A Constitutive Friction Law for Sheet-Bulk Metal Forming

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Friction has a significant effect on metal forming both in economic and technical terms. This is especially true for sheet-bulk metal forming, which is a combination of bulk metal forming and sheet metal forming. Contact loads of sheet forming processes are typically low to moderate. In contrast, very high contact loads occur in bulk metal forming. Therefore, a friction law, which is applicable for a broad range of contact loads, is necessary, if such a process has to be modelled accurately with the finite element method. A friction law fulfilling this demand is identified with the use of an elastic-plastic half-space model. The half-space model is used to determine the normal contact of rough surfaces and it is validated with experiments. In order to highlight the impact of the identified friction law, a typical sheet-bulk metal forming process is taken into account in the framework of the finite element method.

Keywords: constitutive friction law, simulation, half-space, sheet-bulk metal forming, finite element method, orbital forming

1. Introduction

The finite element method (FEM) is a widely used and well-established tool for the analysis, design and optimization of technical processes. As the available computing capacity rises, FEM becomes increasingly essential for the computation of complex interactions in engineering mechanics. Sheet-bulk metal forming (SBMF) is a prime example for such a complex problem. In SBMF metal sheets of a thickness ranging between 1 mm and 5 mm are formed by a bulk forming process [1]. The workpiece of a SBMF process is exposed to both low and high strains as well as locally varying 2- and 3-dimensional stress and strain states. Furthermore, the anisotropic material behaviour of the metal sheets impacts the resulting form of the product. Another point, which increases the complexity not only of SBMF, but of forming processes in general, is friction. For example, Groche [2] and Zhang [3] showed that the product quality and tool durability are affected by friction. The importance and influence of friction especially in SBMF was described by Merklein [4].

The precise representation of friction in FEM demands a mathematical formulation, which accurately describes the contact condition between the contacting partners. The selection of the appropriate friction law depends on the considered forming process. For instance, Coulomb’s law of friction setting the friction force proportional to the applied normal force is suited for sheet forming, but the law is mostly repealed, if elastic recovery occurs or the applied load becomes large. In contrast, Tresca’s law of friction sets the frictional shear stress direct proportional to the yield strength of the weaker material in contact, which makes it suitable for processes with large contact loads, for example bulk forming. However, the overestimation of the friction shear stress for low contact loads is a disadvantage of Tresca’s law.

A more advanced friction models has been proposed by Wanheim et al. who used the slip-line theory to determine a friction model which depends on the friction factor and the real contact of area [5]. Makinouchi and Ike performed FEM calculations in order to investigate the contact between tool and workpiece on a microscopic scale [6]. Furthermore, Wang et al. proposed a friction law which focuses on the friction in dry metal forming [7]. This model also has been validated with a newly developed tribometer.

However, as SBMF combines sheet metal forming with bulk metal forming, the local process loads vary greatly, which makes the choice of an appropriate friction law difficult. In addition, a SBMF process can occur in
several stages. The plastic smoothing of the workpiece can alter the initial contact condition for each stage, which seems to play an important role [8]. Consequently, the development of a suitable constitutive friction law (CFL), that takes into account the particular contact situations in SBMF as well as plastic smoothing, is of critical importance for an eligible numerical simulation of SBMF processes.

The determination of the CFL is performed numerically, because in-situ contact measurements are highly complex and the identification of tribological conditions in SBMF requires up to three different friction tests [9]. The accurate representation of the multi-scale conditions in SBMF requires up to three different friction tests [9]. Consequently, the identification of tribological conditions in SBMF requires up to three different friction tests [9]. Therefore, an elastic-plastic half-space model is chosen for the numerical determination as it only requires a discretization of the contacting surfaces and thereby presents the advantage that the numerical effort is highly reduced compared to standard FEM [10].

Kalker was one of the first who presented a half-space model [11]. The model is founded on the minimization of the complementary potential energy to solve a non-Hertzian elastic problem. The half-space model is also able to consider elastic-plastic contact by limiting the local contact pressure, as mathematical described by Hencky [12] and experimentally validated by Bowden and Tabor [13].

