System Performance Analysis of Three Dimensional Reaction Wheel for the Attitude Control of Microsatellites

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This paper presents a novel attitude control device which is called three dimensional reaction wheel (3DRW). 3DRW consists of only one levitated spherical mass which can rotate around arbitrary axes. This leads to the reduction of the weight and volume of the device as compared to existing reaction wheel. Furthermore, this device has no mechanical contact between rotor and stator, so the failure caused by the mechanical contact would be reduced. In this paper, the results of the analysis and experiment on the dynamics and control of 3DRW are shown. In the experiments of the rotation control, the air bearing system is used. Using this device, the characteristics of rotation of the spherical mass are obtained. To verify the feasibility of the concept of 3DRW, the experiments of angular velocity feedback control are carried out. The results of experiments are applied to the numerical simulation of the attitude control for microsatellites, and the feasibility of 3DRW is verified.

Key Words: Reaction Wheel, Spherical Motor, Microsatellite

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\upsilon)</td>
<td>velocity</td>
</tr>
<tr>
<td>(\omega)</td>
<td>angular velocity of the rotor</td>
</tr>
<tr>
<td>(B)</td>
<td>intensity of magnetic field</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>conductivity constant</td>
</tr>
<tr>
<td>(V)</td>
<td>voltage</td>
</tr>
<tr>
<td>(R)</td>
<td>resistance</td>
</tr>
<tr>
<td>(i)</td>
<td>electrical current</td>
</tr>
<tr>
<td>(F)</td>
<td>Lorentz force</td>
</tr>
<tr>
<td>(T)</td>
<td>torque</td>
</tr>
<tr>
<td>(\Omega)</td>
<td>angular velocity of magnetic field</td>
</tr>
<tr>
<td>(k_i)</td>
<td>coefficient of input torque</td>
</tr>
<tr>
<td>(k_a)</td>
<td>coefficient of the air damper torque</td>
</tr>
<tr>
<td>(c_{sl})</td>
<td>slip coefficient</td>
</tr>
<tr>
<td>(I)</td>
<td>inertia moment of rotor</td>
</tr>
<tr>
<td>(I_{SC})</td>
<td>inertia moment of spacecraft</td>
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</table>

1. Introduction

To realize accurate attitude control, the reaction wheel is commonly used for many satellites. However, it has never used for very small satellite such as micro-class or nano-class satellites. The main reason is that existing reaction wheel is too heavy and take too much space to carry, and also needs too much electricity to work for this class of satellite.

In order for such satellites to have a high precision attitude control capability, a novel attitude control device which is called three-dimensional reaction wheel (3DRW) has been proposed.

3DRW consists of only one levitated spherical mass which can rotate around arbitrary axis, while conventional reaction wheel consists of three or more one-axis rotating mass to realize three axes control as shown in Fig. 1. This leads to the reduction of the weight and volume of the system. The spherical mass consists of conductive metal, and is rotated by rotational magnetic field. This would reduce the failure caused by the mechanical contact. For these reasons, 3DRW has many advantages to conventional reaction wheel.

The principal concept of 3DRW was introduced as spherical flywheel by Muller et al. However, the concept is not formulated analytically and the characteristics of the control device are not evaluated efficiently for the practical use.

Authors had formulated the concept analytically, and verified a feasibility of 1-DOF rotation control with a magnetic levitation system. However, the feasibility of 3-DOF rotation control was not verified sufficiently due to some structural problems and an interference problem between magnetic fields for the levitation and one for the rotation control.

To solve these problems, authors developed a new
experiments include the air levitation device, and verified the feasibility of 3-DOF rotation using this system. In this paper, the results of experiment on the dynamics and control of 3DRW are shown.

First, the concept of 3DRW and analytical formulation is introduced, and the dynamics of the rotation mass in a magnetic field is analyzed. Using the analytical model, the angular velocity feedback controller is designed. The results of the analysis are demonstrated by the experiments. On the experiments of the rotation, air bearing system is used to keep the rotor in a given position. Through the experiments, the characteristics of 3DRW are investigated, and angular velocity feedback control is performed. The results of the experiments of rotation control are compared with numerical simulations, and the effectiveness of the analytical model is verified. Finally, the attitude control of spacecraft using 3DRW is investigated through the numerical simulation.

2. Formulation of 3DRW

Suppose that a magnetic field is rotating around the sphere, and then an eddy current is generated on the surface of the sphere. Due to the interference between these magnetic field and eddy current, the Lorentz force is produced on the sphere and becomes the rotational torque. The rotational magnetic field can be generated around arbitrary axes, consequently, the sphere can be rotated around arbitrary axes. In this section, the torque of 3DRW generated by magnetic field is investigated.

2.1. Torque generated by the rotation magnetic field

Fig. 2. indicates the analytical model of the rotation of the sphere. Suppose that the sphere has superior electrical conductivity and rotates in uniform magnetic field. \( \omega \) is the angular velocity of the sphere about z-axis, and B is intensity of uniform magnetic field.

