Effect of Magnetohydrodynamics and Couple Stress on Steady and Dynamic Characteristics of Plane Slider Bearing

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(Manuscript received 31 August 2015; accepted 25 January 2016; published 15 March 2016)

In this paper, an attempt has been made to study the effects of MHD and couple stress fluid lubricants on the steady state and dynamic behaviour of plane slider bearings by considering the squeezing action. The modified Reynolds equation is derived for the plane slider bearing with squeeze effects for the MHD and couple stress fluid. The dynamic stiffness and damping characteristics of one-dimensional MHD and couple stress plane slider bearings are computed. The effect of MHD and couple stress fluid lubricant improves the steady state and dynamic stiffness and damping characteristics of the plane slider bearings.

Keywords: squeeze film, magnetohydrodynamics, couple-stress fluid, steady and dynamic characteristics

1. Introduction

The MHD lubrication has been developed because the application of electric and magnetic fields increases the load carrying capacity of the liquid metal lubricated bearing. Last few years many types of magnetohydrodynamic (MHD) bearings have been investigated. The MHD lubrication in a slider bearing has been investigated by Agrawal [1] and found that for open-circuit case, load capacity increases due to inertia at small Hartmann number and decreases at large Hartmann number. Kuzma [2] presented the magnetohydrodynamic journal bearing. It is shown that the load carrying capacity is increased by the application of a magnetic field. Magneto-hydrodynamic couple stress squeeze film lubrication of circular stepped plates was studied by Naduvnamani et al., [3]. They observed that the effect of applied magnetic field on the squeeze film lubrication between circular stepped plates with couple stress fluids is to increase the load-carrying capacity significantly and to delay the time of approach considerably as compared to the corresponding non-magnetic case. Dynamic characteristics for wide magneto-hydrodynamic slider bearings with a power-law film profile were presented by Lin [4]. He displayed that, comparing with the non-conducting-fluid power-law film-shape bearing, the MHD bearing provides an increase in the load capacity, and the stiffness and damping coefficients. Comparing with the MHD inclined-plane slider bearing, the MHD parabolic-film

bearing signifies an improvement in the steady performances and the dynamic characteristics. Syeda Tasneem Fathima et al., [5] presented the analysis of Hydromagnetic squeeze films between porous rectangular plates with couple stress fluids. These authors predicted that effect of couple stresses on the MHD squeeze film lubrication between porous rectangular plates is to increase the load carrying capacity significantly and to delay the time of approach as compared to the corresponding non-magnetic case and Newtonian case.

Advances in technology and the severe operational requirements of machine necessitate the development of improved lubricants to ensure smooth operation. Generally the viscosity of the lubricating oils decreases with temperature. For operation under high speeds and heavy loads, oils containing high molecular weight polymers as viscosity index improvers are used to minimize viscosity variation with temperature. Conventionally, the prediction of squeeze film motion assumes that the lubricant behaves as a Newtonian viscous fluid. However, experimental results show that the addition of small amounts of long chained additives to a Newtonian fluid minimizes the sensitivity of the lubricant to change in shear rate and provides beneficial effects on the load-carrying and frictional characteristics [6-7]. Moreover, a base oil blended with additives can stabilize the behaviour of lubricants in elastohydrodynamic contacts and reduce friction and surface damage [8]. To describe the rheological

behaviour of this kind of non-Newtonian lubricant, many microcontinuum theories have been generated [9-10]. The Stokes theory [11] is the simplest generalization of the classical theory of fluids which allows for polar effects such as presence of couplestresses, body couples and non-symmetric tensors. This couplestress fluid is a special case of a non-Newtonian fluid and is intended to take account of particle size effects. On the basis of the microcontinuum theory, this study is intended to predict the effects of couplestresses on the pure squeeze-film behaviour of a finite bearing subject to a time-dependent load. The modified Reynolds equation is obtained by using the Stokes equations to account for the influence of couplestresses resulting from the lubricant blended with various additives. Several types of couplestress bearings have been investigated [12-17].