Half-space models can be used to research non-Gaussian rough surfaces as shown by Kim et al. [14], normal contact between a rough surface and a rigid sphere as shown by Tian and Bhushan [15] or the contact of fractal surfaces as shown by Willner [16].

The half-space model for the identification of the CFL is described in Sec. 2. The model is used to simulate the elastic-plastic contact of rough surfaces and compared with experiments in Sec. 3. That followed, Sec. 4 explains the concluded CFL, which is finally applied in an orbital forming process in Sec. 5.

2. Theory of the elastic-plastic half-space model

The basic principle of the half-space model are the Boussinesq potentials [17]. The Boussinesq solution relates the normal displacement \( u_x \) of a surface point at \((x, y)\) in dependency on the acting pressure \( p \) at \((x', y')\) with

\[
  u_x(x,y) = \frac{1-\nu^2}{\pi E} \int \frac{p(x', y') \, dx' \, dy'}{\sqrt{(x-x')^2 + (y-y')^2}}, \tag{1}
\]

where \( E \) is Young’s Modulus and \( \nu \) is Poisson’s ratio. Replacing \( E/(1-\nu^2) \) with the composite elastic modulus \( E' \) [18] allows to include the elasticity of both contact partners. The composite elastic modulus is given with

\[
  \frac{1}{E'} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}, \tag{2}
\]

where the indices 1 and 2 refer to each contact partner. The model also rests upon the numerical method of Tian and Bhushan [15] and evaluates the elastic deformation of the contacting surfaces with the minimization of the complementary potential energy based on a variational principle. This requires the discretization of the contact area of size \( L_x \times L_y \) of both contact partners into a mesh with \( N_x \times N_y = M \) rectangular elements each with size \( 2a \times 2b \), as shown in Fig. 1. If \( a \) and \( b \) are small, the contact pressure acting on an element may be assumed constant.

The superposition of the deformations due to all surface elements \( l \) that are in contact is applicable to evaluate the surface displacement in normal direction \( u_{z,l} \) of any surface element \( k \), since the Boussinesq solution provides a linear system of equations. The surface displacement is given by

\[
  u_{z,k} = \sum_{l=1}^{M} C_{zkl} p_l, \tag{3}
\]

where \( C_{zkl} \) is the influence coefficient matrix, which is defined as

\[
  C_{zkl} = \frac{1}{\pi E} \int_{-a}^{a} \frac{d\zeta d\eta}{\rho} \tag{4}
\]

with

\[
  \rho = \sqrt{(x_k - x_l - \zeta)^2 + (y_k - y_l - \eta)^2}. \tag{5}
\]

A solution of Eq. (4) was presented by Love [19]. The contact pressure in Eq. (3) is restricted to

\[
  p_l \geq 0 \quad l = 1, \ldots, M, \tag{6}
\]

because the load in normal direction at the surface must not be tensile. The gap equation is identified with the geometrical setting in Fig. 2. The equation describes the gap distance \( g_k \) at \( k \) with

\[
  g_k = h_{max} - h_k + u_{z,k} - u_0, \tag{7}
\]

where \( h_{max} \) is the maximum height coordinate of the discretized surface, \( h_k \) is the local height coordinate and \( u_0 \) is the global approach of the contact surfaces. Eq. (7) is subjected to the complementarity conditions which are

\[
  g_k = 0, \quad p_k > 0, \quad k \in I_0, \tag{8}
\]

\[
  g_k > 0, \quad p_k = 0, \quad k \notin I_0, \tag{9}
\]

where \( I_0 \) includes all elements which are in contact. In addition, the contact pressure distribution has to be in
balance with the globally applied pressure $p_{mean}$. The equilibrium of forces is given as

$$\sum_{l=1}^{M} p_l \cdot 2a \cdot 2b = p_{mean} \cdot L_x \cdot L_y.$$  \hspace{1cm} (10)

Allwood presented in [20] a comparison between different solution methods to solve the elastic contact problem, which is defined with Eqs. (3, 7, 10). Of the methods compared, the active set method in combination with the conjugate gradient method requires the least memory, gives a very precise solution and is fast in solving the elastic contact problem. For the same reasons, this method is also applied here.

