Study on the Characteristics of Two-burn Translunar Trajectory*

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In this paper, the translunar trajectory of the simple sequence is investigated. The trajectory discussed in this paper is a two-burn ballistic trajectory from the low earth parking orbit to the low lunar orbit. The problem is practical in that the geometric relation from the launch site to the moon is fully considered. The paper includes three subjects. First is the structure of the problem and the solution space. The problems are defined and the solutions are grouped by the properties of the lunar transfer sequence. Second are the characteristics of optimal solutions. The topics discussed are the transition of the required velocity increment based on the launch date, and the difference according to the property of the lunar transfer sequence. Third is the sensitivity analysis of a number of items to the deviation of the parameters from the optimal solution.

Key Words: Lunar Mission, Trajectory Design, Lunar Transfer

Nomenclature

LAO :	lunar approach orbit
LEO :	low earth orbit
LSOI :	lunar sphere of influence
LTO :	lunar transfer orbit
PLP :	perilune passage
α :	out of LEO plane angle of LTO injection Δv
β :	in LEO plane angle of LTO injection Δv
Δv :	velocity increment
$\Delta v_x, \Delta v_y, \Delta v_z$:	components of LTO injection Δv in LTO injection coordinate system
θ :	transfer angle on LTO

1. Introduction

With the opening of this new century, the moon has been attracting attention again as the target of space exploration. SMART-1, launched by ESA and orbited and impacted on the moon, is the lead-off visitor to the moon of the new century. Japan has been developing SELENE, which is planned to be launched in summer, 2007. China and India are planning their first mission to the moon, and the United States is refocusing on the human exploration of the moon.

Looking from the point of lunar transfer trajectory, most of the spacecrafts, including scores of spacecrafts in the last century, approach the moon directly. Two exceptions are SMART-1 and the formerly planned LUNAR-A, though their methods are quite different. They take a round-about but efficient way to the moon. Investigated in this paper is the former, a two-burn direct translunar trajectory.

There are many papers which deal with the translunar

trajectory of the actual lunar missions¹⁻⁸⁾. The papers show the results of the trajectory design considering the practical conditions, requirements, and constraints of the respective missions. However, the results are specific to the missions, and general characteristics of the translunar trajectory are not investigated in detail. On the other hand, there are many papers which analyze the general characteristics of the translunar trajectory⁹⁻¹⁷⁾. However, in most of the papers, the scopes of the analyses are restricted and simplified. Widely investigated is the most simplified problem which assumes the two dimensional motion (that is, the motion of the spacecraft is restricted in the moon orbit plane) and the circular orbiting moon⁹⁻¹⁴⁾. Among them, Hiller⁹⁾ and Hantzsche¹⁰⁾ investigated the problem in detail, and the classification of the translunar trajectory in this scope is mostly completed in their works. However, it is natural that an analysis based on this simplified problem has a limitation. For example, it cannot deal with the translunar trajectory which starts from the low earth orbit out of the moon orbit plane, which is often the case in practical missions. As a more realistic problem, the three-dimensional translunar trajectory is analyzed in several papers¹⁵⁻¹⁷⁾. In particular, Breuer¹⁷⁾ considered the launch site geometry and the monthly motion of the moon. The problem definition adopted by Breuer is sufficient to deal with most of the topics faced in the practical mission analysis. However, discussed by Breuer is a very specific topic, that is, the minimum lunar orbit inclination attainable under a certain condition.

In this paper, general characteristics of the translunar trajectory are investigated under the problem definition following Breuer. The problem is practical in that it considers the geometric relation from the launch site to the moon as well as the monthly motion of the moon. The topics mainly discussed are the structure of the solution space and the characteristics of the required Δv , which are widely applicable to practical trajectory design and mission analysis.