Recently researchers Lin et al., [18] are analyzed the Dynamic stiffness and damping characteristics of one-dimensional magneto-hydrodynamic inclined-plane slider bearings. It will be shown that effect of applied magnetic field is to increase the bearing MHD steady load carrying capacity, the MHD dynamic stiffness and the MHD damping coefficient. In this paper, an attempt has been made to study the effects of MHD and couple stress fluid lubricants on the steady state and dynamic behaviour of plane slider bearings by considering the squeezing action and the results are compared with the MHD case studied by Lin et al. [18], and the results are presented in Table 1. The present results are in excellent agreement with the earlier studies Lin et al. [18]. To the

best of the authors' knowledge, no studies are been found in the literatures on MHD steady and dynamic characteristics of plane slider bearings with couple stress fluids. This motivates the present study, where the main objective is to examine the effect of MHD and couple stress on steady and dynamic characteristics of plane slider bearings.

2. Mathematical formulation and solution of the problem

The geometry of the problem under consideration is shown in Fig. 1, in which lower surface have pure tangential sliding motion relative to the other surface. The film thickness in the entry region is h_1 and in the exit region is h_m . The two parallel plates of distance h apart are approaching each other with a normal velocity V ($= \partial h / \partial t$). The uniform magnetic field B_0 is applied perpendicular to the two plates. The basic equations of motion for the steady laminar couple stress fluid flow in the film region in the presence of applied magnetic field are given by

$$\mu \frac{\partial^2 u}{\partial y^2} - \eta \frac{\partial^4 u}{\partial y^4} - \sigma B_0^2 u = \frac{\partial p}{\partial x} + \sigma E_z B_0 \quad (1)$$

$$\frac{\partial p}{\partial y} = 0 \quad (2)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3)$$

Table 1 MHD and couple stress static and dynamic characteristics W_s^* , K_d^* , D_d^* and comparison with the case studied by Lin et al., [18] with $\delta = 1.5$

Lin et al., [18] analysis			Present analysis				
			$l^* = 0$		$l^* = 0.25$		
M_0	$h_m^* = 1$	$h_m^* = 1.5$	$h_m^* = 1$	$h_m^* = 1.5$	$h_m^* = 1$	$h_m^* = 1.5$	
W_s^*	0	0.158017	0.0706415	0.158003	0.0706352	0.169398	0.0734438
	4	0.260152	0.136437	0.260153	0.136437	0.287065	0.147749
	8	0.425999	0.236413	0.426003	0.236415	0.507274	0.278785
K_d^*	0	0.295358	0.0988893	0.295332	0.0988806	0.332989	0.105628
	4	0.401149	0.148891	0.401151	0.148892	0.462482	0.164052
	8	0.593939	0.23806	0.593945	0.238062	0.724509	0.284157
D_d^*	0	0.210689	0.0941886	0.21067	0.0941803	0.225864	0.097925
	4	0.34687	0.181916	0.346871	0.181916	0.382754	0.196999
	8	0.567999	0.315217	0.568004	0.31522	0.676366	0.371714

Fig. 1 Plane slider bearing

$$\int_{y=0}^h (E_z + B_0 u) dy = 0 \quad (4)$$

The film thickness for the inclined plane slider bearing can be written as

$$h(x, t) = h_m(t) + h_s(x) \quad (5a)$$

$$h_s(x) = d \left(1 - \frac{x}{L} \right) \quad (5b)$$

where $h_m(t)$ is the minimum film thickness, d is the difference of steady inlet-outlet thickness.

$$f(h, l, M_0) = \begin{cases} \frac{6h_m^2 h^2}{\mu l M_0^2} \left\{ \frac{A^2 - B^2}{\frac{A^2}{B} \tanh \frac{Bh}{2l} - \frac{B^2}{A} \tanh \frac{Ah}{2l}} - \frac{2l}{h} \right\} & \text{for } 4M_0^2 l^2 / h_m^2 < 1 \\ \frac{6h_m^2 h^2}{\mu l M_0^2} \left\{ \frac{2(\cosh(h/\sqrt{2}l) + 1)}{3\sqrt{2} \sinh(h/\sqrt{2}l) - h/l} - \frac{2l}{h} \right\} & \text{for } 4M_0^2 l^2 / h_m^2 = 1 \\ \frac{6h_m^2 h^2}{\mu l M_0^2} \left\{ \frac{M_0 (\cos B_1 h + \cosh A_1 h)}{h_2 (A_2 \sin B_1 h + B_2 \sinh A_1 h)} - \frac{2l}{h} \right\} & \text{for } 4M_0^2 l^2 / h_m^2 > 1 \end{cases} \quad (11)$$