The elastic contact model is easily extended to consider elastic-plastic contact by limiting the contact pressure with

$$p_l \leq H \quad l = 1, \ldots, M,$$  \hspace{1cm} (11)

where $H$ is the surface hardness of the elastic-plastic surface. The surface hardness depends on the yield stress $\sigma_y$ with $H = 2.8 \cdot \sigma_y$ according to Bowden and Tabor [13].

The consideration of elastic-plastic contact requires the decomposition of the surface displacement $u_{z,k}$ into a plastic part $u_{pl,z,k}$ and an elastic part $u_{el,z,k}$, which alters the gap function to

$$g_k = h_{max} - h_k + u_{pl,k} + u_{el,k} - u_0.$$  \hspace{1cm} (12)

The plastic deformation is evaluated after the determination of the purely elastic contact for $p_{mean}$. Surface elements with a contact pressure greater than the surface hardness are added to a ‘plastic set’ as it is described by Hauer [21]. The contact pressure of elements in the plastic set are limited by Eq. (11). Then, the deformation $u_{pl,k}$ due to the plastic set is evaluated. The gap equation Eq. (12) is updated with the new $u_{pl,k}$ and the evaluation of the elastic problem for $p_{mean}$ subtracted by the load caused by the plastic set is solved. This loop iteratively corrects $u_{pl,k}$ until the load equilibrium in Eq. (10) is converged.

This plasticity model reduces the local height of the surface by removing material. Therewith, the model is not volume conservative. In contrast, the plastic deformation of a metallic surface is normally performed with volume conservation. In addition, experimental research of Pullen and Williamson [22] show that the valleys of rough surfaces are evenly filled with the volume which is displaced due to the plastic deformation of the surface asperities. Therefore, the contact model includes an algorithm which evaluates the plastically removed material. This material is evenly distributed on the surface area which is not in contact to obtain the observation of Pullen and Williamson.

3. Experimental validation

Several experiments were carried out for the verification of the elastic-plastic half-space model [23]. The experimental setup is composed of a polished circular punch with a diameter of 8 mm, which is brought into normal contact with an EDT (Electric Discharge Texturing) treated metal sheet with a thickness of 2 mm. The contact interaction of these surfaces represents a typical contact interaction in SBMF. The material of the punch is a hardened steel with about 60 HRC (hardness Rockwell C). The metal sheet is made of DC04, which is a cold rolled steel strip. The material DC04 is defined with the standard EN 10139. The investigation of the normal contact was performed under dry condition with different nominal contact pressures $p_{mean}$, which ranged from 100 MPa to 600 MPa in steps of 100 MPa. Each normal contact was performed on 5 different samples. The surfaces of the sheets were measured before and after the experimental procedure with the Keyence VK-X105 laser microscope.

Each contact load was also performed with the half-space model. A square-shaped portion of each surface with $L_x = L_y = 2.11$ mm was used in the simulation. In order to perform the numerical investigation in reasonable time, the resolution of the surfaces has been interpolated to $N_x = N_y = 128$ elements. The surface of the punch was simulated solely elastic, whereas the surface of the sheet was considered elastic-plastic. Table 1 shows the material parameters and the centre-line average $R_a$ as well as the root mean square roughness $R_q$ of the initial surfaces, which were passed to the numerical model.

The plastic deformation of the sheet metal surface can be estimated on the basis of the change of both $R_a$ and $R_q$. The experimental results were also interpolated to a resolution with $N_x = N_y = 128$, which makes it able to compare the experimental results with the numerical results. The interpolation is similar to a low-pass filtering and alters $R_a$ and $R_q$ insignificantly.

