Shear Stress Analysis of EHL Oil Films Based on Thermal EHL Theory
- Effect of Inlet Oil Temperature -

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Under the same EHL contact conditions as in the traction experiments carried out by Wedeven et al., the authors performed a non-Newtonian thermal EHL analysis. In the present analysis, the lubricating oil was assumed to behave as a Maxwell fluid. Lubricating oil is the automotive traction oil with the viscosity grade of ISO VG32. Input parameters are maximum Hertzian pressure $P_H=1.5$ GPa, entrainment velocity $u_e=10$ m/s, slide-roll ratio $\Sigma=0\%\sim4.0\%$ and inlet oil temperature $t_0=313$ K$\sim413$ K. The Eyring stress, the elastic shear modulus and the limiting shear stress which are needed to calculate the shear stress components were obtained from reference traction curves. The traction coefficients obtained by the numerical analysis agreed well with the measured values in traction experiments. Furthermore, it was found that the shear stress distribution is affected not only by the inlet oil temperature but also by the rolling-sliding conditions.

Keywords: shear stress, traction oil, thermal EHL, Maxwell model, inlet oil temperature, slide-roll ratio

1. Introduction

Using an automotive traction oil, the authors have examined the rolling contact fatigue life of bearing steel rollers with mirror-like smooth surfaces\textsuperscript{1,2).} And the results were compared with those obtained using a mineral oil with almost the same viscosity grade. In spite of full EHL conditions, the fatigue life with traction oil was shorter than that with mineral oil, and fatigue failures were apt to occur with increase in slip ratio. Assuming that the lubricating oil behaves as a Newtonian fluid, the authors also carried out numerical analysis based on the thermal EHL theory. Incidentally, Liu\textsuperscript{3)} and Kaneta\textsuperscript{4)} showed in their full non-Newtonian thermal EHL analyses that the traction coefficients at relatively low slide-ratios are clearly different from those in the Newtonian fluid model, and suggested that the non-Newtonian behavior of lubricating oil is an important factor to dominate the traction characteristics.

Wedeven et al. measured the traction coefficient with the same commercialized traction oil at a wide range of Hertzian pressures, oil temperatures and rolling speeds using the USCAR testing machine\textsuperscript{5)}. In this paper, a non-Newtonian thermal EHL analysis was performed for the same point EHL contact conditions as in the traction tests. In the analysis, the Maxwell model was applied as a non-Newtonian model. The calculated traction coefficients were compared with the experimental results, and the dependences of the shear stress distribution of EHL oil film on the inlet oil temperature and slide-roll ratio are clarified.

2. Procedure of numerical analysis

Wedeven et al. examined the traction properties at maximum Hertzian pressures between 0.5 GPa and 4.0 GPa, temperatures from 213 K to 413 K and rolling speeds up to 10 m/s\textsuperscript{5)}. The diameters of ball and disk are 20.64 mm and 102 mm, respectively.

Fig. 1 shows the computation domain of numerical analysis. The calculating grid consists of 129 nodes along the $X$ and $Y$ direction. 11 equidistant nodes are across the film. In each solid, 6 non-equidistant nodes along $Z_a$ or $Z_b$ direction are adopted.

In the analysis, it is assumed that the lubricating oil behaves as a non-Newtonian fluid, and the Maxwell model is adopted. The constitutive equations of the Maxwell model are expressed as

\[
\begin{align*}
\frac{\partial u}{\partial z} &= u \frac{\partial \tau_x}{\partial x} \frac{\tau_x}{\eta \tau_x} \sinh \left( \frac{\tau_x}{\tau_0} \right) \\
\frac{\partial v}{\partial z} &= v \frac{\partial \tau_y}{\partial y} \frac{\tau_y}{\eta \tau_y} \sinh \left( \frac{\tau_y}{\tau_0} \right)
\end{align*}
\]
where $\tau = \sqrt{\tau_x^2 + \tau_y^2}$.

The shear stress $\tau_x$, $\tau_y$, the viscosity $\eta$, the elastic shear modulus $G_e$, and the Eyring stress $\tau_0$ are functions of pressure and temperature.