$$A_2 = (B_1 - A_1 \cot \theta), \quad B_2 = (A_1 + B_1 \cot \theta)$$

Using the following non-dimensional quantities

$$x^* = \frac{x}{L}, \quad h^*(x^*) = \frac{h(x)}{h_m}, \quad h_s^*(x^*) = \frac{h_s(x)}{h_m},$$

$$\delta = \frac{\delta}{h_m}, \quad P^* = \frac{p^* h_m^2}{\mu U L},$$

$$l^* = \frac{2l}{h_m}, \quad t^* = \frac{t}{h_m} \quad \text{in equation (10), the non-dimensional MHD and couplestress Reynolds equation is expressed as}$$

The relevant boundary conditions are

At the upper surface $y = h$

$$u = 0, \quad \frac{\partial^2 u}{\partial y^2} = 0, \quad v = \frac{\partial h}{\partial t} \quad (6)$$

At the lower surface $y = 0$

$$u = U, \quad \frac{\partial^2 u}{\partial y^2} = 0, \quad v = 0 \quad (7)$$

Solving equation (1) using the boundary conditions (6), (7) and the condition (4), the velocity component is expressed as

$$u = -\frac{U}{2} g_1 - \frac{h_m^2 h}{2l \mu M_0^2} \frac{\partial p}{\partial x} g_2 \quad (8)$$

Where

$$g_1 = g_{11} - g_{12}, \quad g_2 = g_{13} - g_{14} \quad \text{for } 4M_0^2 l^2 / h_m^2 < 1 \quad (9a)$$

$$g_1 = g_{21} - g_{22}, \quad g_2 = g_{23} - g_{24} \quad \text{for } 4M_0^2 l^2 / h_m^2 = 1 \quad (9b)$$

$$g_1 = g_{31} - g_{32}, \quad g_2 = g_{33} - g_{34} \quad \text{for } 4M_0^2 l^2 / h_m^2 > 1 \quad (9c)$$

Hartmann number $M_0 = B_0 h_m (\sigma/\mu)^{1/2}$ and Couple stress parameter $l = \sqrt{\eta/\mu}$

The associated relations in equations (9a), (9b) and (9c) are given in Appendix A.

Substituting for u in to equation (3) and using the boundary conditions (6) and (7), the MHD and couplestress dynamic Reynolds equation is derived

$$\frac{\partial}{\partial x} \left\{ f(h, l, M_0) \frac{\partial p}{\partial x} \right\} = 6U \frac{\partial h}{\partial x} + 12 \frac{\partial h}{\partial t} \quad (10)$$

where

$$\frac{\partial}{\partial x^*} \left\{ f(h^*, l^*, M_0) \frac{\partial P^*}{\partial x^*} \right\} = 6 \frac{d\{\delta(1-x^*)\}}{dx^*} + 12 \frac{\partial h^*}{\partial t^*} \quad (12)$$

