Mixed Lubrication Analysis of Vane Sliding Surface
in Rotary Compressor Mechanisms
- Influences of Elastic Deformation at Surface End of Vane-Slot -

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In order to investigate the effects of the elastic deformation of the vane-slot on the lubrication characteristics of the vane sliding surface in a rotary compressor, a mixed lubrication analysis considering the elastic deformation has been performed for the vane sliding surface. In this analysis, the modified Reynolds equation and the elastic contact equation, within which the influence of surface roughness is considered, are solved as a coupled problem. The elastic deformation of the vane-slot is calculated by using an FEM model with two-dimensional isoperimetric elements. By comparing the analysis results with that of a rigid-body model, the effects of the elastic deformation of the vane-slot on the lubrication characteristics were clarified. As a result, it is found that the elastic deformation of vane-slot contributes to the smooth reciprocating motion of the vane.

Keywords: mixed lubrication, elastic deformation, rotary compressor, vane, vane-slot

1. Introduction

In the compression mechanism of a rotary compressor for air conditioners, a vane is provided to separate the suction chamber and the compression chamber. The vane that undergoes reciprocating motion is driven by the eccentric rotation of the rolling piston. Severe load and moment generated by the pressure difference between the suction chamber and the compression chamber act on the vane and the vane-slot. When the friction loss and the surface damage of the vane increase, the performance and the reliability of the rotary compressors will decrease. Therefore, the optimum design by which a good lubricating condition on the sliding surface of the vane can be realized is required. And it is important to know the accurate lubrication characteristics of the vane sliding surface1-3).

In a previous study11, a numerical analysis approach for mixed lubrication has been performed in order to investigate the lubrication characteristics of the vane sliding surface. In the analysis, the vane-slot was assumed to be a rigid body. However, solid contact may occur at the surface end of vane-slot, and the elastic deformation of the surface end may become large. So, it is considered that the elastic deformation of vane-slot may influence the lubrication characteristics of vane sliding surface.

In order to investigate the lubrication characteristics of the vane sliding surface accurately, a mixed lubrication analysis considering the elastic deformation has been performed. The modified Reynolds equation and the elastic contact equation, within which the influence of surface roughness is considered, are solved as a coupled problem. The elastic deformation of the vane-slot is calculated by using an FEM model with two-dimensional isoperimetric elements. Moreover, analyses were performed by the rigid-body model as well as by the elastic deformation model. By comparing the analysis results, the effects of the elastic deformation of the vane-slot were clarified.

2. Governing equations

Figure 1 shows a schematic of the compression mechanism in a rotary compressor. The rolling piston driven by the crank rotates eccentrically. Owing to this eccentric rotation, the volume of the compression chamber decreases, and the pressure of refrigerant...
becomes high. The vane separates the suction chamber and the compression chamber. Figure 2 shows the coordinate system of vane. Discharge pressure $p_{dis}$ acts on the tail-end of the vane.

2.1. Equations of motion of vane

As shown in Fig. 1, the vane is subjected to the load due to the differences of pressures $p_{dis}$, $p_{suc}$, and $p_{com}$, the spring force, the friction forces between vane and vane-slot and the friction force between vane and rolling piston. The equation of motion of the vane and the equations of equilibrium of forces and moments are,

$$m_v \ddot{x}_v = F_{v_x} + F_{v_y} + F_{vz} + F_{w} \cos \alpha + F_{w} \sin \alpha - F_s$$

(1)

$$0 = F_{w} + F_{v1} - F_{v2} + F_{w} \cos \alpha - F_{w} \sin \alpha$$

$$+ w_1 \int p_1 \, dx - w_2 \int p_2 \, dx$$

(2)

$$0 = M_{v1} + M_{v2} - M_{v3} + M_{v4} - M_{v5}$$

$$+ w_1 \int xp_1 \, dx - w_2 \int xp_2 \, dx$$

(3)

where $m_v$ is the mass of vane, $x_v$ is the x-directional displacement of vane measured from the center of cylinder $O_v = (x_v + r_v \cos \alpha + r_o \sin \psi, r_v)$ is the radius of vane tip, $r_o$ is the outer radius of piston, $\varepsilon$ is the eccentricity of crank, $\psi$ is the crank angle), $\alpha$ is the angle extended by piston and vane contact point, $\omega_p$ is the angular velocity of piston rotation. $\omega_p$ is calculated by the following equation.

$$\omega_p = \frac{I \dot{\theta}_p}{M_p}$$

(6)

where $I$ is the polar moment of inertia of piston, $M_p$ is the fluid friction moment on the faces of piston end due to revolution. The sliding section between the piston and crank is regard as a hydrodynamic journal bearing. Then, the friction

![Fig. 1 Schematic of compression mechanism in rotary compressor](image1)

![Fig. 2 Coordinate system of vane](image2)

![Fig. 3 FEM model of cylinder on suction side](image3)
moment $M$, can be calculated by the petroff’s equation\(^4\). Here, the viscous friction moment due to the refrigerant gas between the piston and cylinder is negligible small.

