Study on a Lunar Approach Strategy Tolerant of a Lunar Orbit Injection Failure*

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Discussed in this paper is a lunar approach strategy tolerant of a lunar orbit injection (LOI) failure. LOI is one of the most critical events for a lunar orbiting mission. If LOI is not performed, the spacecraft flies by the moon, and in the worst case, it escapes not only from the moon but also from the earth, which leads to mission failure. The proposed strategy is to design a trajectory so as to re-encounter the moon even when LOI is not performed. It provides an opportunity for mission recovery even in the case of an unexpected fly-by. A trajectory design procedure is introduced and an example of the designed trajectory is shown.

Key Words: Lunar Mission, Trajectory Design, Lunar Orbit Injection, Failure Recovery

Nomenclature

i	: inclination of a spacecraft's orbit
m, n	: natural number
$\mathbf{n}_{op}$	: orbit plane vector
P	: orbit period
P_m	: orbit period of the moon
R_m	: radius of the moon orbit
r	: radius of a spacecraft's orbit
r_p	: perilune radius of a spacecraft's orbit
v	: magnitude of $\mathbf{v}$
$\mathbf{v}_m$	: velocity of the moon
$\mathbf{v}_{pre}$	: velocity of a spacecraft before swing-by
$\mathbf{v}_{post}$	: velocity of a spacecraft after swing-by
$\mathbf{v}_{\infty in}$	: incoming excess velocity of a spacecraft
$\mathbf{v}_{\infty out}$	: outgoing excess velocity of a spacecraft
X, Y, Z	: inertial coordinate system
x, y, z	: local coordinate system
x_B, y_B, z_B	: ballistic plane coordinate system
α_{sync}	: angle between $\mathbf{v}_m$ and $\mathbf{v}_{\infty in}$ (or $\mathbf{v}_{\infty out}$)
Δv	: velocity increment
γ	: angle between $\mathbf{v}_m$ and a spacecraft's orbit plane
θ_{max}	: the maximum deflection angle
θ_{tp}	: deflection angle at a swing-by

1. Introduction

With the opening of the new century, the moon is attracting attention again as a target of space exploration. SMART-1 launched by ESA, which orbited and impacted on the moon, is the lead-off visitor to the moon of the new century. Japan is developing SELENE, which is planned to be launched in

summer, 2007. China and India are planning their first missions to the moon, and the United States is refocusing on human exploration of the moon.

A lunar orbit injection (LOI) is one of the most critical events for lunar orbiting missions. If LOI is not performed, the spacecraft flies by the moon, and in the worst case, it escapes not only from the moon but also from the earth. Fig. 1 and Fig. 2 show an example of a lunar transfer sequence, drawn in an earth-centered inertial coordinate system. Z axis is in the direction of the orbit plane vector of the moon, X axis is in the direction of the perigee of the moon, and Y axis is perpendicular to Z and X axes. The three axes are fixed at the beginning of the sequence, and they are not updated during the sequence. As is shown in Fig. 1, if LOI is not performed, the spacecraft on the approaching hyperbolic orbit flies by the moon and escapes from the moon. Moreover, as is shown in Fig. 1, as a result of the lunar swing-by, the spacecraft is accelerated and escapes from the earth. The sequence flow of a typical lunar transfer can be depicted as Fig. 2.

One method to avoid this kind of risk is to use a gravity capture by the moon¹⁾. This method was firstly demonstrated by HITEN, and it was planned to be used by LUNAR-A. In this method, the perturbation from the earth's gravity is effectively utilized to decelerate the spacecraft on the lunar approaching trajectory. Finally, the spacecraft is captured by the moon and orbits around the moon naturally without any active maneuver, which results in a complete avoidance of the risk mentioned above. However, this method is inconvenient because it requires a long process and duration to prepare for the lunar approaching condition to be captured.

Another method to avoid the risk is to design a lunar approaching trajectory considering the case that LOI is not performed. That is to say, to design the trajectory so as to

provide an opportunity for the mission recovery even in the case of an unexpected fly-by. An example of this method is the free return trajectory used in the Apollo program²⁾. By the usage of this trajectory, the astronauts would have been able to come back safely to the earth even in the case of LOI failure. However, as this was the case of a manned mission, returning to the earth was considered as the mission recovery.