As is shown in the next section, the problem is basically

Fig. 1. Lunar transfer sequence.

formulated as parameters optimization under some constraints. Any well known numerical tool will easily converge the parameter set to the stationary point, that is, the local optimum solution. However, it is sometimes problematic to use the result without any knowledge of the structure of the problem and the solution space. Overlooking the better solution is the typical case. In Section 2, the problem is defined and the structure of the problem and the solution space is investigated. As a result, the solutions are grouped by two properties of the lunar transfer sequence. The optimal solution is basically a Hohmann-type transfer connected to the lunar approaching hyperbolic trajectory. However, it has several characteristics to be pointed out. Discussed in Section 3 are the transition of Δv based on the launch date, and the difference according to the property defined in the previous section. Although " Δv optimal" is the most significant factor to be considered, many other items must be taken into account in a practical problem. And sometimes, parameters that deviate from the optimal solution are chosen to improve the other items. Discussed in Section 4 is the effect of the parameters' deviation from the optimum solutions.

2. Problem Definition

Figure 1 describes the lunar transfer sequence investigated in this paper. The sequence starts from the launcher's injection to LEO. After coasting for a while, the spacecraft is injected into the translunar trajectory by impulsive maneuver of the launcher. The spacecraft approaches the moon without any additional maneuver, and the sequence terminates at the spacecraft's PLP.

Figure 2 describes the boundary conditions and the control parameters of the problem. The state at LEO injection is fully given in the earth fixed coordinate system. This means that the launcher, the launch site and the launch trajectory from the launch site to LEO injection is fully specified. The only variable here is the time of LEO injection, which determines the direction of LEO orbital plane in the inertial coordinate system. The time of translunar trajectory injection determines the state

Fig. 2. Boundary condition and control parameters.

Fig. 3. Approximate optimal lunar transfer.

just before the injection, and with Δv of the injection maneuver, the state just after the injection, that is the initial state of the translunar trajectory, is determined. As to the translunar trajectory, the patched conic method is applied¹⁸⁾. The trajectory is divided into two sections. They are LTO from the translunar trajectory injection to LSOI injection, and LAO from LSOI injection to PLP. The two orbits are connected at LSOI injection both in position and velocity. The state at PLP is partially assigned, but the details are discussed later.

The parameters are LEO injection time, LTO injection time, and LTO injection Δv . LTO injection Δv has three degrees of freedom, two for its direction and one for its magnitude. There are five parameters all together; however, possible ranges of their values are strongly restricted from the point of minimizing the energy for the transfer.

Roughly speaking, LTO is the transfer orbit between noncoplanar circular orbits, LEO and the moon orbit. Minimum energy two-impulse transfer of this type is generalized Hohmann transfer¹⁸⁾. The transfer orbit is an elliptical orbit cotangential with the two circular orbits at the apses, and the plane is changed at the apogee maneuver (Fig. 3). This indicates that the direction of LTO injection Δv should be tangential to LEO and the transfer angle on LTO should be 180 deg. for the minimum energy transfer. The terminal of LTO, that is the apogee of LTO, is assigned from the position of the moon at the arrival. This indicates that LEO injection time should be assigned so that the assigned LTO terminal is included on the LEO orbital plane, which is identical to the LTO orbital plane. In addition, LTO injection time should be assigned so that LTO injection position, that is the perigee of LTO, is the opposite side of the assigned LTO terminal. LEO, which is fixed to the earth, rotates around the earth's axis once a day. Therefore, there are two chances of LEO injection per day when the assigned LTO terminal is on the LEO orbital plane (Fig. 4). These two cases are called "short coast" and "long coast" after the length of the coasting on LEO.

The rough investigation above assigns the approximate values of the parameters for the minimum energy transfer. Next, the values of the parameters are adjusted to satisfy the boundary condition at the terminal, PLP. At this point, two of the parameters that define the direction of LTO injection Δv are fixed to the optimal values, that is, tangential to LEO. The

Fig. 4. Short coast and long coast.

reason is that the magnitude of LTO injection Δv is more sensitive to these two parameters. The effect of the deviation from the optimal values is investigated in Section 4 for these two parameters. In addition, the parameter “LTO injection Δv magnitude” is replaced with θ for the handling convenience (Fig. 5). Both parameters determine the shape of the LTO ellipse; however, the latter is more useful in that it can distinguish between the LSOI injections before and after LTO apogee passage. Now, the remaining three parameters, LEO injection time, LTO injection time, and θ , are available to adjust the three boundary conditions at the terminal.

