions for finite width journal bearing obeying the power law model. Authors have plotted the performance results for the power law index values in the range of 0.7 to 1.2, aspect ratio of 0.5, 1.0 and 1.2 at various values of rotational speeds. They have also showed the effects of shear thinning and shear thickening fluids on the performance characteristics of the finite bearing.  

In the present work, the Navier-Stokes equations in cylindrical coordinates for laminar and Newtonian fluids are used. The effect of turbulence is introduced by the application of linear theory proposed by Ng-Pan and the non-Newtonian effect is introduced by modifying the viscosity term using a cubic shear stress law. The static performance characteristics for a finite hydrodynamic journal bearing (\(L / D = 1\)) in turbulent regime using non-Newtonian lubricant have been studied in terms of the load carrying capacity, attitude angle, friction coefficient variable, rise in temperature variable and total oil flow. The results presented in this study are expected to be quite useful to the bearing designers. The bearing geometry is shown in Fig. 1.

2. Analysis

2.1. Turbulence model

The most widely accepted theory of turbulent fluid flow was given by Boussinesq [3]. According to this theory, the effective shear stress for the two-dimensional turbulent flow is expressed by [3]:

\[
\tau_{ij} = \mu \left[ \frac{1}{\nu} \left( \frac{\partial U_j}{\partial x_i} + \frac{\partial U_i}{\partial x_j} \right) \right] \quad (1)
\]

Where \(U\) represents the velocity vector \(i, j, k\) are the directional coordinates \(\theta, r, \) and \(z\) respectively. Reichardt’s suggested the empirical relationship for
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eddy diffusivity, which can be given by [3] following expression:
\[
\left( \frac{\varepsilon}{\nu} = k \left[ Z^+ - \delta^+ \tanh \frac{Z^+}{\delta^+} \right] \right)
\] (2)

Where numerical constants \( k = 0.4, \delta^+ = 10.7 \) have been used in this study, as proposed by [3] and \( \delta^+ \), non-dimensional distance measured from the nearest wall is represented by following expression:
\[
Z^+ = \frac{h}{\nu} \sqrt{\frac{1}{\rho}} \begin{cases} 0 < \xi = \frac{Z}{h} \leq 0.5 \\ (1-\xi) \frac{h}{\nu} \sqrt{\frac{1}{\rho}} \quad 0.5 < \xi = \frac{Z}{h} \leq 1 \end{cases}
\] (3)

The non-dimensional distance measured from the nearest wall is the function of local shear stress \(|\tau|\) and Reynolds number \(Re\). The local shear stress \(|\tau|\) is represented by following expression:
\[
|\tau| = [ (\tau_u - \mu) ]^{1/2} + (\tau_v)^{1/2}
\]

The non-Newtonian lubricant viscosity is expressed by the apparent viscosity \(\mu_a\) and is defined as in terms of shear strain rate \(\dot{\gamma}\).
\[
\mu_a = (\tau / \dot{\gamma})
\] (8)

2.3. Finite element analysis

The Navier-Stokes equation (in cylindrical coordinates), which govern the Newtonian lubricant flow in the clearance space of a finite journal bearing, may be modified in the following non-dimensional form, and can be written as:

\[
\frac{R_u}{r} = \mu_a (1 + K) \left\{ \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{1}{R_e} \frac{\partial^2 u}{\partial z^2} + \frac{2}{r^2} \frac{\partial u}{\partial \theta} \right\}
\] (9a)

\[
\frac{R_v}{r} = \mu_u (1 + K) \left\{ \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{1}{r^2} \frac{\partial^2 v}{\partial \theta^2} + \frac{1}{R_e} \frac{\partial^2 v}{\partial z^2} \right\}
\] (9b)

\[
\frac{R_w}{r} = \mu_w (1 + K) \left\{ \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} + \frac{1}{R_e} \frac{\partial^2 w}{\partial z^2} \right\}
\] (9c)

The continuity equation in non-dimensional form is given by:
\[
\frac{1}{r} \frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} + \frac{1}{r} \frac{\partial w}{\partial z} = 0
\] (9d)

Where, \(\mu_a\) is the apparent viscosity and \(K = (\varepsilon / \nu)\) is the eddy diffusivity or turbulent viscosity coefficient and \(R_e = (\tilde{R} / C)\). The \(u, v, w\) are the velocities in \(\theta, r, z\) directions respectively.