(a) The lunar transfer

(b) A close up of the lunar approach
Fig. 1. An example of a lunar transfer sequence.

Fig. 2. The sequence flow of a typical lunar transfer.

(a) The injection into MSO

(b) In the case of a transient fault

(c) In the case of a permanent fault

(d) The definition of Robust LOI

Fig. 3. The sequence flow of the proposed concept.

Discussed in this paper is a lunar approach strategy based on the latter risk avoidance method. That is to say, a trajectory is designed to re-encounter the moon even in the case that LOI is not performed (Fig.3(a)). In other words, a trajectory is designed so that the spacecraft is injected into the moon synchronous orbit (MSO) after the fly-by even in the case of LOI failure. In the case that LOI failure is due to a transient anomaly (such as an operation failure, a configuration set up delay, etc.), to re-encounter the moon provides another opportunity for LOI, which leads to an almost perfect mission recovery (Fig.3(b)). In the case that the injection failure is due to a permanent anomaly (such as main engine trouble, etc.), a drastic restructure of the trajectory may be required for the mission recovery. Even in this case, to re-encounter the moon provides the most effective means of an orbit maneuver, that

is, a lunar swing-by (Fig.3(c)). From this point of view, proposed in this paper is a lunar approach strategy tolerant of LOI failure. In this strategy, the condition of injection into the mission orbit, and the condition of injection into MSO are to be satisfied simultaneously. LOI which satisfies these conditions simultaneously is called “Robust LOI” hereafter (Fig.3(d)). Restrictions and expense to adopt this strategy are discussed in the following sections.

2. Robust LOI at First Encounter

2.1. Injection into MSO

MSO is a geocentric orbit whose P is expressed as

$$P = \frac{m}{n} P_m \quad (1)$$

where m and n are assumed to be irreducible. If a spacecraft is injected into MSO at the position of the moon, it will re-encounter the moon after $m \times P_m$ (here, the gravity of the moon and the other perturbations are neglected.) Even if LOI is not performed normally, if the spacecraft is injected into MSO after the fly-by, it will re-encounter the moon and will be able to retry LOI.

To investigate the condition to inject into MSO, a local coordinate system is defined as Fig. 4. The origin of the frame is the moon, x axis is in the direction of the earth, y axis is perpendicular to x axis and the north pole direction of the moon, and z axis is perpendicular to x and y axes. For a simplification, y axis is assumed to be exactly opposite to the velocity of the moon, and z axis is assumed to be identical with the north pole direction of the moon.

Fig. 4. Definition of a local coordinate system.

Fig. 5. A simplified model of a lunar swing-by.

Fig. 5 gives a simplified model of a lunar swing-by, projected on xy plane. $\mathbf{v}_m$ is in $-y$ direction. $\mathbf{v}_{pre}$ is also in $-y$ direction, assuming that the spacecraft is at the apogee

of a long elliptical lunar transfer orbit. $\mathbf{v}_{\infty \text{ in}}$ is deflected by the swing-by to be $\mathbf{v}_{\infty \text{ out}}$ after the swing-by. $\mathbf{v}_{\text{post}}$ is obtained by adding $\mathbf{v}_{\text{m}}$ and $\mathbf{v}_{\infty \text{ out}}$. θ_{max} is determined from the three parameters, the gravity constant of the moon, $\mathbf{v}_{\infty \text{ in}}$, and the possible minimum perilune radius. θ_{max} limits the ranges of $\mathbf{v}_{\infty \text{ out}}$ and $\mathbf{v}_{\text{post}}$ attainable by a single swing-by.

On the other hand, if a target $\mathbf{v}_{\text{post}}$ is given, $\mathbf{v}_{\infty \text{out}}$ is obtained by subtracting $\mathbf{v}_m$ from $\mathbf{v}_{\text{post}}$. Assuming a free (not powered) swing-by, $v_{\infty \text{out}}$ is constrained to be equal to $v_{\infty \text{in}}$ (that is, the top of $\mathbf{v}_{\text{post}}$ is constrained to be on the circle, whose center is the top of $\mathbf{v}_m$, and whose radius is $v_{\infty \text{in}}$). The relation between $\mathbf{v}_{\infty \text{out}}$ and $\mathbf{v}_{\infty \text{in}}$ gives the swing-by condition to attain the target $\mathbf{v}_{\text{post}}$. For example, the velocities to inject a spacecraft into MSO are shown in Table 1. R_m is assumed to be constant here ($\approx 3.844 \times 10^8 \text{ m}$). Drawn in Fig. 6 are the contours of $\mathbf{v}_{\text{post}}$ to inject a spacecraft into MSO, overwritten on the swing-by diagram of Fig. 5. $\mathbf{v}_{\infty \text{out}}$ to attain MSO are obtained as the intersections of the contours and the circle of attainable $\mathbf{v}_{\infty \text{out}}$.