Evans and Johnson derived the Eyring stress from traction experiments\(^{6}\). In the present study, the value was obtained by the same method. The relationship of Eyring stress with average Hertzian pressure $P_{av}$ and inlet oil temperature $t_0$ is described as

\[
\tau_0 = 0.9395 \times 10^7 \times \exp(0.1671 \times 10^4 P_{av} + 11.20 \times 10^{-9} t_0)
\]  

(2)

The elastic shear modulus was calculated using the method proposed by Muraki and Dong\(^{7}\). The elastic shear modulus is represented in the following empirical equation

\[
G_e = 2.446 \times 10^7 \times \exp(0.1944 \times 10^4 P_{av} - 30.25 \times 10^{-8} t_0)
\]  

(3)

Owing to the estimation of viscosity, the following equation based on the free volume theory\(^{8}\) was adopted.

\[
\log_{10} \eta = 7 - \frac{11.3(t - t_{av})(206/t_{av})}{35.9 + (t - t_{av})(206/t_{av})}
\]  

(4)

where the viscoelastic transition temperature $t_{av}$ is described as

\[
t_{av} = 206 + 209 \log(1 + 1.46 p)
\]  

(5)

Integrating the force equilibrium equation in a thin oil film with respect to film thickness yields:

\[
\tau_x = \tau_{xw} + z \frac{\partial p}{\partial x}, \quad \tau_y = \tau_{yw} + z \frac{\partial p}{\partial y}
\]  

(6)

where $\tau_{xw}$ and $\tau_{yw}$ are the shear stresses on surface A. Integrating Eqs.(1) over film thickness $h$ gives

\[
\int_0^h \left[ \frac{u}{G_e} \frac{\partial \tau_x}{\partial x} + \frac{\tau_0}{\eta} \frac{\partial u}{\partial x} \sinh \left( \frac{\tau_x}{\tau_0} \right) \right] \, dz = u_a - u_b
\]

\[
\int_0^h \left[ \frac{v}{G_e} \frac{\partial \tau_y}{\partial y} + \frac{\tau_0}{\eta} \frac{\partial v}{\partial y} \sinh \left( \frac{\tau_y}{\tau_0} \right) \right] \, dz = 0
\]  

(7)

where $u_a$ and $u_b$ are the velocities of solid surface A and B, respectively.

The analytical scheme for the shear stresses $\tau_x$ and $\tau_y$ is basically the same as that developed by Yang and Wen\(^{9}\) and Liu et al\(^{10}\). Namely, Eqs.(7) are solved with a conventional Newton-Raphson method for $\tau_{xw}$ and $\tau_{yw}$, and then the values of $\tau_x$ and $\tau_y$ are obtained by using Eqs.(6). If $\tau_x$ exceeds the limiting shear stress $\tau_x$, $\tau_y = \tau_{yw}$.

In this case, the shear stresses components $\tau_x$ and $\tau_y$ are obtained by

\[
\tau_x = \frac{u}{\sqrt{u^2 + v^2}} \tau_x, \quad \tau_y = \frac{v}{\sqrt{u^2 + v^2}} \tau_y
\]  

(8)

\[\text{Fig. 1 Computation domain for numerical analysis}\]

The limiting shear stress was calculated by the products of the maximum traction coefficient to the average Hertzian pressure. Taking into account of the decrease in traction coefficient caused by the phase transition of lubricating oil\(^{9}\), the relationship of limiting shear stress $\tau_\ell$ with the average pressure $P_{av}$ and the mean film temperature $t_\ell$ was obtained by the least squares method. The mean temperature $t_\ell$ is defined as the sum of the inlet temperature $t_0$ and the mean temperature rise predicted using Crook's formula\(^{11}\). The limiting shear stress in solid state ($t_{av} > t_\ell$) is shown as the following formula

\[
\tau_\ell = \left[ \frac{32.39 + 176.1 \times 10^5 P_{av}}{0.079114 + 0.2047 \times 10^{-3} P_{av}} \right] \times t_\ell \times 10^6
\]  

(9)

While, in liquid state ($t_{av} \leq t_\ell$)

\[
\tau_\ell = \left[ \frac{89.85 \times 10^5 P_{av} - 0.8307 t_\ell + 334.5 \times 10^5}{0.079114 + 0.2047 \times 10^{-3} P_{av}} \right] \times t_\ell \times 10^6
\]  

(10)

In this analysis, when the Eyring stress $\tau_0$, the elastic shear modulus $G_e$, and the limiting shear stress $\tau_\ell$ are calculated by Eqs.(2),(3),(9) and (10), the pressure $p$ at each point is used instead of the average Hertzian pressure $P_{av}$, and the absolute temperature $t$ is substituted for $t_0$ and $t_\ell$ in these formulas.