The solution of eqs. (9a to 9d), using suitable boundary conditions, gives velocities \(u, v, w\) and distribution of pressure \(p\) from which the steady state parameters can be calculated. In the present study, the local and convective
inertia terms are neglected, the turbulence and non-Newtonian effect of lubricant is considered. The following equations are written after applying the orthogonality condition of Galerkin’s technique [31] to eqs. (9a to 9d), for an element in the positive pressure oil film region of the journal bearing:

\[
\int \int \int N_i^r \left[ \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{1}{R_i} \frac{\partial^2 u}{\partial z^2} + \frac{2}{r^2} \frac{\partial^2 v}{\partial \theta^2} - \frac{u}{r^2} \right] r \phantom{d} \theta \phantom{d} z \phantom{d} dz = 0
\]

(10a)

\[
\int \int \int N_i^r \left[ \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{1}{r^2} \frac{\partial^2 v}{\partial \theta^2} + \frac{1}{R_i} \frac{\partial^2 v}{\partial z^2} - \frac{v}{r^2} \right] r \phantom{d} \theta \phantom{d} z \phantom{d} dz = 0
\]

(10b)

\[
\int \int \int N_i^r \left[ \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} + \frac{1}{R_i} \frac{\partial^2 w}{\partial z^2} \right] r \phantom{d} \theta \phantom{d} z \phantom{d} dz = 0
\]

(10c)

\[
\int \int \int N_i^r \left[ \frac{\partial u}{\partial r} + \frac{\partial v}{\partial r} + \frac{\partial w}{\partial z} \right] r \phantom{d} \theta \phantom{d} z \phantom{d} dz = 0
\]

(10d)

where,

\[
u' = \sum_{i=1}^{20} N_i^r u_i, \quad v' = \sum_{i=1}^{20} N_i^r v_i, \quad w' = \sum_{i=1}^{20} N_i^r w_i, \quad p' = \sum_{i=1}^{k} N_i^r p_i
\]

(11)

In the present study, three-dimensional isoparametric elements having parabolic shape functions for velocity and linear shape function for pressure have been used to discretise the positive pressure fluid-film region of the clearance space of the finite journal bearing system [32,33,34]. By integrating eqs. (10a to 10d), with suitable substitutions, the equation in the matrix form can be expressed for an element as:

\[
\begin{bmatrix} K^v & K^r \\ K^r & 0 \end{bmatrix} \begin{bmatrix} \Phi^v \\ \Phi^p \end{bmatrix} = 0
\]

(12)

Where \(K^v,K^r,\Phi^v,\Phi^p\) are the sub-matrices for viscous terms, pressure terms, continuity equation terms, nodal velocities and nodal pressures respectively. The following boundary conditions [Fig. 1] have been applied at the element level to reduce the dimension of the global fluidity matrix:

\[
p(\theta,z) = 0 \quad \text{at} \quad \theta = 0,\theta_r \quad \text{and at} \quad z = \pm \left( \frac{L}{D} \right)
\]

\[
\frac{\partial p}{\partial \theta}(\theta,z) = 0 \quad \text{at} \quad \theta = \theta_r \quad \text{and at} \quad z = 0
\]

(13)

\[
u = 1 \quad \text{at} \quad r = R
\]

\[
u = v = w = 0 \quad \text{at} \quad r = R + h
\]

\[
v = w = 0 \quad \text{at} \quad r = R
\]

The element fluidity matrices, after adopting the modifications for boundary conditions, have been assembled for every elements to obtain the global system equation:

\[
[K] \Phi = b
\]

(14)

The solution of equation (14) provides the nodal pressure and nodal velocities for entire assemblage.

