2A19 Learning ability of stochastic feed-forward network
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1. Introduction
Many people have been interested in finding an efficient learning algorithm which would be capable of solving many complicated problems. The error-back propagation, and Boltzmann machine are powerful examples of the basic learning tools of the field; but they are not applicable for the association problem with strongly correlated output patterns.

In this paper, we study the learning abilities of the stochastic feed-forward network with the learning algorithm derived by minimizing the relative entropy, for the inverse XOR problem in which correlated target patterns are given by conditional probabilities corresponding to each input pattern.

2. Contents
We consider a feed-forward neural network consisting of $L$ layers. The neurons on the $l-1$ th layer are connected to neurons on the $l$ th layer via synaptic interactions. A state of $j$ th neurons on the $l$ th layer is represented by $\{s_{jl}\}$. When neurons on the $l-1$ th layer are at the state of $\{s_{j(l-1)}\}$, the neurons on the $l$ th layer take a state of $\{s_{jl}\}$ with a conditional probability,

$$P(\{s_{jl}\}|\{s_{j(l-1)}\}) = \prod_j \frac{e^{\beta s_{jl}(h_{jl}^{(l)})}}{2 \cosh \beta h_{jl}^{(l)}},$$

where $\beta$ is the inverse temperature of the network system, and $h_{jl}^{(l)}$ is an internal field at the $j$ th neuron on the $l$ th layer which is given by

$$h_{jl}^{(l)} = \sum_i w_{jl}^{(l)} s_{ji}^{(l-1)} + \theta_j^{(l)}.$$

Here $w_{jl}^{(l)}$ is the synaptic efficacy from $s_{ji}^{(l-1)}$ to $s_{jl}^{(l)}$, and $\theta_j^{(l)}$ is the threshold of the neuron $s_{jl}^{(l)}$.

We adopt the relative entropy $S$ as an error measure to be minimized in the learning process,

$$S = - \sum_j p(\mu) \sum_{\nu} Q(\nu|\mu) \ln \frac{P(\nu|\mu)}{Q(\nu|\mu)}.$$ 

The decrease of $S$ in the learning process is realized by the following change of the weights as:

$$\Delta w_{jl}^{(l)} = -\eta \frac{\delta S}{\delta (\beta w_{jl}^{(l)})}, \Delta \theta_j^{(l)} = -\eta \frac{\delta S}{\delta (\beta \theta_j^{(l)})},$$

where $\eta$ is the learning coefficient.

We applied this algorithm to the inverse XOR problem, and studied its learning abilities for several target probabilities $Q(\nu|\mu)$ taking the following values in the table as:

| $\mu$ | $\nu$ | $Q(\nu|\mu)$ |
|-------|-------|--------------|
| $(1, 1)$ | $(-1, 1)$ | $0.0$ |
| $(1, -1)$ | $(-1, -1)$ | $0.0$ |
| $(-1, 1)$ | $(1, 1)$ | $0.0$ |
| $(-1, 1)$ | $(1, -1)$ | $0.0$ |
| $(-1, -1)$ | $(1, 1)$ | $0.5$ |

A typical learning result is shown for the case $Q((1, 1)|(-1, 0)) = 0.49$, $Q((1, -1)|(-1, 0)) = 0.51$ in the graph as:

3. Results and Conclusion
It has been shown that the stochastic feed-forward network is able to learn the inverse XOR problem even with any arbitral ratio of the conditional probabilities of the target patterns. Analytical thought gives us the minimum structure of the network to solve the inverse XOR problem in general. It is, however, unknown how the network realizes any ratio of target probabilities. Details of the learning mechanism will be presented.

4. References