2.2. Reaction forces of oil film between vane and vane-slot

In this analysis approach, the vane sliding surface is treated as a surface having infinite width along the direction perpendicular to the plane of Fig. 2.

The oil film pressures on discharge side and suction side between the vane and vane-slot $p_1, p_2$ are calculated by the modified Reynolds equations as follows\(^5\),

$$\frac{\partial}{\partial x} \left( \Phi \frac{h}{\eta} \frac{\partial p}{\partial x} \right) = 6U \frac{\partial h}{\partial x} + 6U \frac{\partial \Phi}{\partial x} + 12 \frac{\partial h}{\partial t}$$

(7)

where, $h$ is the average oil film thickness, $h_T$ is the local oil film thickness, $\eta$ is the oil viscosity, $U$ is the sliding velocity of vane, $\sigma$ is the standard deviations of composite roughness, $\Phi$ is the pressure flow factor, and $\Phi_k$ is the shear flow factor. The vane sliding surface is treated as a surface having the longitudinal type of surface roughness. The direction parameter of the surface roughness used here is 3.0.

The oil film thicknesses on discharge side and suction side between the vane and vane-slot are, $\bar{h}_1 = c_i - kx$, $\bar{h}_2 = \bar{h}_0 + kx + \delta$$^9$

(8), (9)

where, $\bar{h}_1$ and $\bar{h}_2$ are the average oil film thicknesses on discharge side and suction side, $c_i$ is the clearance between vane and vane-slot, $\bar{h}_0$ is the average oil film thickness at the lower end of vane-slot ($x=0$ location in Fig. 2) on suction side, $k$ is the inclination of vane. In $\bar{h}_1$, the elastic deformation of the vane-slot $\delta$ is considered.

2.3. Contact forces between vane and vane-slot

For calculating the contact forces between the vane and vane-slot, Patir and Cheng’s approximate expression based on Greenwood and Tripp’s theory are used\(^6,7\),

$$p_x = \begin{cases} k_E' \times 4.4086 \times 10^{-3} \left( 4 - \frac{\bar{h}}{\sigma} \right)^{6.804} \left( \bar{h} < 4\sigma \right) \\ 0 \hspace{5cm} \left( \bar{h} \geq 4\sigma \right) \end{cases}$$

(10)

where, $p_x$ is the contact pressure between the vane and vane-slot, $k_E'$ is a constant in force-compliance relationship, and $E'$ is the equivalent elastic modulus.

2.4. Elastic deformation of vane-slot on suction side

In this analysis, the elastic deformation of vane-slot on suction side is calculated by using FEM. As shown in Fig. 3, mesh division of cylinder on suction side is carried out. The elastic deformation of node on the vane-slot due to the oil film force and the contact force is calculated. The elastic deformation of vane-slot $\delta$ is expressed as follows,

$$\delta = u_j$$

(11)

where, $u_x$, $u_y$ are the coordinates of elastically deformed surface of vane-slot. The displacement vector $\{u\}$ containing $u_x$ and $u_y$ is expressed as follows,

$$\{u\} = [C]\{f\}$$

(12)

where, $[C]$ is the Influence coefficient matrix, and $\{f\}$ is the external load vector acting on nodes that is calculated from the pressure distribution on the vane-slot. $[C]$ is the inverse matrix of the rigid matrix $[K]$ of the FEM model\(^8\).

3. Analysis procedure and conditions

3.1. Analysis procedure

Figure 4 shows the analysis flow. The Influence coefficient matrix $[C]$ used in the mixed lubrication analysis is calculated by the FEM model of cylinder.