However, to keep the transfer energy within reasonable level, the values of the parameters cannot be far from that assigned for the minimum energy transfer. Therefore, the possible range of PLP state is limited. For example, two of the orbital elements of LAO, the semimajor axis and the direction of the nodes with the lunar equatorial plane, are determined by the relative velocity of the spacecraft against the moon at LSOI injection. However, the relative velocity at LSOI injection is almost fixed for the minimum energy transfer. Its magnitude is approximately in the range from 800m/s to 900m/s and its direction is almost against the moon’s orbital motion. As a result, the values of these two orbital elements are almost fixed for the transfer with reasonable energy.

The combination of three boundary conditions is adaptable. The examples of possible combinations are to assign the values of three variables, or to assign the values of two state variables and minimize one performance index, and so on. Boundary conditions used in the following discussion are, to assign the altitude and the inclination at PLP and minimize the velocity at PLP. This is general type of boundary condition for a

Fig. 5. Definition of θ (transfer angle on LTO).

Fig. 6. Ascending approach and descending approach.

lunar orbiting mission. For a given relative velocity vector at LSOI injection, there are two LAO orbital planes that include the velocity vector. Two cases are called “ascending approach” and “descending approach” after the direction of the motion at PLP (Fig. 6).

Discussion as to the structure of the problem and the solution space is summarized as follows. There are five parameters all together; however, at this point, two of them are fixed to the optimal values. Therefore, the number of free parameters turns out to be three. The number of boundary conditions to be satisfied is also three. This indicates that the number of parameter sets that satisfy the conditions is finite. Two properties characterize the lunar transfer sequence. The first is the property as to the coasting on LEO, that is, “short coast” and “long coast”. The second is the property as to the approaching direction of LAO, that is, “ascending approach” and “descending approach”. These properties can be selected independently; therefore, there are four possible sets of parameters (per day) that satisfy the boundary conditions.

3. Optimal Solution Characteristics

Discussed in this section are the characteristics of the solutions obtained under the condition defined in the previous section.

Previous to the discussion, the conditions used in the analysis are described in the following. As to the initial condition, LEO, it is assigned as the circular orbit with the altitude 290km, launched eastward from Tanegashima space center (131.0deg.E, 30.4deg.N). For the convenience of handling “short coast” and “long coast”, the injection point is assumed to be the north edge of the orbit. As to the connection of LTO and LAO, the radius of LSOI is assigned as 66280km. As to the terminal condition, PLP, the altitude is assigned as 100km and the inclination is assigned as 90deg. against the moon equatorial plane.

Firstly discussed is the transition of Δv based on the launch date. Figure 7 shows the transition of two Δv 's by the launch date from July to September in 2005. The two Δv 's are LTO injection Δv (solid squares) and the low lunar orbit (LLO) injection Δv (solid circles). LLO is the circular orbit with the

Fig. 7. LTO injection Δv and LLO injection Δv .

altitude of 100km. The information as to the position of the moon at LSOI injection is also marked in the figure. These marks express the relation with the apses of the moon orbit (perigee and apogee) and the relation between the nodes of the moon orbit and the earth equatorial plane (ascending node and descending node). The figure shows the result of the case “short coast” and “descending approach”. However, the latter property doesn’t matter so much for these Δv ’s, since these Δv ’s are determined mainly by LTO, whereas the latter property is mainly related to LAO.

The transition of LTO injection Δv is simple. Since this Δv is determined mainly from the size of LTO (note that the direction of LTO injection Δv is fixed to be tangential to LEO), Δv is large when the moon is far (at the apogee) and is small when the moon is near (at the perigee).

