Short Communication

Shear Stress Analysis of EHL Oil Films Based on Thermal EHL Theory - In the Case of Elliptical Contact Condition -

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Under the elliptical contact conditions with an ellipticity parameter of $K_e=1.4$, a non-Newtonian thermal EHL analysis was carried out. The lubricating oil was assumed to behave as a Maxwell fluid. Lubricating oil is the automotive traction oil with the viscosity grade of ISO VG32. Input parameters are maximum Hertzian pressure $P_H=1.5$ GPa, entrainment velocity $u_e=10$ m/s, slide-roll ratio $\Sigma=0\%\sim 4.0\%$ and inlet oil temperature $t_0=413$ K. The shear stress distribution under elliptical contact conditions showed a similar tendency to that obtained in point contact conditions, and it was found that the traction coefficients at maximum Hertzian pressure of $P_H=1.5$ GPa are scarcely affected by the shape of contact area. Furthermore, although both the pressure distributions and the temperature distributions hardly changed up to the slide-roll ratio of $\Sigma=1.0\%$, the considerable variation in the shear stress distributions was recognized.

Keywords: shear stress, traction oil, thermal EHL, Maxwell model, elliptical contact, slide-roll ratio

1. Introduction

Wedeven et al. measured the traction coefficient with an automotive traction oil by the USCAR testing machine¹⁾. The authors performed a non-Newtonian thermal EHL analysis for the same point EHL contact conditions as in the traction tests²⁾, and the effect of inlet temperature on the shear stress of oil films was clarified.

In the present paper, the authors carried out the non-Newtonian thermal EHL analysis under the same elliptical contact conditions as in the traction tests which can be examined using USCAR testing machine, and the relationship of shear stress distribution with slide-roll ratio was investigated.

2. Procedure of numerical analysis

Assuming that the lubricating oil behaves a non-Newtonian fluid, the Maxwell model is adopted. The constitutive formulas of the Maxwell model are described as

$$\begin{aligned}\frac{\partial u}{\partial z} &= \frac{u}{G_e} \frac{\partial \tau_x}{\partial x} + \frac{\tau_x}{\eta} \frac{\tau_0}{\tau_e} \sinh\left(\frac{\tau_e}{\tau_0}\right) \\ \frac{\partial v}{\partial z} &= \frac{v}{G_e} \frac{\partial \tau_y}{\partial y} + \frac{\tau_y}{\eta} \frac{\tau_0}{\tau_e} \sinh\left(\frac{\tau_e}{\tau_0}\right)\end{aligned}\quad (1)$$

$$\text{where } \tau_e = (\tau_x^2 + \tau_y^2)^{0.5}.$$

The shear stress τ_x , τ_y , the viscosity η , the elastic shear modulus G_e and the Eyring stress τ_0 are functions of pressure and temperature.

With regards to the Eyring stress τ_0 and the elastic shear modulus G_e , the empirical equations used in the analysis under point contact conditions²⁾ are adopted. In order to estimate the viscosity η , the equation based on the free volume theory³⁾ is employed.

Integration of the force balance equation in a thin lubricant film with regard to film thickness gives

$$\tau_x = \tau_{xa} + z \frac{\partial p}{\partial x}, \quad \tau_y = \tau_{ya} + z \frac{\partial p}{\partial y} \quad (2)$$

where τ_{xa} and τ_{ya} are the shear stresses on solid surface A. Integrating Eqs. (1) over film thickness h yields

$$\begin{aligned}\int_0^h \left[\frac{u}{G_e} \frac{\partial \tau_x}{\partial x} + \frac{\tau_x}{\eta} \frac{\tau_0}{\tau_e} \sinh\left(\frac{\tau_e}{\tau_0}\right) \right] dz &= u_a - u_b \\ \int_0^h \left[\frac{v}{G_e} \frac{\partial \tau_y}{\partial y} + \frac{\tau_y}{\eta} \frac{\tau_0}{\tau_e} \sinh\left(\frac{\tau_e}{\tau_0}\right) \right] dz &= 0\end{aligned}\quad (3)$$

where u_a and u_b are the surface velocities of solid A and B, respectively.

The analytical scheme developed by Yang and Wen⁴⁾

and Liu et al.⁵⁾ is applied to calculate the shear stresses τ_x and τ_y . First, τ_{xa} and τ_{ya} are obtained by solving Eqs. (3), and then τ_x and τ_y are calculated using Eqs. (2). If τ_e is larger than the limiting shear stress τ_L , τ_e is fixed at τ_L ⁶⁾, and the shear stress components are determined by

$$\tau_x = \frac{u}{\sqrt{u^2 + v^2}} \tau_L, \quad \tau_y = \frac{v}{\sqrt{u^2 + v^2}} \tau_L \quad (4)$$

The limiting shear stress was obtained by the same formula used in the recent work²⁾.