In the first step, Eqs.(1)-(3) and Eqs.(7), (10) are solved as a coupled problem, and the inclination of vane $k$, the average oil film thickness at the lower end of vane slot $\bar{h}_0$ and the normal force acting on vane at piston contact $F_{in}$ are numerically calculated by

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Newton-Raphson method. By Eq. (1) and (4), it can be seen that $F_{\text{in}}$ can be expressed as a function of $k$ and $\vec{h}_n$. And in the numerical calculation this function is used. The time differential of $k$ and the time differential of $\vec{h}_n$ used in the calculation of the squeeze terms in the modified Reynolds equations are approximately calculated by the following equation.

\[
\frac{\dot{k}}{k} = \frac{1}{\Delta t} \left[ k_{\text{in},w} - k_{\text{in},w}^0 \right] \tag{13}
\]

In the second step, the elastic deformation of vane-slot $\delta$ due to the oil film pressure and the contact pressure on the suction side is calculated recursively by Eq. (11). $\delta$ is obtained as the convergence value of the following corrected elastic deformation $\delta^n$.

\[
\delta^n = \delta^{n-1} + \beta(\vec{\delta}^n - \delta^{n-1}) \tag{14}
\]

where $\vec{\delta}^n$ is the $n$-th calculation estimate by Eq. (11), $\beta$ ($<1$) is the relief coefficient. $\vec{\delta}^n$ is compared with the previous estimate $\delta^{n-1}$, and the calculation is repeated until a convergence condition is fulfilled. After the calculation of second step, the time step is advanced and the same convergence calculation is repeated.

3.2. Analysis conditions

The analysis conditions are shown in Table 1. The radius of cylinder is 31.5 mm. The outer radius of rolling piston is 26.2 mm. The eccentricity of crank is 5.3 mm. The suction pressure is 1.27 MPa. The discharge pressure is 4.25 MPa. The rotor rotating frequency is 60 Hz. The oil viscosity is $2.83 \times 10^{-3}$ Pa·s. The material of cylinder is cast-iron. Its material constants are used for the calculation of [C]. In this analysis, the modulus of longitudinal elasticity is 120 GPa, and Poisson’s ratio is 0.27. Figure 5 shows the relationship between the crank angle and the pressure of compression chamber, which was used as the analysis condition. It contains the effects of over-compression.

4. Results and discussion

4.1. Effects of clearance between vane and vane-slot

Figure 6 shows the analysis results of the elastic deformation model and the rigid-body model. The horizontal axis is the crank angle $\psi$. Figure 6(a) shows the variation of the inclination of vane $k$ with respect to the crank angle through one revolution of the crank. It can be seen that $k$ is always larger than $0^\circ$. That is, the vane always inclines in the clockwise direction as shown in Fig. 2. Consequently, the solid contact between the vane and vane slot occurs at the lower end ($x=0$ location in Fig. 2) on suction side and at upper end ($x=l_v$ location in Fig. 2) on discharge side. It can be seen that, in the neighborhood of $\psi=210^\circ$, the crank angle at which $k$ approximately reaches a maximum value, $k$ of the elastic deformation model is larger than that of the rigid-body model. This is because the elastic deformation of vane-slot increases at the lower end where the solid contact occurs. Figure 6(b) shows the variation of the minimum oil film thickness at the lower end of vane slot on suction side in the form of oil film parameter $\Lambda_\psi$ ($=h_0/\sigma$) through one revolution of the crank. Because the inclination $k$ is always larger than $0^\circ$ as shown in Fig. 6, it can be seen from the geometry of vane (Fig. 2) that $h_0$ becomes the minimum oil film thickness on suction side between the vane and vane slot. It can be seen that $\Lambda_\psi$ increases between $\psi=0^\circ$ and $\psi=180^\circ$. This is because the oil film pressure on suction side is raised by the wedge film effect. Because the wedge film effect disappears when the motion of vane reverses at $\psi=180^\circ$, $\Lambda_\psi$ decreases drastically from $\psi=180^\circ$. It can be seen that in the neighborhood of $\psi=210^\circ$, the crank angle at which $\Lambda_\psi$ approximately reaches a minimum value, $\Lambda_\psi$ of the elastic deformation model is larger than that of the rigid-body model. This is because, by considering the effect

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**Table 1** Analysis conditions