In order to apply the generalized Reynolds equation\(^{9}\) to the Maxwell model, the effective viscosities $\eta_\ell$, $\eta_0$ are introduced, which are defined as

\[
\eta_\ell = \frac{\tau_\ell}{\partial u/\partial z}, \quad \eta_0 = \frac{\tau_0}{\partial v/\partial z}
\]  

(11)

The generalized Reynolds equation can then be written as
\[
\frac{\partial}{\partial x} \left[ \left( \frac{\rho}{\eta} \right)_o h^2 \frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \left( \frac{\rho}{\eta} \right)_o h^2 \frac{\partial p}{\partial y} \right] = 12u_h \frac{\partial}{\partial x} \left( \rho^* h \right)
\]  
(12)

where,

\[
\left( \frac{\rho}{\eta} \right)_o = 12 \left( \frac{\eta o \rho^o}{\eta^o} - \rho^o \right)
\]

\[
\left( \frac{\rho}{\eta} \right)_h = 12 \left( \frac{\eta h \rho^h}{\eta^h} - \rho^h \right)
\]

\[
\rho^* = \left[ \left( \rho_o \eta_o (u_x-u_y) + \rho_h u_y \right) \right] / u_x
\]

\[
\rho^* = \left( \frac{1}{\eta} \right) \frac{1}{h} \int \rho dz
\]

\[
\rho^* = \left( \frac{1}{\eta} \right) \frac{1}{h} \int \eta dz
\]

\[
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\]

\[
\rho^* = \left( \frac{1}{\eta} \right) \frac{1}{h} \int \eta dz
\]

\[
\frac{\partial}{\partial x} \int q u dz' + \frac{\partial}{\partial y} \int q v dz'
\]

The film thickness is given by

\[
h(x,y) = h_o + \frac{h^*}{2} \frac{h^*}{R} R
\]

\[
+ \frac{2}{\eta^o} \int \int \frac{\rho(x',y')}{(x-x')^2 + (y-y')^2} dx'dy'
\]  
(14)

The other governing equations (the force balance equation, the energy equations of solids and the heat flux continuity conditions) and the boundary conditions are the same as those used in the previous works [1,2].

In their traction tests [3], a synthetic traction oil with the viscosity grade of ISO VG32 (kinematic viscosity \( \nu \): 32.2 mm²/s at 313 K, 5.31 mm²/s at 373 K, pressure viscosity coefficient \( \alpha \): 31.2 GPa⁻¹ at 313 K, specific gravity 288/277 K: 0.962) was used. The density of the traction oil was also appraised using the formula based on the free volume theory [4].

The numerical schemes are basically the same as that developed by Guo et al. [2] and Liu et al. [3]. According to the same procedure as in previous studies [1,2], the numerical analysis was carried out.

### 3. Results of numerical calculation

Material properties of lubricating oil and contacting solids used in the numerical analysis are shown in Tables 1. Input parameters are: maximum Hertzian pressure \( P_h=1.5 \) GPa, entrainment velocity \( u_e=10 \) m/s, slide-roll ratio \( \Sigma=0\% \sim 4.0\% \) and inlet oil temperature \( T_0=313 \) K, 353 K, 373 K and 413 K. Table 2 shows the dimensionless load parameter \( W \), speed parameter \( U \), and material parameter \( G \) under these conditions.

Fig. 2 shows the map of lubrication regimes [5] for the ellipticity parameter \( K_e=1.0 \). According to the study by Ohno et al., it is found that the transition from visco-elastic to visco-elastic solid of lubricating oil occurs at \( \alpha P_{av}=13 \) and the phase transition from visco-elastic solid to elastic-plastic solid occurs at \( \alpha P_{av}=25 \) [6]. As shown in Fig. 2, the phase of lubricating oil under EHL conditions differs with different inlet oil temperature \( T_0 \).