With a view to applying the generalized Reynolds equation⁴⁾ to the Maxwell fluid model, the effective viscosities η_x^* , η_y^* are introduced. These are defined as

$$\eta_x^* = \frac{\tau_x}{\partial u / \partial z}, \quad \eta_y^* = \frac{\tau_y}{\partial v / \partial z} \quad (5)$$

Because the effective viscosities needs to be positive, the numerators in Eqs. (5) must have the same sign as the respective denominators given by the sum of elastic term and viscous term as shown in Eqs. (1). If the effective viscosity is not positive, the elastic term is neglected. When the elastic term is zero, the effective viscosity is the same as that with Eyring model⁵⁾.

Yang and Wen⁴⁾ derived the fluid velocity from the adoption of the effective viscosity. The velocity components u and v are given by

$$u = u_a + \frac{\partial p}{\partial x} \left[\int_0^z \frac{z' dz'}{\eta_x^*} - \frac{\eta_{ex}}{\eta_x'} h \int_0^z \frac{z' dz'}{\eta_x^*} \right] + \frac{\eta_{ex}(u_b - u_a)}{h} \int_0^z \frac{z' dz'}{\eta_x^*} \quad (6)$$

$$v = \frac{\partial p}{\partial y} \left[\int_0^z \frac{z' dz'}{\eta_y^*} - \frac{\eta_{ey}}{\eta_y'} h \int_0^z \frac{z' dz'}{\eta_y^*} \right] \quad (7)$$

where,

$$1/\eta_{ex} = (1/h) \int_0^h (1/\eta_x^*) dz, \quad 1/\eta_{ey} = (1/h) \int_0^h (1/\eta_y^*) dz$$

$$1/\eta_{ex}' = (1/h^2) \int_0^h (z/\eta_x^*) dz, \quad 1/\eta_{ey}' = (1/h^2) \int_0^h (z/\eta_y^*) dz$$

The other governing equations (the generalized Reynolds equation, the energy equation of the oil film, the film thickness equation, the force balance equation, the energy equations of solids and the heat flux continuity conditions) and the boundary conditions are the same as those used in the previous works^{2,7)}.

In their traction tests¹⁾, an automotive traction oil with the viscosity grade of ISO VG32 (kinetic viscosity ν : 32.2 mm²/s at 313 K, 5.31 mm²/s at 373 K, pressure viscosity coefficient α : 31.2 GPa⁻¹ at 313 K, specific gravity 288/277 K: 0.962) was used. The density of the traction oil was also appraised using the formula based on the free volume theory³⁾.

The numerical schemes are almost the same as that

Fig. 1 Computation domain for numerical analysis

developed by Liu et al.⁵⁾, and the process for calculating the effective viscosity is only added to the numerical procedure for the Newtonian thermal EHL analysis^{7,8)}.

Fig. 1 shows the computation domain of numerical analysis. The dimensionless coordinates are as follows: $X=x/b$, $Y=y/b$, $Z=z/h$, $Z_a=z_a/b$, $Z_b=z_b/b$ (b : minor radius of contact ellipse). The calculating grid consists of 129 nodes along the X and Y direction. 11 equidistant nodes are used across the film. In each solid, 6 non-equidistant nodes along Z_a or Z_b direction are adopted.

3. Results of numerical calculation

In the USCAR testing machine, a roller specimen can be substituted for a ball specimen¹⁾. Thus it is possible to perform the traction tests in elliptical contact. The roller diameter in rolling direction and the disk diameter are 20.64 mm and 102 mm, respectively. When the ellipticity parameter K_e is 1.4, the roller specimen has the radius of about 17.11 mm in an axial direction. Material properties of lubricating oil and contacting solids used in the numerical analysis are the same as in previous studies^{2,7)}. Other input parameters are the maximum Hertzian pressure $P_H = 1.5$ GPa, the entrainment velocity $u_e = 10$ m/s, the slide-roll ratio of $\Sigma = 0\% \sim 4.0\%$, the inlet oil temperature $t_0 = 413$ K, the dimensionless load parameter $W = 9.77 \times 10^{-6}$, the dimensionless velocity parameter $U = 1.00 \times 10^{-11}$ and the dimensionless material parameter $G = 1.42 \times 10^3$.