Where

$$h^*(x^*, t^*) = h_m^*(t^*) + h_s^*(x^*) \quad (13a)$$

$$h_s^*(x^*) = \delta(1-x^*) \quad (13b)$$

$$f^*(h^*, l^*, M_0) = \begin{cases} \frac{12h^{*2}}{l^*M_0^2} \left\{ \frac{(A^{*2} - B^{*2})}{\frac{A^{*2}}{B^*} \tanh \frac{B^*h^*}{l^*} - \frac{B^{*2}}{A^*} \tanh \frac{A^*h^*}{l^*}} - \frac{l^*}{h^*} \right\} & \text{for } M_0^2 l^{*2} < 1 \\ \frac{12h^{*2}}{l^*M_0^2} \left\{ \frac{1 + \cosh(\sqrt{2}h^*/l^*)}{(3/\sqrt{2}) \sinh(\sqrt{2}h^*/l^*) - (h^*/l^*)} - \frac{l^*}{h^*} \right\} & \text{for } M_0^2 l^{*2} = 1 \\ \frac{12h^{*2}}{l^*M_0^2} \left\{ \frac{M_0(\cos B_1^*h^* + \cosh A_1^*h^*)}{A_2^* \sin B_1^*h^* + B_2^* \sinh A_1^*h^*} - \frac{l^*}{h^*} \right\} & \text{for } M_0^2 l^{*2} > 1 \end{cases} \quad (14)$$

$$A^* = \left\{ \frac{1 + (1 - l^{*2}M_0^2)^{1/2}}{2} \right\}^{1/2}, \quad B^* = \left\{ \frac{1 - (1 - l^{*2}M_0^2)^{1/2}}{2} \right\}^{1/2}$$

$$A_1^* = \sqrt{2M_0/l^*} \cos(\theta^*/2), \quad B_1^* = \sqrt{2M_0/l^*} \sin(\theta^*/2), \quad \theta^* = \tan^{-1}(\sqrt{l^{*2}M_0^2 - 1})$$

$$A_2^* = (B_1^* - A_1^* \cot \theta^*), \quad B_2^* = (A_1^* + B_1^* \cot \theta^*)$$

Integrating equation (12) w. r. t. x^* twice, and using the pressure boundary conditions $P^* = 0$ at $x^* = 0, 1$ gives the MHD and couplestress film pressure

$$P^* = 6G_1(0, x^*) + 12V^*G_2(0, x^*) - \left\{ \frac{6G_1(0, 1) + 12V^*G_2(0, 1)}{G_3(0, 1)} \right\} G_3(0, x^*) \quad (15)$$

where $V^* = dh_m^*/dt^*$ is the non-dimensional squeezing velocity, and

$$G_1(0, x^*) = \int_{x^*=0}^{x^*} \frac{\delta(1-x^*)}{f^*(h^*, l^*, M_0)} dx^*, \quad G_1(0, 1) = \int_{x^*=0}^1 \frac{\delta(1-x^*)}{f^*(h^*, l^*, M_0)} dx^*$$

$$G_2(0, x^*) = \int_{x^*=0}^{x^*} \frac{x^*}{f^*(h^*, l^*, M_0)} dx^*, \quad G_2(0, 1) = \int_{x^*=0}^1 \frac{x^*}{f^*(h^*, l^*, M_0)} dx^*$$

$$G_3(0, x^*) = \int_{x^*=0}^{x^*} \frac{1}{f^*(h^*, l^*, M_0)} dx^*, \quad G_3(0, 1) = \int_{x^*=0}^1 \frac{1}{f^*(h^*, l^*, M_0)} dx^*$$

The load per unit width is given by

$$w = b \int_0^L p dx$$

The non dimensional load carrying capacity is given by

$$\begin{aligned} W^* &= \frac{wh_m^2}{\mu UL^2b}, = \int_{x^*=0}^1 P^* dx^* \\ W^* &= 6 \int_0^1 \int_{x^*=0}^{x^*} \frac{\delta(1-x^*)}{f^*(h^*, l^*, M_0)} dx^* dx^* + 12V^* \int_0^1 \int_{x^*=0}^{x^*} \frac{x^*}{f^*(h^*, l^*, M_0)} dx^* dx^* \\ &\quad - \left\{ \frac{6G_1(0, 1) + 12V^*G_2(0, 1)}{G_3(0, 1)} \right\} \int_0^1 \int_{x^*=0}^{x^*} \frac{1}{f^*(h^*, l^*, M_0)} dx^* dx^* \end{aligned} \quad (16)$$