The change of $R_a$ and $R_q$ of both the experiments and the numerical simulations are shown in Figs. 3 and 4, respectively. The results of the simulation coincide very well with the experimental results over the whole range.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>DC04</th>
<th>Hardened Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$</td>
<td>205000 MPa</td>
<td>$E_2 = E_1$</td>
</tr>
<tr>
<td>$\nu_1$</td>
<td>0.30</td>
<td>$\nu_2 = \nu_1$</td>
</tr>
<tr>
<td>$H$</td>
<td>800 MPa</td>
<td>$R_p = 0.102 \mu m$</td>
</tr>
<tr>
<td>$R_q$</td>
<td>1.267 $\mu m$</td>
<td>$R_q = 0.160 \mu m$</td>
</tr>
</tbody>
</table>
4. Constitutive Friction Law (CFL)

If two surfaces are brought into initial contact with a low to moderate nominal contact load, only their surface peaks are in actual contact. The contact area due to these surface asperities is called real contact area if hydrodynamic effects are neglected. The ratio \( A_{\text{real}}/A_0 \) is expressed as \( \alpha_c \) and can be estimated with the half-space model. Figure 5 shows \( \alpha_c \) in dependency on \( p_{\text{mean}} \), which was determined from the results of the numerical simulations.

For initial contact the parameter \( \alpha_c \) rises linearly in dependency on \( p_{\text{mean}} \) as this contact condition is elastic. If the load \( p_{\text{mean}} \) becomes larger than \( p_0 \), \( \alpha_c \) rises again linearly, since this contact condition is elastic-plastic again.

A local friction law of Tresca in \( A_{\text{real}} \) was assumed for the formulation of the constitutive friction law. For initial contact this gives a mathematical expression, which is similar to the findings of Shaw [24] and given by

\[
\tau_r = \mu \cdot p_{\text{mean}},
\]

(14)

where \( \tau_r \) is the resulting friction shear stress, \( k \) is the shear yield strength of the weaker material, \( m \) is the friction coefficient according to Tresca’s law and \( C_1 \) as well as \( n_1 \) are parameters, which are identified with the half-space model. If \( p_{\text{mean}} \) is low to moderate, Eq. (13) is linearly rising and similar to Coulomb’s law of friction which is defined as

\[
\tau_r = m \cdot k \cdot \alpha_c \cdot \sqrt{\frac{p_{\text{mean}} \cdot C_1}{H}} \cdot \alpha_{c}(p_h).
\]

(15)

In the case of un-/reloading higher friction stresses are transmitted, since \( A_{\text{real}} \) is increased. For this case, the friction law is

\[
\tau_r = m \cdot k \cdot \alpha_c \cdot \sqrt{\frac{p_{\text{mean}} \cdot C_1}{H \cdot \alpha_{c}(p_h)}} \cdot \alpha_{c}(p_h).
\]

(16)

Eq. (16) is similar to Eq. (13), but differs in the parameters \( C_2 \) and \( n_2 \), which are again identified with the half-space model. Furthermore, Eq. (16) takes into account the ratio \( A_{\text{real}}/A_0 \), which is determined for \( p_0 \) and denoted as \( \alpha_{c}(p_0) \).

5. Application of the CFL in a FEM simulation

In order to highlight the differences between
conventional friction laws, i.e., Coulomb’s law of friction and Tresca’s law of friction, and the CFL, a well-established SBMF process is used. It is an orbital forming process applied on sheet metal as it is explained in [25]. The software Simufact.forming 12.0.1 was used for the numerical simulation of the forming process. The software supports user subroutines written in FORTRAN programming language, which makes it able to implement the CFL into the FE-framework. The workpiece used in the process is a metal sheet with a thickness of 2 mm and a diameter of 100 mm. The sheet consists of the material DC04, which was also used for the verification of the half-space model. The material parameters are $E = 212000$ MPa and $\nu = 0.30$. In addition, a flow curve of DC04 is already provided by the FE-software, which was also used in this simulation and is shown in Fig. 6. The material is modelled isotropic, since the experimental outcome did not show significant anisotropy, and a von Mises yield criterion is considered. The forming force was set to 2000 kN and the tumbling angle $\phi$ is ranged between 0° and 1°. Due to the high contact loads the simulation can be conducted using Tresca’s law and $m = 0.1$ according to [25]. Due to the transition from sliding friction to rolling friction, the orbital forming process is also feasible with Coulomb’s law [26]. For this case the friction coefficient is $\mu = 0.1$.