Fig. 2 shows the traction curve calculated by the numerical calculations. The numerical results agree well not only with the experimental results of traction tests¹⁾ but also with the analytical results in point contact²⁾. Kato et al.⁹⁾ performed the traction tests in the point contact condition or the elliptical contact conditions with $K_e = 2.5$ or 5.6, and they also found that the traction curve is not greatly influenced by the contact geometry of test speci-

Fig. 2 Traction curves obtained by numerical calculation and experimental data¹⁾

mens under high pressure conditions.

Fig. 3 shows the oil film shapes on the plane of $Y=0$ (H : dimensionless film thickness, R'_x : curvature of radii in X direction). Both the film thickness and the film shape are hardly influenced by the slide-roll ratio. The oil films are remarkably thin, and the film thicknesses are almost constant through the area of contact ellipse.

The distributions of dimensionless pressure $P=p/P_H$ (p : hydrodynamic pressure) under slide-roll ratio of $\Sigma=0.1\%$ and 4.0% are shown in Fig. 4. The pressure distributions are independent of the slide-roll ratio, and the shapes of pressure distribution are similar to that of Hertzian pressure distribution.

Fig. 5 shows the distributions of dimensionless temperature $T=t/t_0$ (t : absolute temperature) in the middle layer of oil film. The temperature rise is not found in the range of the slide-roll ratio from $\Sigma=0.1\%$ to $\Sigma=1.0\%$. In the case of $\Sigma=4.0\%$, although the slight change in temperature is recognized over the whole of contact area, the maximum temperature T_{max} is only 1.04 ($t \approx 430$ K).

Fig. 6 shows the distributions of dimensionless shear stress given by τ_x/P_H on the solid surface A. The relation between the shear stress distribution and the slide-roll ratio is similar to the numerical results under point contact conditions²⁾, and the shear stress distributions change markedly depending on the slide-roll ratio. Namely, in the case of slide-roll ratio $\Sigma=0.1\%$, the shear

Fig. 3 Oil film shapes on the plane of $Y=0$

Fig. 4 Dimensionless pressure distributions (dimensionless pressure $P=p/P_H$)

stress increases almost linearly in the direction of the outlet area of contact ellipse. The effect of limiting shear stress increases with increase in the slide-roll ratio. The shear stress distributions at the part subjected to large pressure are similar to the pressure distributions.

The fluid velocity is calculated using Eqs.(6) and (7). The contour maps of dimensionless velocity $U_e=u/u_e$ (u_e : entrainment velocity, $(u_a+u_b)/2$) on the plane of $Y=0$ are shown in Fig. 7. In the contour maps, differences in U_e between contours are (a) 1.25×10^{-4} and (b) 5.0×10^{-3} ,

Fig. 5 Dimensionless temperature distributions in mid-film (dimensionless temperature $T=t/t_0$)

Fig. 6 Dimensionless shear stress distributions on the surface of solid A

respectively. The fluid velocity u is affected by the effective viscosity η_x^* . In spite of the high inlet temperature of $t_0=413$ K, the viscosity η of the traction oil is large at the part which receives high pressure, and so the elastic term in Eqs. (1) becomes a dominant factor in the effective viscosity. When the slide roll ratio is $\Sigma=0.1\%$, the effective viscosities η_x^* hardly change across the oil film due to the small shear stress and the invariable temperature, and the velocity distribution is almost constant. On the other hand, in the case of $\Sigma=4.0\%$, owing to the limiting shear stress, the elastic term at the center of contact ellipse is zero, and then the effective viscosities η_x^* in the rearward of contact area subjected to high pressure are not positive. In the above area, the elastic term is invalid and the effective viscosities are dependent on the viscous term. The viscous term is so sensitive to temperature that the effective viscosity η_x^* decreases even with slight increase in temperature. Therefore, the fluid flows discontinuously in the vicinity of center of contact area.

4. Conclusions

The shear stress distributions obtained by the non-Newtonian thermal EHL analysis under the elliptical contact condition with the elliptical parameter of $K_e=1.4$ show a similar tendency to those in the point contact conditions, and the shape of contact area has only a negligible effect on the traction coefficients. Furthermore, although the variations in pressure and temperature were hardly recognized up to the slide-roll ratio of $\Sigma=1.0\%$, the shear stress distribution varied remarkably.

5. References

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 Fig. 7 Contour maps of dimensionless velocity $U_e=u/u_e$ on the plane of $Y=0$

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