The transition of LLO injection Δv reflects the transition of the relative velocity of the spacecraft against the moon at LSOI injection. The relative velocity is determined not only from the difference in velocity magnitude, but also from the difference in velocity direction. The difference in velocity magnitude is large when the moon is at the perigee and is small when the moon is at the apogee. The difference between the velocities of the moon at these two points overtakes the difference between the velocities of the spacecraft at these two points. The difference in the velocity direction is mainly determined from the angle between the LTO orbital plane and the moon orbital plane at their intersection. In the case of “short coast” here, the spacecraft is injected into LTO in the descending direction (note that the discussion here is the case that the LEO injection point is in the northern hemisphere), and it moves in the ascending direction at the terminal of the LTO (See Fig. 4). Therefore, the angle between the velocities at the intersection is large when the moon is at the descending node and is small when the moon is at the ascending node. The transition of LLO injection Δv reflects the transition of these two factors. There are peaks at the perigee and the descending node. On the other hand, the bottoms at the ascending node are obtuse, and they are combined with the adjacent bottoms at the apogee. Therefore, they appear to be combined bottoms of the ascending node and the apogee. It is observed that the interval between the two peaks extends little by little. This is the effect of the perturbation of the solar gravity to the moon orbit.

Fig. 8. LLO injection Δv of short/long coast.

Secondly discussed is the difference according to the property of “short coast” and “long coast”. Since these two cases differ from the start of the sequence, difference resulting from this property can be found in many aspects. However, the essence is the difference of LTO orbital plane and the difference of LTO injection point.

The most important effect caused by the difference of LTO orbital plane is the difference in the transition of LLO injection Δv by the launch date. In contrast to the case of “short coast” discussed before, in the case of “long coast”, the spacecraft moves in the descending direction at the terminal of LTO (See Fig. 4). Therefore, the transition of the difference in the velocity direction between the spacecraft and the moon shows a trend inverse to that observed in the case of “short coast”. That is to say, the difference in the velocity direction is large when the moon is at the ascending node and is small when the moon is at the descending node. On the other hand, the transition of the difference in the velocity magnitude between the spacecraft and the moon shows the same trend as that observed in the case of “short coast”. The transitions of these two factors result in the transition of LLO injection Δv shown in Fig. 8. In the figure, the transitions of LLO injection Δv of the two cases are shown (“short coast” in solid circles and the “long coast” in solid squares) with the marks that signify the position of the moon at LSOI injection. In the case of “long coast”, there are large peak results from the combination of the adjacent peaks of the perigee and the ascending node. In addition, for about half of a month, the LLO injection Δv in the case of “long coast” is less than that in the case of “short coast”.

Other examples of the effect caused by the difference of LTO orbital plane are the difference in the visibility of the spacecraft on LTO from the ground station in the northern/southern hemisphere, the difference in the argument of perilune at PLP, and so on.

Next, the most important effect caused by the difference of LTO injection point is the duration of the coasting on LEO. In most cases, the launcher is still in operation on LEO. Therefore, the performance of the launcher (for example, the amount of the gas for the attitude control thrusters, or the accuracy of the inertial navigation system) usually limits the duration of the coasting. In practice, “long coast” is often not applicable from this limit. The difference of LTO injection point itself is also

Fig. 9. LAO of ascending/descending approach.

important. The factors to be considered are the visibility of the LTO injection maneuver from the ground station, the eclipse right after the spacecraft's separation from the launcher, and so on.

Finally discussed in this section is the difference according to the property of “ascending approach” and “descending approach”. The difference by this property is limited compared to the difference by the former property. The reason is that this property is mainly related to LAO, the final phase of the sequence, and basically it does not affect the former phase. (Of course the parameters in the former phase are adjusted to satisfy the boundary condition at the terminal; the effect of the difference in LAO caused by this property is small.) The difference by this property pointed out here is the difference of the argument of perilune at PLP. If the spacecraft is injected directly into the circular LLO, this value is not important. However, if the spacecraft is injected into the elliptical orbit, this value is closely related with the effect of the perturbation of the earth gravity.

Figure 9 shows LAO of the two cases projected on the x - z plane of the moon-centered moon orbital plane coordinate system. The green line is LAO in the case of “ascending approach” and the orange line is LAO in the case of “descending approach”. The launch date is August 4, 2005, and the property as to the coasting on LEO is “short coast”. Also drawn in the

Fig. 10. Definition of LTO injection Δv angle α and β .

figure are the straight lines and the ellipses to clarify the relation between the major axis of LAO and the moon orbital plane. The straight dash-and-dotted line is the major axis of LAO, and the ellipse in dashed line is the elliptical orbit with eccentricity 0.4 whose perilune point is identical with that of LAO. (The ellipse looks warped since the orbit is not viewed from the right in front of its orbital plane.) It indicates that the effect of the perturbation of the earth to the elliptical orbit is different between the two cases.