By letting the minimum film height be constant and letting squeezing velocity be zero in equations (15) and (16), the dimensionless steady film pressure P_s^* and the dimensionless steady load carrying capacity W_s^* is given by

$$P_s^* = 6G_1(0, x^*) - 6 \left\{ \frac{G_1(0, 1)}{G_3(0, 1)} \right\} G_3(0, x^*) \quad (17)$$

$$W_s^* = 6 \int_0^1 \int_0^{x^*} \frac{\delta(1-x^*)}{f^*(h^*, l^*, M_0)} dx^* dx^* - 6 \left\{ \frac{G_1(0, 1)}{G_3(0, 1)} \right\} \int_0^1 \int_0^{x^*} \frac{1}{f^*(h^*, l^*, M_0)} dx^* dx^* \quad (18)$$

Differentiating MHD film force partially with respect to dimensionless minimum film thickness and thereafter taking the value under steady state gives the dimensionless linear dynamic stiffness (K_d^*) is given by

$$\begin{aligned} K_d^* &= - \left(\frac{\partial W_s^*}{\partial h_m^*} \right) \\ &= -6 \int_0^1 \int_0^{x^*} h_s^*(-1) f^{*-2} \frac{\partial f^*(h^*, l^*, M_0)}{\partial h_m^*} dx^* dx^* \\ &\quad + 6G \int_0^1 \int_0^{x^*} \frac{1}{f^*(h^*, l^*, M_0)} dx^* dx^* \\ &\quad + 6 \left\{ \frac{G_1(0, 1)}{G_3(0, 1)} \right\} \int_0^1 \int_0^{x^*} (-1) f^{*-2} \frac{\partial f^*(h^*, l^*, M_0)}{\partial h_m^*} x^* dx^* \\ G &= \frac{G_3(0, 1) \left(\int_0^1 h_s^*(-1) f^{*-2} \frac{\partial f^*(h^*, l^*, M_0)}{\partial h_m^*} dx^* \right) - G_1(0, 1) \left(\int_0^1 (-1) f^{*-2} \frac{\partial f^*(h^*, l^*, M_0)}{\partial h_m^*} dx^* \right)}{\{G_3(0, 1)\}^2} \end{aligned} \quad (19)$$

where

$$\frac{\partial f^*(h^*, l^*, M_0)}{\partial h_m^*} = \frac{12}{l^* M_0^2} \left\{ \left(\frac{2h^* \xi_1 - h^{*2} \xi_2}{\xi_1^2} \right) - l^* \right\} \quad (20)$$

$$\xi_1 = \xi_{11}, \quad \xi_2 = \xi_{12} \quad \text{for } M_0^2 l^{*2} < 1 \quad (21a)$$

$$\xi_1 = \xi_{21}, \quad \xi_2 = \xi_{22} \quad \text{for } M_0^2 l^{*2} = 1 \quad (21b)$$

$$\xi_1 = \xi_{31}, \quad \xi_2 = \xi_{32} \quad \text{for } M_0^2 l^{*2} > 1 \quad (21c)$$

The associated relations in equations (21a), (21b) and (21c) are given in Appendix A.

Differentiating MHD film force partially with respect to dimensionless squeezing velocity and thereafter taking the value under steady state gives the dimensionless linear damping coefficient (D_d^*) is given by

$$\begin{aligned} D_d^* &= - \left(\frac{\partial W_s^*}{\partial V^*} \right) \\ &= -12 \int_0^1 \int_0^{x^*} \frac{x^*}{f^*(h^*, l^*, M_0)} dx^* dx^* + \frac{12G_2(0, 1)}{G_3(0, 1)} \int_0^1 \int_0^{x^*} \frac{1}{f^*(h^*, l^*, M_0)} dx^* dx^* \end{aligned} \quad (22)$$

3. Results and discussion

The dynamic characteristics of plane slider bearings lubricated with an electrically conducting couple stress fluid with transverse magnetic field are analysed. Both the steady state performance and the dynamic characteristics are analysed. The present results are compared with one dimensional magneto-hydrodynamic inclined slider bearings studied by Lin et al. [18]. The dynamic characteristics of plane slider bearings lubricated with an electrically conducting couple stress

fluid with transverse magnetic field are analysed with respect to the following various non-dimensional parameters:

$M_0 = 0, 4, 8, l^* = 0.025, 0.5, \delta = 0.1 \text{ to } 3.5, h_m^* = 0.4 \text{ to } 1.0$.