3. Static characteristics

The static characteristics of the finite circular journal bearing system in terms of bearing load capacity, friction coefficient variable, rise in temperature variable and total oil flow can be obtained by following expressions.

3.1. Bearing load capacity

The journal bearing load capacity is computed by putting together the oil film force components of all the elements along and perpendicular to the line of centers:

\[
W = \sum W^e_r, \quad W_\theta = \sum W^e_\theta
\]

where,

\[
W^e_r = - \int \int (p)^e \cos \theta \phantom{d} r \phantom{d} \theta \phantom{d} dz
\]

\[
W^e_\theta = \int \int (p)^e \sin \theta \phantom{d} r \phantom{d} \theta \phantom{d} dz
\]

The resultant load-carrying capacity is computed by following expression:

\[
W = \left( W^e_r + W^e_\theta \right)^{0.5}
\]

(15)

The Sommerfield number is expressed as:

\[
S = \left( \frac{2L}{\pi D W} \right)
\]

(16)

and the attitude angle is calculated by:

\[
\varphi = \tan^{-1} \left( \frac{W_\theta}{W_r} \right)
\]

(17)

3.2. Friction coefficient

The frictional force on the journal surface is computed by addition of the frictional force on all the elements, which expressed as:

\[
F = \sum F^e_j
\]

Where,

\[
F^e_j = \int \int \left[ \mu \left( 1 + \frac{\epsilon}{v} \right) \frac{1}{H} + \frac{H}{2} \frac{\partial p}{\partial \theta} \right] r \phantom{d} \theta \phantom{d} dz
\]

The friction coefficient variable is computed from following expression:
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\[ f\left( \frac{R}{C} \right) = \left( \frac{F_j}{W} \right) \]  

(18)

3.3. Total oil flow

The journal bearing side leakage is calculated by summing up the axial direction flows of all the elements on each side of the journal bearing.

\[ Q_s = \sum q' \quad q' = \iint (w)' r \ d\theta \ dr \]  

(19)

3.4. Rise in temperature variable

The rise in temperature variable is obtained by following expression:

\[ \Delta T = \left( \frac{F_j}{Q_s} \right) \]  

(20)

4. Solution procedure

In the beginning, the nodal velocities \( u, v, w \) and nodal pressure \( p \) are to be computed for laminar flow conditions, ignoring the eddy diffusivity or turbulence viscosity coefficient \((K = \frac{\varepsilon}{\nu})\). The local shear stress \((\tau)\) computed from the laminar flow condition is used to calculate the initial value of \( K = \frac{\varepsilon}{\nu} \). By utilising the initial value of \( K = \frac{\varepsilon}{\nu} \), the solution of equation (5) is established. The values of the eddy diffusivity or turbulence viscosity coefficient \((K = \frac{\varepsilon}{\nu})\) are further upgraded for a particular value of Reynolds number \((Re)\) and the corresponding new value of shear stresses are computed. First to upgrade the modified viscosity and then to upgrade the local shear stress \((\tau)\) which is a function of Reynolds number, the direct iteration method [32] with nested iteration loops are employed. These iterations are continued until convergence is reached. The iteration method used to compute the solution of equations (9a to 9d) considering the effect of turbulence is shown in Fig. 2. The system of three non-dimensional momentum equations and the continuity equation [9a to 9d] governing the lubricant flow field in the clearance space of the finite journal bearing and the resulting global system equations (14) are non-linear in the nature due to variation in viscosity of the lubricant. The direct iteration method [32] with nested iteration loops is employed to solve the equations. The solution for Newtonian lubricant condition is taken as the initial trial solution for the non-Newtonian lubricant condition. For the non-Newtonian lubricant, the shear strain rate \((\gamma)\) is computed by using Eq. (7), and the Newton-Raphson method is used to calculate the shear stress \((\tau)\) by using Eq. (6). The apparent viscosity at each Gaussian integration point is then calculated from the eq. (8). The non-Newtonian lubricant solution is upgraded using apparent viscosity.