![Fig. 2 Map of lubrication regimes for \( K_e=1.0 \)](image)
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Fig. 3 Traction curves obtained by numerical calculation and experimental data.

Fig. 3 shows the traction curves obtained by the numerical analysis. The numerical results agree well with the measured results of the traction tests. However, in the case of \( t_0 = 413 \) K, some differences between the numerical and experimental results for the range of the slide-roll ratio from \( \Sigma = 0.8\% \) to \( \Sigma = 2.0\% \) are recognized.

Fig. 4 shows the dimensionless pressure distributions on the plane of \( Y = 0 \) under \( \Sigma = 0.1\% \) and \( \Sigma = 4.0\% \). In the case of \( t_0 \geq 353 \) K, the pressure distributions were hardly influenced by the inlet oil temperature and the slide-roll ratio. On the other hand, when the inlet oil temperature is 313 K, the variations of pressure distribution in the outlet area of contact circle are larger than those with \( t_0 \geq 353 \) K. Especially, the pressure shows the tendency to change with increase in the slide-roll ratio.

Fig. 5 shows the oil film shapes on the plane of \( Y = 0 \). Both the EHL oil film thickness and the film shape depend strongly on the inlet oil temperature. In the case of \( t_0 = 313 \) K, the constrictions of oil film are found at the end of contact area, and these are associated with the occurrence of pressure spike as shown in Fig. 4. The film thickness decreases with increase in the inlet oil temperature, and the film shape tends to be flattened through the contact area.

The dimensionless temperature distributions in the middle layer of oil film are shown in Fig. 6. In the case of \( t_0 = 313 \) K, the temperature rise is small at the outlet of the fluid flow even under the slide-roll ratio of \( \Sigma = 0.1\% \), and same tendency is observed in \( \Sigma = 4.0\% \). According to the numerical results, it is suggested that these heating at the outlet of contact area under low slide-roll ratio conditions are related to the thermal expansion coefficient of the free volume. On the other hand when the slide-roll ratio increases to \( \Sigma = 4.0\% \), the temperature rise is recognized over the whole of contact area and then the maximum temperature is \( T_{\text{max}} \approx 1.25 \). In the case of \( t_0 = 353 \) K, the temperature rise in oil film is remarkably small compared with that at \( t_0 = 313 \) K.

Fig. 6 Dimensionless temperature distributions in mid-film (dimensionless temperature \( T = t/t_0 \))

\[ T = \frac{t}{t_0} \]
Fig. 7  Shear stress distributions on the surface of solid A ($t_0=413$ K)

Fig. 7 shows the dimensionless shear stress distributions on the solid surface A at $t_0=413$ K. It is found that the shear stress distributions change remarkably in the range from $\Sigma=0.1\%$ to $\Sigma=1.0\%$. Namely, in the case of $\Sigma=0.1\%$, the shear stress increases almost linearly toward the outlet of fluid flow. As the slide-roll ratio increases, the effect of limiting shear stress increases, and then these at the part subjected to a high normal pressure resemble the pressure distributions. It was confirmed that the relation between the shear stress distribution and the slide-roll ratio is similar to the numerical results presented by Bair et al[14] and the shear stress of EHL oil film is clearly different from that given by applying the Amonton’s law of friction.

Fig. 8 shows the distributions of shear stress on the plane of $Y=0$ under $t_0=413$ K, 353 K and 313 K. The size and shape of distribution curves are affected by the inlet oil temperature. Especially, in the high temperature of $t_0=413$ K where the traction coefficients are comparatively small, the distributions are obviously different from those under the lower temperature conditions. Furthermore, in the case of $t_0=313$ K, relatively small differences between two solid surfaces are found at the inlet and outlet of contact circle. However, except for them, the shear stresses on both the surfaces become the almost same due to the dominant influence of shear flow under EHL conditions.