3.1. Steady film pressure

Figure 2 depicts the variation of non-dimensional steady film pressure, P_s^* with x^* for different values of Hartmann number M_0 and the couple stress parameter l^* . It is observed that, P_s^* increases for increasing values of Hartmann number M_0 and couple stress parameter l^* . The

variation of non-dimensional maximum steady film pressure $P_s^*(M)$ with non-dimensional minimum film height h_m^* for different values of M_0 and l^* is shown in Fig. 3. It is observed that the non-dimensional maximum steady film pressure decreases with increasing values of non-dimensional minimum film height, further the non-dimensional maximum steady film pressure

Fig. 2 Variation of steady film pressure P_s^* with x^* for different values of M_0 and l^* with $h_m^* = 1$, $\delta = 1.5$

Fig. 3 Variation of steady maximum film pressure $P_s^*(M)$ with h_m^* for different values of M_0 and l^* with $\delta = 1.5$

Fig. 4 Variation of steady maximum film pressure $P_s^*(M)$ with δ for different values of M_0 and l^* with $h_m^* = 1$

increases with increasing values of Hartmann number M_0 and couplestress parameter l^* . Figure 4 shows the variation of steady maximum film pressure $P_s^*(M)$ with δ for different values of M_0 and l^* . It is observed that the steady maximum film pressure $P_s^*(M)$ increases with increasing values of δ until maximum is obtained, there after decreases with increasing values of δ .

3.2. Steady load carrying capacity

The variation of non-dimensional steady load carrying capacity W_s^* with δ for different values of M_0 and l^* is shown in Fig. 5. It is observed that the non-dimensional steady load carrying capacity W_s^* increases with increasing values of δ until maximum is obtained, there after decreases with increasing values of δ . Figure 6 shows the variation of non-dimensional steady load carrying capacity W_s^* with non-dimensional minimum film height h_m^* for different values of M_0 and l^* . It is observed that the non-dimensional steady load carrying capacity W_s^* decreases with increasing values of h_m^* .

Fig. 5 Variation of steady load carrying capacity W_s^* with δ for different values of M_0 and l^* with $h_m^* = 1$

Fig. 6 Variation of steady load carrying capacity W_s^* with h_m^* for different values of M_0 and l^* with $\delta = 1.5$

3.3. Dynamic stiffness

Figure 7 depicts the variation of Variation of dynamic stiffness K_d^* with δ for different values of M_0 and l^* . It is observed that the dynamic stiffness increases with increasing values of δ until maximum is obtained, there after decreases with increasing values of δ . Figure 8 shows the variation of dynamic stiffness K_d^* with non-dimensional minimum film height h_m^* for different values of M_0 and l^* . It is observed that the dynamic stiffness decreases with increasing values of h_m^* .

3.4. Damping coefficient

Figure 9 depicts the variation of Variation of damping coefficient D_d^* with δ for different values of M_0 and l^* . It is observed that the damping coefficient decreases with increasing values of δ . Figure 10 shows the variation of damping coefficient D_d^* with non-dimensional minimum film height h_m^* for different values of M_0 and l^* . It is observed that the damping coefficient decreases with increasing values of h_m^* .

4. Conclusions

The effect MHD on couple stress fluid lubricated inclined slider bearing is presented. The modified

Fig. 7 Variation of dynamic stiffness K_d^* with δ for different values of M_0 and l^* with $h_m^* = 1$

Fig. 8 Variation of dynamic stiffness K_d^* with h_m^* for different values of M_0 and l^* with $\delta = 1.5$

Reynolds equation is derived using Stokes micro-continuum theory and the magneto-hydrodynamic thin film lubrication theory. From the results obtained the following conclusions can be drawn.