![Flow diagram for the solution procedure](image-url)
5. Results and discussion

Based on the performance analysis and solution algorithm, a computer program has been developed to compute the static performance characteristics of finite circular journal bearing for combined turbulence and non-Newtonian effect. The static performance characteristics have been computed in terms of load carrying capacity, Sommerfeld number, attitude angle, friction coefficient variable, rise in temperature variable and oil flow for aspect ratio \((L / D) = 1.0\), various eccentricity ratio \((\varepsilon)\). The results for static performance characteristics have been computed for different values of Reynolds number varies from laminar to turbulence \((Re = \text{laminar, 3326, 8314 & 13300})\) and various values of non-linear factor \((k = 0.0, 0.10, 0.58 & 1.0)\) of non-Newtonian lubricant cubic shear stress model separately and jointly.

To check the accuracy and validity of the computer algorithm, results have been computed and compared with the published literature of ref. [4] and ref. [22]. The results have been computed for combined turbulence and non-Newtonian model for different values of Reynolds number \((Re)\) and non-linear factor \((k)\). The results are in good agreement.

Fig. 3 shows the variation of attitude angle with eccentricity ratio obtained by the linearized turbulence theory, cubic shear stress law and present analysis at \(Re = 3326\) and \(k = 0.1\). The results of present analysis are slightly larger than those from the linearized turbulence theory and cubic shear stress law.

The Fig. 3 also shows higher attitude angles for the combined effect of turbulent flow and non-Newtonian as compared to pure laminar, pure turbulence and pure non-Newtonian flow in the working range of journal bearing eccentricities. It is evident from the figure that combining the turbulence and non-Newtonian effect ultimately improves the wedge effect of the oil film and equilibrium shaft position of the journal bearing.

Fig. 4 shows the variation of the non-dimensional load carrying capacity with eccentricity ratio obtained by the linearized turbulence theory, cubic shear stress law and by the present analysis. With an increase in the non-linear factor of non-Newtonian lubricant from \(k = 0.1\) to 1.0 and \(Re = 3326\), the non-dimensional load carrying capacity of the finite hydrodynamic bearing, in general, increase with increase in eccentricity ratio and it is found that the values of the load carrying capacity for the combined effect of turbulence and non-Newtonian lubricant are falls in between the values of the laminar cond., linearised theory and cubic shear stress law.

By keeping the value of Reynolds number constant and increasing the value of non-linear factor of non-Newtonian lubricant, decreases the load carrying capacity. The physical reason for this trend is that as the non-linear factor \((k)\) tends to higher values, the apparent viscosity \((\mu_a)\) decreases and the turbulent viscosity coefficient \(K = (\varepsilon / \nu)\) is not much affected due to constant value of Reynolds number \((Re)\).

Fig. 5 shows the variation of the non-dimensional load carrying capacity with eccentricity ratio obtained by the linearized turbulence theory, cubic shear stress law and by the present analysis. In this fig. the values of Reynolds number changes from \(Re = 3326, 8314 & 13300\) by taking constant value of non-linear factor as \(k = 0.1\) of cubic shear stress model. The combined effect of turbulence and non-Newtonian lubrication on the non-dimensional load carrying capacity is very significant and the load capacity increases as compared to the other combination of turbulence \((Re = 3326)\) and non-linear factor \((k = 0.1)\) specially at \(Re = 13300\) and \(k = 0.1\).