The setup of the simulation is shown in Fig. 7. The sheet represents the deformable workpiece, which is arranged on top of the counterpunch and under the punch. The die encircles both punches as well as the sheet. Its purpose is to constrain the material flow of the sheet in radial direction. The punch starts in its original position with $\phi = 0°$. Four tumbling rotations are performed to angle the punch to $\phi = 1°$ followed by six tumbling rotations with a constant tumbling angle $\phi = 1°$. The highest contact loads are observed in the latter phase. Finally, four more tumbling rotations are needed to position the punch back to its original angle $\phi = 0°$. In order to retrace the deformation of the workpiece, Figs. 8 and 9 show the setup before and after the process, respectively.

Due to the change of $\phi$ and the circular motion of the punch, the acting contact varies greatly in dependency on the simulation progress. Therefore, it is difficult to highlight the differences across the whole simulation. However, the seventh tumbling rotation was used to compare the friction laws on the basis of the occurring contact forces, because the seventh tumbling rotation belongs to the process stage with the maximum contact loads.

Figure 10 shows the contact pressure (a) and the friction stress (b) on the workpiece surface given by the exertion of Coulomb’s law. The top limit of the contact pressure is about 3500 MPa. The resulting friction stress is around a tenth of the contact pressure as defined by Coulomb’s friction law. Figure 10(c) shows the corresponding yield stress. The flow stress increased from its initial value 178 MPa to around 600 MPa and the shear yield stress reaches 346 MPa. Therefore, the maximum friction stress calculated with Coulomb’s law is on the threshold to be valid.

Figure 11 shows the results of the same simulation, if Tresca’s law is used. The friction stress presented in Fig. 11(b) is up to 35 MPa and mostly constant across the contact area. The friction stress can be retraced with the yield stress given in Fig. 11(c). Furthermore, the maximum friction stress is now observable at the exterior perimeter of the disc. The estimated contact pressure in Fig. 11(a) is also significantly lower, if Tresca’s law is used and achieves a maximum of around 2050 MPa. As
the simulation is load-adjusted, this comes with an increase of the contact area.

Figure 12 presents the contact pressure (a) and the friction stress (b) given with the CFL implementation. The contact pressure is nearly equal compared with Tresca’s law, which is plausible as the contact loads are high and the CFL approaches Tresca’s law with rising contact load. For the same reason, the maximum of the friction stress calculated with the CFL does not differ much from the maximum of Tresca’s law. However, the distribution of the friction stress is quite different, which is exemplarily observable at the exterior perimeter of the workpiece. There, the friction stress with the CFL is less pronounced than the friction stress calculated with Tresca’s law.

The influence of the currently used friction law is also reflected by the true strain. Figure 13 shows the forming degree, which is equivalent to the true strain, at a cross section of the workpiece after the simulation with Coulomb’s law. The maximum forming degree is at the radius \( r = 10 \) mm with 2.10 and decreases with rising \( r \) to 0.70. The forming degree is low at the cavity, i.e. at \( r \geq 40 \) mm.

Figure 14 shows the forming degree at the same section, but with the use of Tresca’s law. The maximum forming degree is about 2.40 and located at the beginning of the cavity at \( r = 40 \) mm. Furthermore, local maxima are observable at the cross-section for \( r \approx 7 \) mm and \( r \approx 29 \) mm, whereas the region at \( r \approx 16 \) mm shows a local minimum.