4. Sensitivity Analysis

Discussed in this section is the sensitivity of a number of items to the deviation of the parameters from the optimal solutions. However, the discussion is not for the estimation of the influence of some unintentional deviation, but for the estimation of the effect of some intentional deviation.

Firstly discussed is the deviation of the two parameters that define the direction of LTO injection Δv from the optimal values, that is, tangential to LEO. Two parameters, α and β , are introduced to express the direction of LTO injection Δv (Fig. 10). The LTO injection coordinate system is defined as Fig. 10. The origin of the frame is LTO injection, y axis is in the direction of the spacecraft's velocity just before LTO injection, z axis is in the direction of the orbit plane vector of LEO, and x axis is perpendicular to y , z axes. α , β are defined as

$$\alpha = \text{atan}\left(\Delta v_z / \sqrt{(\Delta v_x)^2 + (\Delta v_y)^2}\right) \quad (1)$$

$$\beta = \text{atan}(\Delta v_x / \Delta v_y) \quad (2)$$

For the optimal solution, where LTO injection Δv (and LTO as well) is tangent to LEO, the values of these two parameters are zero. Deviation of -5.0° for α and β is assumed and their effects are investigated. The value of deviation is selected so that the increment of LTO injection Δv from the optimal solution becomes about 10m/s, that is approximately the maximum difference of LTO injection Δv by the launch date (See Fig. 7). In the analysis, one of α and β is assigned the value of deviation mentioned above, and the other is assigned zero. The remaining three free parameters are used to adjust the three boundary conditions at the terminal. The properties of the sequence used in this analysis are “short coast” and “ascending approach”.

The following five items are selected to evaluate the effect of the deviation.

- LEO injection time
- LTO injection position
- PLP time
- Argument of perilune at PLP
- LLO injection Δv

These items are all closely related to practical operation. LEO injection time is related to the time of launch, the position of LTO injection is related to the visibility of the LTO injection maneuver, PLP time is related to the visibility of the LLO injection maneuver, the argument of perilune is related to the perturbation of the earth, and LLO injection Δv is the item of the most interest.

Firstly shown is the result as to the deviation of α . The effect

Table 1. Effect of deviation of α .

α	-5.0deg.	0.0deg.
(a)	2005/07/28 04:43:27	2005/07/28 04:45:16
(b)	104.7deg.W / 27.5deg.S	104.8deg.W / 27.4deg.S
(c)	2005/08/02 05:13:26	2005/08/02 05:24:17
(d)	331.5deg.	331.3deg.
(e)	777.9m/s	777.3m/s

of the deviation varies by the launch date. Table 1 shows the result of the launch date July 28, 2005, the launch date on which the effect is the largest. The position of the moon at the LSOI injection is about 90 deg. from the ascending node of the moon orbit, that is, the moon is at the north edge of its orbit. (a) to (e) are the selected items listed before. Obviously, the effect of the deviation of α is small even with this largest case. The effect by the deviation of α obtained in expense of 10m/s loss in LTO injection Δv are about 2 minutes in LEO injection time and 10 minutes in PLP time. LTO injection position, the argument of perilune at PLP, and LLO injection Δv hardly change.

Next is the result as to the deviation of β . The effect of the deviation varies by the launch date. Table 2 shows the result of the launch date July 31, 2005, the launch date on which the effect is the largest. The position of the moon at the LSOI injection is about the apogee of the moon orbit. The effect of the deviation of β is larger than in the case of α . The effect by the deviation of β obtained in expense of 10m/s loss in LTO injection Δv is about 15 minutes in LEO injection time, 3deg. in LTO injection position longitude, and 25 minutes in PLP time. The argument of perilune at PLP and LLO injection Δv hardly change.

Although the effect of the deviation of these two parameters is small, the deviation imposes no penalty, that is, the increase of LLO injection Δv , on the spacecraft (note that LTO injection maneuver is generally the role of the launcher). Therefore, for example, a few minutes' launch delay can be recovered by the launcher alone with small additional Δv , and without any effect on the spacecraft.