1. For the plane slider bearing, the non-dimensional steady film pressure, non-dimensional steady load carrying capacity, Dynamic stiffness and Damping coefficient increases with the increasing values of the Hartmann number M_0 and the couple stress parameter l^* as compared to the non magnetic case ($M_0 = 0$) and the Newtonian case ($l^* = 0$).
2. For the plane slider bearing, the non-dimensional maximum steady film pressure, non-dimensional steady load carrying capacity and Dynamic stiffness increases with increasing values of δ until maximum is obtained, there after decreases with increasing values of δ .
3. For the plane slider bearing, Damping coefficient decreases with increasing values of δ .
4. For the plane slider bearing, the non-dimensional steady film pressure, non-dimensional steady load carrying capacity, Dynamic stiffness and

Fig. 9 Variation of damping coefficient D_d^* with δ for different values of M_0 and l^* with $h_m^* = 1$

Fig. 10 Variation of damping coefficient D_d^* with h_m^* for different values of M_0 and l^* with $\delta = 1.5$

Damping coefficient decreases with the increasing values of non-dimensional minimum film height h_m^* .

Nomenclature

B	width of the bearing
B_0	applied magnetic field
d	inlet-outlet thickness difference ($h_1 - h_m$)
D_d^*	damping coefficient
F	frictional force
F^*	non-dimensional frictional force ($= -Fh_{m0}/\mu UL$)
F_s^*	non-dimensional steady frictional force
h	film thickness
h_1	inlet film thickness
h_1^*	non-dimensional inlet film thickness
h_m	outlet film thickness
h^*	non-dimensional film thickness ($= h/h_{m0}$)
K_d^*	dynamic stiffness
l	couple stress parameter (η/μ) ^{1/2}

l^*	non-dimensional couple stress parameter ($2l/h_{m0}$)
L	Bearing length
M_0	Hartmann number ($= B_0 h_{m0} (\sigma/\mu)^{1/2}$)
p	pressure in the film region
P^*	non-dimensional pressure ($= ph_{m0}^2/\mu UL$)
P_s^*	dimensionless steady film pressure
x, y	rectangular coordinates
x^*	non-dimensional rectangular coordinates ($x^* = x/L$)
u, v	velocity components in film region
w	load carrying capacity
W^*	non-dimensional load carrying capacity ($= -wh_{m0}^2/\mu UL^2$)
W_s^*	dimensionless steady load carrying capacity
δ	non-dimensional inlet-outlet thickness difference (d/h_{m0})
η	material constant characterizing couple stress
μ	viscosity coefficient
σ	electrical conductivity

Appendix A

$$g_{11} = \frac{B^2}{(A^2 - B^2)} \left\{ \frac{\sinh(Ah/l) - \sinh(Ay/l) - \sinh A(h-y)/l}{\sinh(Ah/l)} \right\} \quad (A1a)$$

$$g_{12} = \frac{A^2}{(A^2 - B^2)} \left\{ \frac{\sinh(Bh/l) - \sinh(By/l) - \sinh B(h-y)/l}{\sinh(Bh/l)} \right\} \quad (A1b)$$

$$g_{13} = \frac{B^2 \{ \sinh(Ah/l) - \sinh(Ay/l) + \sinh A(h-y)/l \}}{\sinh(Ah/l) \{ (B^2/A) \tanh(Ah/2l) - (A^2/B) \tanh(Bh/2l) \}} \quad (A1c)$$

$$g_{14} = \frac{A^2 \{ \sinh(Bh/l) - \sinh(By/l) + \sinh B(h-y)/l \}}{\sinh(Bh/l) \{ (B^2/A) \tanh(Ah/2l) - (A^2/B) \tanh(Bh/2l) \}} \quad (A1d)$$

$$A = \left[\frac{1 + \{ 1 - (4l^2 M_0^2 / h_{m0}^2) \}^{1/2}}{2} \right]^{1/2} \quad (A1e)$$

$$B = \left[\frac{1 - \{ 1 - (4l^2 M_0^2 / h_{m0}^2) \}^{1/2}}{2} \right]^{1/2} \quad (A1f)$$

$$g_{21} = \frac{\sinh \{ (y-h)/\sqrt{2l} \} + \sinh(y/\sqrt{2l}) - \sinh(h/\sqrt{2l})}{\sinh(h/\sqrt{2l})} \quad (A2a)$$