Here in Fig. 5 the value of non-linear factor of non-Newtonian lubricant is kept constant i.e. \(k = 0.1\) and values of Reynolds number increases that variation ultimately increases the load carrying capacity of the finite bearing. The reason for this observation is that as the Reynolds number \((Re)\) increases, the turbulence in the flow of lubricant increases and turbulent viscosity coefficient \(K = (\varepsilon / \nu)\) changes very sharply. During this type of change the apparent viscosity \((\mu_a)\) is not much affected and this gives the better performance of finite bearing as compared to laminar and Newtonian flow conditions.
Fig. 6 shows the variation of the friction coefficient variable with eccentricity ratio obtained by the linearized turbulence theory, cubic shear stress law and by the present analysis. With an increase in the non-linear factor of non-Newtonian lubricant from $k = 0.1$ to 1.0 and $Re = 3326$, the friction coefficient variable of the finite bearing, in general, decreases with an increase in eccentricity ratio and also found that the values of the friction coefficient variable for the combined effect of turbulence and non-Newtonian lubricant are higher than the values of the laminar cond., linearised theory and cubic shear stress law.

The Fig. 6, reveals that the friction coefficient variable, which involves the ratio of frictional force and load support, is greatly affected by combination of non-linear factor and Reynolds number. As the non-linear factor ($k$) increases and keeping the Reynolds number constant, then the apparent viscosity ($\mu_a$)
decreases, which mainly responsible for the changes in the value of friction coefficient variable for the finite circular journal bearing.

Fig. 7 shows the variation of the friction coefficient variable with eccentricity ratio obtained by the linearized turbulence theory, cubic shear stress law and by the present analysis. In this Fig. the values of Reynolds number changes from $Re = 3326$, $8314$ & $13300$ by taking constant value of non-linear factor as $k = 0.1$ of cubic shear stress model. As the values of Reynolds number increases keeping value of non-linear factor as constant, the friction coefficient variable decreases very rapidly.

It is found that the friction coefficient variable $f(R/C)$ decreases with increment in the value of Reynolds number with particular value of $k$, when all other
conditions remain unaltered. The effect of $Re$ is to reduce the frictional parameter.

Figs. 8 and 9 show the variation of the rise in temperature variable with eccentricity ratio obtained by the linearized turbulence theory, cubic shear stress law and by the present analysis at various values of the Reynolds number ($Re = 3326, 8314$ & $13300$) and non-linear factor ($k = 0.1, 0.58$ & $1.0$). This parameter, in general, decreases with the increase in eccentricity ratio and increases with increasing Reynolds number and non-linear factor jointly.

The rise in temperature variable, which is the ratio of frictional force $F_j$ and bearing oil flow $Q_z$, decreases in the same way as the friction coefficient variable with increase in the values of non-linear factor and Reynolds number. This parameter is little affected by keeping the
value of $Re$ constant and only changing the non-linear factor and same trend is observed as of laminar and newtonian flow. As the value of $Re$ increases by keeping the value of $k$ as constant, the rise in temperature variable is higher than the pure laminar and newtonian flow conditions because temperature rise is observed when the flow inside the bearing changes from laminar to turbulent.

Figs. 10 and 11 show the variation in the oil flow with eccentricity ratio obtained by the linearized turbulence theory, cubic shear stress law and by the present analysis at various values of the Reynolds number ($Re = 3326$, $8314$ & $13300$) and non-linear factor ($k = 0.1$, $0.58$ & $1.0$). The oil flow, in general, increases with the increase in eccentricity ratio and also increases with increasing Reynolds number and non-linear factor combinely. The overall effect of combined turbulence and non-Newtonian lubricant on
the bearing oil flow is insignificant.

The bearing oil flow $Q_z$, which depends upon the pressure gradient and viscosity of the lubricant, is little affected in comparison to the trends obtained by the pure laminar, pure non-Newtonian and pure turbulence flow conditions. Moreover, the non-Newtonian lubricant and turbulent effect is preponderant at higher values of non-linear factor ($k$) and Reynolds number ($Re$).