Table 1. The velocities to attain MSO.

Period of MSO	Velocity at R_m
$1 \times P_m$	1018.3m/s
$(1/2) \times P_m$	654.1m/s
$2 \times P_m$	1191.9m/s
$(2/3) \times P_m$	845.6m/s
$(2/5) \times P_m$	404.7m/s

Fig. 6. $v_{\infty \text{out}}$ to attain MSO (with the period of $1 \times P_m$).

2.2. A mission requirement and MSO injection

To discuss the approaching condition to attain MSO, the ballistic plane coordinate system is introduced. y_B axis is in the direction of $\mathbf{v}_{\infty in}$, x_B axis is perpendicular to y_B axis and the north pole direction of the moon, and z_B axis is perpendicular to x_B and y_B axes. Under the simplified assumption introduced in Section 2.1, the three axes are identical with those of the local coordinate system. An approaching trajectory is specified as a point (target point) on $x_B z_B$ plane of the ballistic plane coordinate system (B-plane.) The target point is the intersection of B-plane and the asymptote of the approaching hyperbola.

Under this simple assumption, the approaching condition to attain MSO after the swing-by (MSO condition) forms circles with its center at the origin. In Fig. 7, they are drawn in thin lines, and are labeled with MSO's orbit periods.

On the other hand, the main task of LOI is to inject the spacecraft into the mission orbit. In the following discussions,

as a typical case, it is assumed that the mission requirement assigns r_p and i . The approaching condition to attain the mission requirement can be also discussed on B-plane. Assuming that target r_p is 1838km and target i is 90deg. (in the selenographic coordinate system), the approaching condition to attain target r_p and i forms a thick circle and a thick line, respectively, in Fig. 7. The intersections of the thick circle and the thick line are the target points to attain the mission requirement. Apparently, they are not on the circles of MSO condition, that is, the mission requirement and MSO condition are not satisfied simultaneously (in other words, Robust LOI condition is not satisfied.)

Fig. 7. The mission requirement and MSO condition (in the simplified model).

It is true that the symmetric shape of MSO condition shown in Fig. 7 is the result of the adoption of the simplified assumption. However, the situation does not change drastically even in the practical condition. Fig. 8 is drawn based on the approaching condition of the sequence shown in Fig. 1. The relation between $\mathbf{v}_m$ and $\mathbf{v}_{\infty in}$ (or between $\mathbf{v}_m$ and y_B axis) is essential, and in this case, $\mathbf{v}_m$ is (227.3m/s, -1038.7m/s, 6.0m/s) on the ballistic plane coordinate system (note that, in the simplified model, $\mathbf{v}_m$ is exactly in $-y$ direction.) The circles of MSO condition slightly warp and shift; however, the relation between the mission requirement and MSO condition does not change very much.

These are general characteristics as far as adopting a near-optimal lunar transfer. In a near-optimal lunar transfer, the lunar transfer orbit is a highly elliptical orbit whose apogee radius is approximately R_m . The magnitude of the spacecraft's velocity around the apogee, which is expressed as v_{pre} in Fig. 5, is much smaller than v_m . Therefore, $\mathbf{v}_{\infty in}$ is almost in the opposite direction of $\mathbf{v}_m$. That is, the spacecraft always approach from the front side of the moon. In summary, MSO condition on B-plane changes if the approaching direction changes. However, as far as a near-optimal lunar transfer is adopted, the approaching direction does not change very much, and consequently the relation between the mission requirement and MSO condition does not change very much.

Fig. 8. The mission requirement and MSO condition (in a practical condition).

Fig. 9. The sequence flow of an indirect injection.

Fig. 10. An example of an intermediate target.