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<tr>
<td>Radius of cylinder [mm]</td>
<td>31.5</td>
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<tr>
<td>Outer radius of piston [mm]</td>
<td>26.2</td>
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<tr>
<td>Eccentricity of crank, $\epsilon$ [mm]</td>
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<td>Dimensionless clearance between vane and vane slot, $C_v$ (=$c_v/d$) [10^{-3}]</td>
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<tr>
<td>Dimensionless length of vane slot, $L_v$ (=$l_v/d$)</td>
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<tr>
<td>Rotor Rotating Frequency, N [Hz]</td>
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<td>Discharge pressure, $P_{\text{dis}}$ [MPa]</td>
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<tr>
<td>Suction pressure, $P_{\text{suc}}$ [MPa]</td>
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<td>Oil viscosity, $\eta$ [10^{-3} Pa·s]</td>
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<td>Poisson’s ratio of cylinder</td>
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<tr>
<td>Young’s modulus of cylinder [GPa]</td>
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<td>Standard deviations of composite roughness, $\sigma$ [µm]</td>
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<tr>
<td>Coefficient of friction between vane and piston, $\mu_f$</td>
<td>0.12</td>
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Fig. 5 Pressure of compression chamber
of elastic deformation, the clearance between the vane and the vane-slot increases at the lower end. The minimum oil film parameter of the elastic deformation model is about twice that of the rigid-body model. Figure 6(c) shows the variation of reaction forces of oil film per unit width of cylinder $w_c$ on suction side between vane and vane slot through one revolution of the crank. Figure 6(d) shows the variation of contact forces per unit width of cylinder $w_c$ on suction side between vane and vane slot through one revolution. It can be seen that the lubrication on suction side between the vane and vane slot is in a hydrodynamic lubrication zone from $\psi=0^\circ$ to $\psi=180^\circ$. From $\psi=180^\circ$, the reaction forces of oil film on suction side decreases drastically, and thus the contact forces occur. It can be seen that for the rigid-body model, the reaction forces of oil film decrease drastically from $\psi=180^\circ$, and for the elastic deformation model, the reaction forces of oil film decrease from $\psi=210^\circ$. Therefore, the contact force of the elastic deformation model is smaller than that of the rigid-body model from $\psi=180^\circ$ to $210^\circ$. This difference is caused by the wedge film effect and squeeze effect due to the elastic deformation of the vane-slot at the lower end. Figure 6(e) shows the variation of reaction forces of oil film per unit width of cylinder $w_c$ on discharge side between vane and vane-slot through one revolution.

Fig. 6  Comparison of Elastic deformation model and Rigid-body model
revolution of the crank. Figure 6(f) shows the variation of contact forces per unit width of cylinder $w_c$ on discharge side between vane and vane-slot through one revolution of the crank. There is almost no difference between the two models. The influences of the elastic deformation are not obvious on discharge side.

For the elastic deformation model, oil film force on suction side is still high after the motion of vane reversed. As a result, solid contact does not occur at the moment of reversion of the vane ($\psi=180^\circ$), and the vane can move smoothly. So, it is found that the elastic deformation of vane-slot contributes to the smooth motion of vane. Therefore, the mixed lubrication analysis considering the elastic deformation is required in order to investigate the lubrication characteristic of the vane sliding surface accurately.

4.2. Effects of vane-slot length

Figure 7 shows the elastic deformation of the vane-slot on suction side. Figure 8 shows the distributions of oil film pressure and contact pressure at $\psi=210^\circ$. The horizontal axis is the dimensionless distance $X (= x/l_v)$ along x direction. And, the compression pressure reaches the maximum at $\psi=210^\circ$. Figure 7 (a) shows the elastic deformation at $\psi=0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$, $120^\circ$ and $150^\circ$. Figure 7(b) shows the elastic deformation at $\psi=180^\circ$, $210^\circ$, $240^\circ$, $270^\circ$, $300^\circ$ and $330^\circ$. At $\psi=210^\circ$, $240^\circ$ and $270^\circ$, the maximum value of the elastic deformation is about eight times that from $\psi=0^\circ$ to $120^\circ$. It is found that, as the solid contact is dominant in the discharge process, the elastic deformation of the vane-slot increases.

Figure 8(a) shows the distributions of oil film pressure and contact pressure of the elastic deformation model. Figure 8(b) shows the distributions of oil film pressure and contact pressure of the rigid-body model. It can be seen that the oil film at the lower end (from $X=0$ to $X=0.2$) is not formed in the rigid-body model and the load exerted on vane sliding surface is supported by the contact force. Moreover, it is found that, in the elastic deformation model, oil film possessing load carrying capacity exists at the lower end and the oil film pressure is large. As a result, the maximum contact pressure of the elastic deformation model is about 60% of that of the rigid-body model.

From the results of Fig. 7 and Fig. 8, it can be seen that the clearance at the lower end increases owing to the elastic deformation. And by this effect, the film formation ability of the vane sliding surface increases.
5. Conclusion

The mixed lubrication analysis considering the elastic deformation has been performed in order to investigate the effects of elastic deformation of vane-slot on the lubrication characteristics of the vane sliding surface in a rotary compressor. The following results have been obtained.

1. The solid contact does not occur at the reverse motion of vane owing to elastic deformation of the vane-slot.
2. When the compression pressure has a peak value, the ability of oil film generation is raised because that the clearance between the vane and the vane-slot at the lower end increases by the elastic deformation.

It can be seen that under the severe lubrication condition, the lubrication characteristics of the vane sliding surface are improved owing to elastic deformation. So, the mixed lubrication analysis considering elastic deformation is required in the optimum design of the vane.

6. References