Figs. 12 to 15 shows the effect of non-linear factor ($k$) of non-Newtonian lubricant at $Re = 8314$ with different values of eccentricity ratio. These figures provide the impact of non-linearity of operating fluid in conjunction with turbulence effect on the static performance of the finite journal bearing. The rheology of the lubricant depends on the governing equation of the cubic shear stress model (Eq. 6). The behaviour of
the shear thinning or shear thickening fluid depends on the non-linearity factor.

Fig. 12 shows the variation of load capacity with non-linear factor ($\kappa$) of non-Newtonian lubricant at Reynolds number of $Re = 8314$ and different values of eccentricity ratio ranging from 0.2 to 0.8. It is observed that as the non-linear factor ($\kappa$) increases from 0.1 to 1.0, the load capacity of the bearing increases in the Newtonian regime and gradually decreases in the non-Newtonian lubrication regime due to fact that the increment in the non-linearity reduces the viscosity of the operating fluid and ultimately the load carrying capacity.

The effect of non-linear factor ($\kappa$) of non-Newtonian lubricant at Reynolds number of $Re = 8314$ and different values of eccentricity ratio on the friction
coefficient variable is shown in Fig. 13. It can be discerned from the figure that at any eccentricity ratio and Re value, an increase in the non-linear factor \( k \) increases the friction coefficient variable as \( k \) value reaches to 1. The variation in the friction coefficient variable at eccentricity ratio \( c = 0.8 \) with specific value of \( Re \) is minimum as compared to other values of the eccentricity ratio. The physical reason for this observation is that at the same condition load capacity is higher than at other eccentricity ratios.

Fig. 14 shows the plot of rise in temperature variable as a function of non-linear factor \( k \) of non-Newtonian lubricant at Reynolds number of \( Re = 8314 \) for different values of eccentricity ratio. The temperature variable initially at lower \( k \) values has the increasing trend and at higher \( k \) values it becomes nearly constant. The temperature variable is the function of frictional force \( (F_r) \) and bearing oil flow \( (Q_z) \). This increasing trend is more predominant at lower values of eccentricity ratios.

Effect of non-linear factor \( k \) on the bearing oil flow when eccentricity ratio \( c \) at particular value of \( Re \) is taken as a parameter can be studied from Fig. 15. For any value of eccentricity ratio, oil flow decreases with increase of \( k \) but variation in oil flow with increment of non-linear factor is negligible. For any value of \( k \), an increase of eccentricity ratio tends to increase the bearing oil flow. This observation is due to the fact that an increase of eccentricity ratio \( c \) creates higher oil film pressure gradient in the axial direction and hence higher bearing oil flow.

6. Conclusions

The linearized turbulence theory proposed by Ng and Pan [4] with cubic shear stress law of non-Newtonian lubrication has been successfully employed, with a finite element technique to compute and calculate the static performance characteristics of the finite hydrodynamic journal bearing under non-Newtonian lubrication in the turbulent flow regime. The following conclusions are evident from the analysis -