4. Conclusions

A non-Newtonian thermal EHL analysis based on the Maxwell fluid model was performed. The relationship between the coefficient of traction and the slide-roll ratio obtained by the numerical calculation agreed well with that in traction tests. Although large variations of pressure and temperature were hardly recognized up to the slide-roll ratio of $\Sigma=1.0\%$, the shear stress distribution changed considerably. It was also found that the size and the shape of distribution curves of shear stress are influenced by the inlet oil temperature.

5. Acknowledgments

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6. Notations

- $b$ = radius of Hertzian contact, m
- $c$ = specific heat of oil, J/kgK
- $d$ = dimensionless thickness of temperature calculation domain of solids
- $E'$ = equivalent elastic modulus of temperature
- $G$ = material parameter, $aE'$
- $Ge$ = elastic shear modulus, Pa
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\[
H = \text{dimensionless film thickness for results, } hR'/b^2
\]
\[
h = \text{film thickness, m}
\]
\[
h_{00} = \text{rigid central film thickness, m}
\]
\[
K_e = \text{ellipticity parameter}
\]
\[
k = \text{thermal conductivity of oil, W/mK}
\]
\[
P = \text{dimensionless pressure, } p/P_H
\]
\[
p = \text{hydrodynamic pressure, Pa}
\]
\[
P_H = \text{maximum Hertzian pressure, Pa}
\]
\[
P_{av} = \text{average Hertzian pressure, Pa}
\]
\[
R' = \text{curvature of radii, m}
\]
\[
T = \text{dimensionless temperature, } t/t_0
\]
\[
t = \text{absolute temperature, K}
\]
\[
t_0 = \text{inlet temperature, K}
\]
\[
t_f = \text{mean film temperature, K}
\]
\[
t_w = \text{viscoelastic transition temperature, K}
\]
\[
U = \text{velocity parameter, } u_H/E'R'_x
\]
\[
u = \text{oil velocity in } x\text{-direction, m/s}
\]
\[
u_a = \text{velocity of surfaces A and B, m/s}
\]
\[
u_e = \text{entrainment velocity, } (u_a+u_b)/2 \text{ m/s}
\]
\[
v = \text{oil velocity in } y\text{-direction, m/s}
\]
\[
W = \text{load parameter, } w/E'R'_x
\]
\[
w = \text{applied load, N}
\]
\[
X = \text{dimensionless coordinate, } x/b
\]
\[
x = \text{coordinate along the entrainment, m}
\]
\[
x_{in}, x_{out} = \text{domain boundaries in } x\text{-direction, m}
\]
\[
x_{in}, X_{out} = \text{dimensionless domain boundaries in } x\text{-direction, } x_{in}/b, x_{out}/b
\]
\[
Y = \text{dimensionless coordinate, } y/b
\]
\[
y = \text{coordinate perpendicular to the entrainment, m}
\]
\[
y_{in}, y_{out} = \text{domain boundary in } y\text{-direction, m}
\]
\[
y_{in}, Y_{out} = \text{dimensionless domain boundaries in } y\text{-direction, } y_{in}/b, y_{out}/b
\]
\[
Z = \text{dimensionless coordinate across the film, } z/b
\]
\[
z = \text{coordinates across the film, m}
\]
\[
z_a, z_b = \text{coordinates in solids A and B, m}
\]
\[
Z_{in}, Z_{out} = \text{dimensionless coordinates in solids A and B, } z_{in}/b, z_{out}/b
\]
\[
\eta = \text{absolute viscosity of oil, Pa·s}
\]
\[
\rho = \text{densities of oil, kg/m}^3
\]
\[
\tau_0 = \text{Eyring stress, Pa}
\]
\[
\tau_c = \text{module of the shear stress vector, Pa}
\]
\[
\tau_s = \text{limiting shear stress, Pa}
\]
\[
\tau_o, \tau_r = \text{shear stress along the } x \text{ and } y \text{ direction, Pa}
\]
\[
\tau_{ax}, \tau_{ay} = \text{shear stress acting on surface A along the } x \text{ and } y \text{ direction, Pa}
\]
\[
\Sigma = \text{slide-roll ratio, } (\tau_{ax}-\tau_{by})/\tau_{ax} \times 100\%
\]

7. References