Thirdly discussed is the effect of the deviation of θ . In the analysis up to now, this parameter was used with the other two parameters, LEO injection time and LTO injection time, to adjust three boundary conditions at the terminal. However, in the analysis here, θ is assigned a fixed value that deviates from the value of the optimal solution. The two parameters that define the direction of LTO injection Δv are again fixed to the optimal values, that is, tangential to LEO. Therefore, the number of the remaining free parameters turns out to be two and the two boundary conditions here are to assign the altitude and the inclination at PLP. The properties of the sequence used in this analysis are "short coast" and "ascending approach".

Deviation of 5deg. for θ is assumed and the effects are

Table 2. Effect of deviation of β .

β	-5.0deg.	0.0deg.
(a)	2005/07/31 08:45:06	2005/07/31 09:00:54
(b)	129.4deg.W / 18.1deg.S	126.5deg.W / 19.5deg.S
(c)	2005/08/05 09:34:23	2005/08/05 09:09:51
(d)	337.5deg.	337.5deg.
(e)	795.9m/s	795.9m/s

Table 3. Effect of deviation of θ .

θ	+5.0deg.	0.0deg.
(a)	2005/07/31 11:57:42	2005/07/31 09:00:54
(b)	152.6deg.W / 4.5deg.S	126.5deg.W / 19.5deg.S
(c)	2005/08/08 00:10:54	2005/08/05 09:09:51
(d)	335.4deg.	337.5deg.
(e)	829.1m/s	795.9m/s

investigated. The value of deviation, "5deg.", is selected so that the increment of LLO injection Δv from the optimal solution becomes about 40m/s, that is approximately the maximum difference of LLO injection Δv by the launch date (See Fig. 7). The increment of LTO injection Δv from the optimal solution for this deviation is about 5m/s.

The effect of the deviation varies by the launch date. Table 3 shows the result for the launch date July 31, 2005, the launch date on which the effect is the largest. The position of the moon at the LSOI injection is about the apogee of the moon orbit. (a) to (e) are the selected items listed before.

The effect of the deviation of θ is much larger than in the former two cases of α and β . The effects by the deviation of θ obtained in expense of 40m/s loss in LLO injection Δv (and 5m/s loss in LTO injection Δv) are about 3 hours in LEO injection time, 25deg. in LTO injection position longitude, 3 days in PLP time, and 2 deg. in the argument of perigee at PLP.

Although the effect of the deviation of θ is large, the deviation imposes a penalty on the spacecraft, that is, the increase of LLO injection Δv . However, the value of the deviation used here, 5 deg., is an extreme case. If a smaller effect is sufficient, the increase of LLO injection Δv will be smaller. The result here indicates that the conditions on these listed items are tradable with LLO injection Δv in a wide range.

5. Conclusion

In this paper, the characteristics of a translunar trajectory are investigated based on the patched conic method. The trajectory discussed in this paper is a two-burn ballistic trajectory from the low earth parking orbit to the low lunar orbit, which fully considers the geometric relation from the launch site to the moon.

Firstly investigated is the structure of the problem and the solution space. It is shown that the solutions are grouped by two properties of the lunar transfer sequence. They are the property as to the coasting on LEO ("short coast" and "long coast") and the property as to the approaching direction of LAO ("ascending approach" and "descending approach"). By the combination of these two properties, there are four possible sets of parameters (per day) that satisfy the given boundary condition and Δv optimal condition.

Secondly investigated are the characteristics of the optimal solutions. They are analyzed from two aspects, the transition by the launch date and the difference by the property of the sequence. An important result is that LLO injection Δv is strongly related to the velocity direction difference between the spacecraft and the moon at LSOI injection. The velocity direction difference is the key factor to understanding and explaining LLO injection Δv 's transition by the launch date and

the difference between “short coast” and “long coast”.

Finally investigated is the sensitivity of a number of items to the deviation of the parameters from the optimal value. It is shown that the effect of the deviation of θ is far larger than that of α and β , and it comes out that to change θ is the most effective way to control the items considered in practical operations.

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