GA-based Design of 2-D State-Space Digital Filters

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1. Introduction

It is very difficult to design 2-D recursive digital filters with very small approximation error by traditional optimization methods, because of multi-modal error functions used in optimization. To overcome the above problem, we propose a design method of 2-D state-space digital filters using a GA.

2. Formulation of design problem

Consider the following transfer function of a 2-D separable denominator state-space digital filter represented by Roesser’s local state-space model [1]:

\[ H(z_1, z_2) = c[z_1 I_{N_1} \oplus z_2 I_{N_2} - A]^{-1} b + d \]  

(1)

where \( \oplus \) denotes the direct sum of matrices, and \( I_{N_1} \) and \( I_{N_2} \) are \( N_1 \times N_1 \) and \( N_2 \times N_2 \) identity matrices, respectively, and \( A, b, c \) and \( d \) are real matrices of appropriate dimensions. The design problem is to find a set of coefficient matrices \( A, b, c \) and scalar \( d \) which minimize the error criterion as follows:

\[ E_2 = \sum_m \sum_n |(H_d(\omega_{1m}, \omega_{2n})| - |H(\omega_{1m}, \omega_{2n})|)^2 \]  

(2)

subject to the constraint that the designed filter is stable, where \( m \) and \( n \) are sample points in the 2-D frequency plane, and \( H_d \) and \( H \) are the given magnitude response and the magnitude response of the resultant filter, respectively.

3. Coding and Fitness function

All coefficients \( a_{ij}, b_i, c_j \) of matrices \( A, b, c \) and \( d \) are encoded into the binary codes of \( B \) bits inside the interval \((-1, 1)\) according to the following binary representation:

\[ a_{ij}^{(k)} = \frac{(-1)^{a_{ij}+n}}{2^{B-1}} \sum_{b=0}^{B-1} \alpha_{ij,b} 2^{b-1} \]  

(3)

where \( i = 1, \cdots, N_1 + N_2 \), \( j = 1, \cdots, N_1 + N_2 \) and \( k = 1, \cdots, N; N \) is a population size. The remaining coefficients \( b_i \), \( c_j \) and \( d \) are also encoded in the same manner defined in Eq. (3). The fitness of a chromosome is defined as

\[ f(k) = \begin{cases} \text{ranking}(-E_2), & \text{if the filter is stable} \\ F_s, & \text{if the filter is unstable} \end{cases} \]  

(4)

where \( k = 1, 2, \cdots, N \) and the parameter \( F_s \) is a predefined small fitness value, and \( \text{ranking}(\cdot) \) represents rank based fitness scaling procedure [4].

4. Procedure of a GA

The design procedure of a GA used in the proposed method is similar to that of SGA [3]. The difference between two procedures is that the crossover operation used in our method is uniform crossover [4]. In addition, an elite strategy where one chromosome or a few of the best chromosomes are copied into the succeeding generation is used.

5. Stability Test

The stability of 2-D separable denominator state-space digital filters is checked by using the following two conditions:

\[ |\lambda_i(A_1)| < 1, \quad i = 1, 2, \cdots, N_1 \]  

(5)

\[ |\lambda_j(A_4)| < 1, \quad j = 1, 2, \cdots, N_2 \]  

(6)

where \( \lambda_i(A_1) \) and \( \lambda_j(A_4) \) denote the eigenvalues of state transition matrices \( A_1 \) and \( A_4 \), respectively. In addition, to maintain enough stable chromosomes and to obtain a stable filter with small approximation error, we confine the search range of the coefficients of \( A_1 \) and \( A_4 \) to \((-0.7, 0.7)\).

6. Illustrative Example

We consider the magnitude specification of a 2-D filter given by [2]. For the given specification, when we select the population size \( N = 200 \), crossover rate \( P_c = 0.85 \), mutation rate \( P_m = 0.02 \), bit length \( B = 16 \), elite number \( S_e = 2 \) and termination condition \( C_{tc} = 100 \) as GA parameters in this design procedure, we obtain a \((4, 4)^{th}\)-order digital filters. Fig. 1 shows the magnitude response of the resultant filter. The normalized approximation error \( e_2 \) is 8.87%.

References