The highest forming degree is obtained with the use of the CFL, as shown in Fig. 15. The forming degree is rising with the radius from 1.5 to 2.7 at the beginning of the cavity. The forming degree at the cavity decreases, but still shows the highest values of all three friction laws.

The forming degree is also in accordance with the observed friction stress. In general, a high friction stress inhibits material flow more than low friction stress. Since the simulation with Coulomb’s law shows the highest friction stress, the forming degree is the lowest values of the three simulations. Furthermore, the friction stress is high for \( r \geq 40 \) mm. This is also the region with the lowest forming degree. Analogously, the forming degree at low \( r \) is increased as this region exhibits lower friction stresses. In contrast, the simulations with Tresca’s law and the CFL result in significantly lower friction stresses which result in an increased forming degree, because the
material flow is less inhibited due to friction. As already stated, the maximum friction stress of the simulation with Tresca’s law is observed at the exterior perimeter of the disc. Compared with Tresca’s law the CFL yields at this region of the workpiece lower friction stresses which might be a reason for the increased forming degree for \( r \geq 40 \) mm with the CFL.

The strain hardening of the semi-finished products was also investigated by measuring the Vickers hardness as it was performed in [27]. A slightly modified version of this experiment for different load cases was also taken into account in [28] indicating qualitatively a similar outcome. Regions with a large Vickers hardness value can be interpreted as a region in which a large true strain occurred, since the hardness of the material rises with the true strain. The experiments show an increase of the Vickers hardness as the radius of the sheet increases. Therefore, a low Vickers hardness is observed at the centre of the workpiece and a high Vickers hardness is observed at the perimeter of the workpiece. Furthermore, the maximum of the Vickers hardness was measured at the beginning of the cavity, i.e. at \( r \approx 40 \) mm.

The comparison of the forming degree with Coulomb’s law friction law and the Vickers hardness of the experiments are conflicting, since the forming degree at \( r \geq 40 \) mm shows its minimum, although the Vickers hardness shows at this location the highest values. On the other hand, the forming degree with Tresca’s law and with the CFL coincides better with the Vickers hardness. Both simulations depict a region with a large forming degree at the cavity. Furthermore, the Vickers hardness experiments show decreased values for \( 10 \text{mm} \leq r \leq 40 \) mm which is similar to the results of Tresca’s law and the CFL. Since the forming degree with Tresca’s law shows a local maximum for \( r \approx 7 \) mm and \( r \approx 29 \) mm which was not observed by the Vickers hardness experiment, the CFL seems to reflect the strain hardening better.

6. Conclusion

The presented half-space model is able to take into account the multi-scale character of rough surfaces. It is a fast and efficient method, as only the contact surface has to be discretized. Although, more advanced models have been proposed, which take into account three dimensional plastic deformation, for example the one by Jacq [29], the simplistic realization of elastic-plastic contact of the here presented half-space model still enables it to accurately determine the surface deformation due to normal contact. The experimentally validated half-space model was used to determine the change of the real contact area in dependency on the contact load. The result is a mathematical expression, which expresses the ratio of the real contact area to the global contact area in dependency on the acting contact load. Assuming a local law of Tresca in the real contact area, a constitutive friction law similar to the findings by Shaw was determined, which is additionally able to take into account plastic smoothing of the contact surface. The constitutive friction law is ought to predict the contact interaction in the framework of SBMF. This subject considers local highly varying contact loads and incremental forming processes. These characteristics are reflected in an orbital forming process. It is shown that the contact loads, which are calculated with the CFL, differ from the numerical results acquired with the well-established friction law of Coulomb or the friction law of Tresca. Considering the forming degree, the CFL seems to reflect the contact interactions accurately and presents an improvement compared to the conventional friction laws. However, there are no experimental results available that focus on both orbital forming and friction. Hereafter, a direct comparison with the CFL method could not yet be carried out.

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References

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