1. At particular value of the Reynolds number and non-linear factor of non-Newtonian lubricant, favors the increment in attitude angle, exhibiting better wedge effect of the oil film.
2. The combined effect of turbulent flow and non-Newtonian lubricant is to reduce the values of Sommerfield number as both Reynolds number and non-linear factor increases. This effect gives better opportunity in achieving the equilibrium condition at different shaft positions.
3. The load carrying capacity reduces as the values of non-linear factor \( k \) increases at constant value of Reynolds number \( (Re = 3326) \). On the other hand, the effect of increment in Reynolds number \( (Re) \) is to increase the load capacity when the non-linear factor is taken as constant. In general, the combination of turbulence and non-Newtonian lubrication exhibit a better load capacity than a Newtonian fluid and turbulent flow.
4. The effect of increase in non-linear factor is to reduce the friction coefficient variable, when the Reynolds number is taken as constant. On the other hand, reduction in the friction coefficient variable is observed as Reynolds number increases, when the non-linear factor is taken as constant. Like the condition of laminar and Newtonian lubrication, the combined effect also manifests a beneficial effect in frictional characteristics of finite bearing.
5. The variation of rise in temperature variable is also significant on the selection of combined mode of turbulence and non-Newtonian lubricant. The values of rise in temperature variable \( (\Delta T) \) are higher than those of laminar and Newtonian condition. This is due to the fact that as the Reynolds number increases, the turbulence in flow also increases and this adds to temperature variation.
6. The variation of oil flow of the bearing with the change in Reynolds number of flow and non-linear factor of non-Newtonian lubricant follows the similar trend of that of laminar and Newtonian flow condition. However, an increase of Reynolds number \( (Re) \) and non-linear factor \( (k) \) causes an insignificant reduction in oil flow at any value of eccentricity ratio. But the effect of increase of eccentricity ratio is to increase oil flow at different values of Reynolds number and non-linear factor.
7. The variation of non-linear factor \( (k) \) with Reynolds number affects the rheological performance of the finite journal bearing quite significantly.

Nomenclature

\[
\begin{align*}
\epsilon & \text{ radial clearance} \\
E & \text{ bearing eccentricity} \\
r & \text{ radius of journal} \\
L & \text{ length of bearing} \\
D & \text{ diameter of journal} \\
\bar{u} & \text{ tangential velocity} \\
\bar{v} & \text{ radial velocity} \\
\bar{w} & \text{ axial velocity} \\
\bar{p} & \text{ fluid film pressure} \\
\mu & \text{ absolute viscosity of lubricant} \\
h & \text{ oil film thickness} \\
\bar{r} & \text{ radial coordinate measured from journal center} \\
\bar{z} & \text{ axial coordinate measured from bearing center}
\end{align*}
\]
Nondimensional parameters

- \( p \) fluid film pressure
- \( \tau_e \) effective shear stress (eq. 1)
- \( k \) constant = 0.4 (eq. 2)
- \( \delta_e \) constant = 10.7 (eq. 2)
- \( Z_e \) non-dimensional distance measured from the nearest wall (eq. 2)
- \( \xi \) radial coordinate measured from the surface of journal
- \( \varepsilon \) or \( ep \) eccentricity ratio = \( e / c \)
- \( \phi \) attitude angle (deg.)
- \( r \) radial coordinate = \( \varphi / c \)
- \( k \) non-linear factor (eq. 6)
- \( \dot{\gamma} \) shear strain rate (eq. 7)
- \( \mu_e \) apparent viscosity (eq. 8)
- \( \tau \) shear stress defined in non-Newtonian law
- \( S \) Sommerfield number
- \( z \) axial coordinate measured from the mid plane of the bearing
- \( \theta_r \) Reynolds boundary, extent of positive pressure lubricant film of the bearing
- \( W_r \) oil film force along the line of centers
- \( W_\theta \) oil film force perpendicular to the line of centers
- \( W \) resultant load carrying capacity
- \( f (R / C) \) friction coefficient variable
- \( \Delta T \) rise in temperature variable
- \( N_i^p \) parabolic shape functions for velocity at node \( i \)
- \( \nu \) (\( \varepsilon / \nu \)) eddy diffusivity or turbulent viscosity coefficient (eq. 2)
- \( R_i \) clearance ratio = \( (R / C) \)
- \( R_e \) Reynolds number = \( (\rho \omega Rc / \mu) \)
- \( u, v, w \) velocities in \( \theta, r, z \) directions respectively.
- \( Q_x \) bearing oil flow
- \( F_f \) frictional force

Matrices

- \( [K^v] \) sub-matrix for viscous terms for element equation
- \( [K^r] \) sub-matrix for continuity equation terms for element equation
- \( [K^p] \) sub-matrix for pressure terms for element equation

References