It is a possible option not to inject the spacecraft directly into the mission orbit. That is to say, the spacecraft is firstly injected into an intermediate orbit that satisfies MSO condition (but does not satisfy the mission requirement), and

then transfers to the mission orbit. The sequence flow of this option is shown in Fig. 9.

Fig. 10 shows an example of a possible intermediate target in the condition of Fig. 9. The intermediate target satisfies MSO condition. Target r_p is 5360km and target i is 90deg (in the selenographic coordinate system). This option is feasible; however, an injection at the large r_p results in an increase of LOI Δv . Assuming that the mission orbit is the circular orbit whose r is 1838km and i is 90deg. (in the selenographic coordinate system), total Δv required for this indirect injection is approximately 270m/s larger than that required for the direct injection.

3. Robust LOI at Second Encounter

3.1. The basic concept

As is discussed in Section 2, MSO condition on B-plane changes if the approaching direction changes. To change the direction of the relative velocity of a spacecraft against the moon, the efficient way is to use a swing-by. Of course, after the direction of the relative velocity is changed by the swing-by, the spacecraft must re-encounter the moon to perform the actual LOI. That is, the spacecraft must be injected into MSO after the first swing-by.

To summarize the concept, at the first encounter with the moon, the spacecraft is “planned” to fly by the moon. By the swing-by, the spacecraft is injected into MSO; at the same time, the approaching direction at the second encounter is set so that Robust LOI condition is satisfied. At the second encounter, if LOI is performed normally, the spacecraft is injected into the mission orbit. And even if LOI is not performed normally, the spacecraft is injected into MSO after the fly-by, and it will re-encounter the moon and will be able to retry LOI. The sequence flow of the concept is shown in Fig. 11.

Fig. 11. The sequence flow of Robust LOI at the second encounter.

3.2. The conditions for the second encounter

Investigated in this section is the approaching direction at the second encounter to attain Robust LOI condition.

Here we use the simplified model introduced in Section 2.1. Additionally, the periods of MSO before and after the second encounter are assumed to be the same, and $v_{\infty in}$ at the second encounter and $v_{\infty out}$ of the first encounter are assumed to be the same. The latter assumption means that the orbit of the moon and MSO are both non-perturbed elliptical orbits.

Firstly discussed is the orbit plane of the approaching hyperbola which satisfies MSO condition and the mission requirement for r_p (Fig. 12.) α_{sync} is determined from MSO condition. Additionally, the mission requirement for r_p determines θ_{rp} . Then, γ is obtained from α_{sync} and θ_{rp} using the spherical geometry,

$$\cos \gamma = \frac{\cos \alpha_{sync}}{\cos(\theta_{rp}/2)} \quad (2)$$

Fig. 12. The orbit plane to satisfy Robust LOI condition.

There are two possible n_{op} s for a given orbit plane. The angles between v_m and the two n_{op} s are $\gamma + \pi/2$ and $\gamma - \pi/2$ respectively. On the other hand, n_{op} is also required to satisfy the mission requirement as to i . That is, the angle between n_{op} and z axis are required to be i . In summary, n_{op} is determined from the following two conditions.

- n_{op} is inclined with v_m in the angle of $\gamma + \pi/2$ or $\gamma - \pi/2$.
- n_{op} is inclined with z axis in the angle of i .

From these conditions, the four possible n_{op} s are obtained as shown in Fig. 13.

Fig. 13. The four possible solutions for Robust LOI.

Once n_{op} is obtained, $v_{\infty in}$ (or $v_{\infty out}$) is determined from the following two conditions.

- $v_{\infty in}$ (or $v_{\infty out}$) is inclined with v_m in the angle of α_{sync} .
- $v_{\infty in}$ (or $v_{\infty out}$) is perpendicular to n_{op} .

$v_{\infty in}$ and $v_{\infty out}$ are set so that $v_{\infty in} \times v_{\infty out}$ is in the direction of n_{op} .

Obtained by the procedure above is $v_{\infty in}$ at the second encounter which satisfies Robust LOI condition. From the assumption of the simplified model, $v_{\infty in}$ at the second encounter is the same as $v_{\infty out}$ of the first encounter. The approaching condition at the first encounter is determined to attain this $v_{\infty out}$ after the swing-by.