$$g_{22} = \frac{y \cosh \{ (y-h)/\sqrt{2l} \} + y \cosh(y/\sqrt{2l}) - h \coth(h/2\sqrt{2l}) \sinh(h/\sqrt{2l})}{2\sqrt{2l} \sinh(h/\sqrt{2l})} \quad (A2b)$$

$$g_{23} = \frac{y \sinh \{ (y-h)/\sqrt{2l} \} + y \sinh(y/\sqrt{2l}) - h \sinh(y/\sqrt{2l})}{6l \sinh(h/\sqrt{2l}) - \sqrt{2}h} \quad (A2c)$$

$$g_{24} = \frac{2 \cosh \left\{ (y-h)/\sqrt{2l} \right\} + 2 \cosh \left(y/\sqrt{2l} \right) - 2 \cosh \left(h/\sqrt{2l} \right) - 2}{3\sqrt{2} \sinh \left(h/\sqrt{2l} \right) - (h/l)} \quad (A2d)$$

$$g_{31} = \frac{\cosh A_1 y \cos B_1 (y-h) - \cos B_1 y \cosh A_1 (y-h)}{(\cosh A_1 h - \cos B_1 h)} \quad (A3a)$$

$$g_{32} = \frac{\cot \theta \{ \sinh A_1 y \sin B_1 (y-h) - \sin B_1 y \sinh A_1 (y-h) \} + (\cosh A_1 h - \cos B_1 h)}{(\cosh A_1 h - \cos B_1 h)} \quad (A3b)$$

$$g_{33} = \frac{\cot \theta \{ \sin B_1 y \sinh A_1 (y-h) + \sinh A_1 y \sin B_1 (y-h) \} + (\cos B_1 h + \cosh A_1 h)}{(B_1 - A_1 \cot \theta) \sin B_1 h + (A_1 + B_1 \cot \theta) \sinh A_1 h} \quad (A3c)$$

$$g_{34} = \frac{\cos B_1 y \cosh A_1 (y-h) + \cosh A_1 y \cos B_1 (y-h)}{(B_1 - A_1 \cot \theta) \sin B_1 h + (A_1 + B_1 \cot \theta) \sinh A_1 h} \quad (A3d)$$

$$A_1 = \sqrt{M_0 / lh_{m0}} \cos(\theta/2) \quad (A3e)$$

$$B_1 = \sqrt{M_0 / lh_{m0}} \sin(\theta/2) \quad (A3f)$$

$$\theta = \tan^{-1} \left(\sqrt{4l^2 M_0^2 / h_{m0}^2 - 1} \right) \quad (A3g)$$

$$\xi_{11} = \frac{(A^{*2}/B^*) \tanh(B^* h^*/l^*) - (B^{*2}/A^*) \tanh(A^* h^*/l^*)}{A^{*2} - B^{*2}} \quad (A4a)$$

$$\xi_{12} = \frac{(A^{*2}/l^*) \operatorname{sech}^2(B^* h^*/l^*) - (B^{*2}/l^*) \operatorname{sech}^2(A^* h^*/l^*)}{A^{*2} - B^{*2}} \quad (A4b)$$

$$\xi_{21} = (3/\sqrt{2}) \tanh(h^*/\sqrt{2}l^*) - (h^*/2l^*) \operatorname{sech}^2(h^*/\sqrt{2}l^*) \quad (A5a)$$

$$\xi_{22} = \operatorname{sech}^2(h^*/\sqrt{2}l^*) \{ (1/l^*) + (h^*/\sqrt{2}l^{*2}) \tanh(h^*/\sqrt{2}l^*) \} \quad (A5b)$$

$$\xi_{31} = \frac{(B^* - A^* \cot \theta^*) \sin B^* h^* + (A^* + B^* \cot \theta^*) \sinh A^* h^*}{M_0 (\cos B^* h^* + \cosh A^* h^*)} \quad (A6a)$$

$$\xi_{32} = \frac{2}{l^*} \left\{ \frac{1 + \cot \theta^* \sin B^* h^* \sinh A^* h^* + \cos B^* h^* \cosh A^* h^*}{(\cos B^* h^* + \cosh A^* h^*)^2} \right\} \quad (A6b)$$

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