Fig. 14 shows an example of the targeting at the first encounter. The approaching condition at the first encounter is the same as that of Fig. 8. The periods of MSO before and after the second encounter are assumed to be P_m . No perturbation is considered for the orbit of the moon or MSO. Four targeting points are placed corresponding to the four possible solutions shown in Fig. 13. Of course, they are on the circle of $(1 \times P_m)$ MSO condition.

Fig. 14. An example of the targeting at the first encounter.

Fig. 15. The mission requirement and MSO condition (at the second encounter).

Fig. 15 shows B-plane at the second encounter, in the case of adopting Target #4 in Fig. 14. MSO condition on B-plane

drastically changes due to the change of the approaching direction. Now, the mission requirement and MSO condition are satisfied simultaneously (labeled “Target” in the figure,) that is, Robust LOI.

3.3. An example sequence

Using the result of the previous section (Target #4 in Fig. 14) as an initial estimate, a consistent sequence is constructed under the multi-body model. The perilune passage condition at the second and the third encounter are adjusted by several midcourse correction maneuvers. Note that the maneuver Δv is not completely optimized (minimized), where a space for the reduction still remains.

(a) Projection on XY plane

(b) Projection on XZ plane

Fig. 16. An example sequence of Robust LOI.

Fig. 16 shows the trajectory of the sequence in the earth-centered inertial coordinate system. As explained in Section 1, the three axes of this coordinate system are defined based on the moon orbit. The three axes are fixed at the beginning of the sequence, and they are not updated during

the sequence (in practice, the moon orbit is perturbed during the sequence, and it can be observed in Fig. 16 (a).) Fig. 16 (a) shows the projection of the trajectory on XY plane, that is the orbit plane of the moon, and Fig. 16 (b) shows the projection of the trajectory on XZ plane.

Fig. 17. The sequence of Robust LOI (close up of the second encounter).

Fig. 18. The sequence of Robust LOI (close up of the third encounter).

Firstly, a spacecraft transfers to the moon in almost the same trajectory as that of Fig. 1. By the swing-by at the first encounter with the moon, the spacecraft is injected into MSO. MSO is carefully chosen to satisfy Robust LOI condition at the second encounter with the moon. About one month later, with 9m/s of the midcourse maneuver, the spacecraft re-encounters the moon. The close up of the trajectory around

the second encounter is shown in Fig. 17. It is the trajectory projected on YZ plane. At the perilune passage, normally, LOI is performed and the spacecraft is injected into the mission orbit, whose r_p is 1838km and i is 90deg. (in the selenographic coordinate system.) In the case that LOI is not performed for some reason, the spacecraft will just fly by the moon at the second encounter. However, even in this case, the spacecraft will be injected into MSO again, and about one month later, with 45m/s of the midcourse maneuver, the spacecraft will re-encounter the moon. The close up of the trajectory around the third encounter is shown in Fig. 18. It is the trajectory projected on YZ plane. At the perilune passage, LOI is performed and the spacecraft is injected into the mission orbit.

4. Conclusion

Discussed in this paper is a lunar approach strategy tolerant of LOI failure. The proposed strategy is to design a trajectory so as to inject a spacecraft into MSO even when LOI is not performed. It provides an opportunity for mission recovery even in the case of unexpected fly-by. The concept is named Robust LOI, and possible sequences to attain Robust LOI are introduced.

Firstly discussed is Robust LOI at the first lunar encounter. It is shown that, from general characteristics of a near-optimal lunar transfer, MSO condition is strongly constrained. Therefore, Robust LOI condition is not satisfied generally. One possible option is not to inject the spacecraft directly into the mission orbit; however, it is shown that it requires the respectively large additional Δv .

Secondly discussed is Robust LOI at the second lunar encounter. In this concept, the spacecraft is planned to fly by the moon at the first lunar encounter. By the swing-by, the spacecraft is injected into MSO; at the same time, the approaching direction at the second encounter is set so that Robust LOI condition is satisfied. A procedure to achieve Robust LOI condition at the second lunar encounter is introduced and an example of the designed trajectory is shown.

The result shows that Robust LOI at the second lunar encounter provides an option to reduce the risk of mission failure with relatively small Δv increase at the cost of one month's delay of the moon arrival.

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