For the small element on the surface of the sphere, let \( r \) the vector from the center of the sphere to the element, \( \phi \) the angle between z-axis and the \( r \) vector, and \( \theta \) the angle between x-axis and orthogonal projection of \( r \) for x-y plane.

The voltage induced by the magnetic field in the element is expressed by

\[
dV = (\mathbf{v} \times \mathbf{B}) \cdot l = -r^2 \omega B \cos \theta \sin^2 \phi \, d\phi.
\]  

(1)

Assuming that electric current flows on the surface of the sphere along the \( \pm z \) direction, resistance of element is expressed by

\[
dR = \frac{l}{\sigma A} = \frac{d\phi}{\sigma \delta d\theta}
\]  

(2)

where \( \delta \) is the effective volume of the element. The electrical current along z-axis is derived as follows:

\[
di = \frac{dV}{dR} = -\sigma r^2 \omega B \delta \cos \theta \sin^2 \phi \sin \theta \, d\phi
\]  

(3)

Lorentz force is induced on the element by the interaction between the current and the magnetic field as follow:

\[
dF = -dl(\mathbf{i} \times \mathbf{B})
\]

\[
= -\sigma r^2 \omega B^2 \delta \cos \theta \sin^2 \phi \sin \theta \, d\phi
\]

(4)

By integrating Eq. (4) for entire surface of the sphere, the following equation is derived:

\[
T = \int_{\phi=0}^{\pi/2} \int_{\theta=0}^{2\pi} r \times dF \, d\phi \, d\theta = -\sigma r^4 \omega B^2 \delta \left[ \begin{array}{c} 0 \\ 0 \\ 3\pi^2/8 \end{array} \right].
\]  

(5)

Eq. (5) indicates that the torque is generated against the direction of the rotation.

In the actual 3DRW, the sphere rotates in inverse direction of the rotating magnetic field. Therefore the sphere rotates in the same direction of the rotation magnetic field. Suppose that the magnetic field rotates relative to the sphere, the torque is expressed as follows:

\[
T = \frac{3}{8} \pi^2 \sigma r^5 (\Omega - \omega) B^2.
\]  

(6)

2.2. Torque generated by magnetic field orthogonal to the rotational axis

If the magnetic field is parallel to the rotational axis of the sphere, the disturbance torque would be induced. The torque generated for this case is derived in a similar way to the previous section.

\[
T = \int_{\phi=0}^{\pi/2} \int_{\theta=0}^{2\pi} r \times dF \, d\phi = -\sigma r^4 \omega B^2 \delta \left[ \begin{array}{c} 0 \\ 0 \\ \pi^2/4 \end{array} \right].
\]  

(7)
Eq. (7) implies that the torque is generated in a direction opposite to the rotation. Thus, the magnetic field parallel to the rotation axis reduces the velocity of rotation.

### 2.3. Equation of motion

Suppose that the sphere rotates at \( \omega_{\text{in}} = [\omega_x, \omega_y, \omega_z]^T \) in a coordinate system \( \Sigma_{\text{in}} \), remain stationary relative to magnetic field. From the Eq. (5) and (7), the torque is expressed in the coordinate system as

\[
T(t) = -\frac{1}{8} \pi^2 \sigma^4 \delta \left[ \begin{array}{c} B_x(t)^2 \\ B_y(t)^2 \\ B_z(t)^2 \end{array} \right] \left[ \begin{array}{c} \omega_x \\ \omega_y \\ \omega_z \end{array} \right]
\]

where \( E \) is unit matrix, and \( B \) satisfies following equation:

\[
B = \sqrt{B_x(t)^2 + B_y(t)^2 + B_z(t)^2} = \text{const}.
\]

Suppose that \( \Sigma_{\text{R}} \) rotates at \( \Omega \) relative to the inertial coordinate system and set the coordinate transform matrix \( A_{\text{R}i} \) from inertial coordinate system to \( \Sigma_{\text{R}} \) as

\[
A_{\text{Rli}} = \begin{bmatrix} \cos \Omega t & \sin \Omega t & 0 \\ -\sin \Omega t & \cos \Omega t & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

where \( \Omega, \varphi \) and \( \theta \) are 3-2-3 Euler angles. The rotational magnetic field \( B \) is applied as follows:

\[
B = B \begin{bmatrix} \cos(\Omega t + \varphi) & \cos(\Omega t + \varphi) & -\sin \theta \\ \cos(\Omega t + \varphi) & \cos(\Omega t + \varphi) & \cos \theta \\ \sin \theta & -\sin \theta & 1 \end{bmatrix}
\]

The angular velocity vector of magnetic field \( \omega_{\text{in}} \) is described as follows:

\[
\omega_{\text{in}} = \Omega \begin{bmatrix} \sin \varphi \cos \theta \\ \sin \varphi \sin \theta \\ \cos \varphi \end{bmatrix}.
\]

Using

\[
\omega_{\text{Ri}} = \omega_{\text{in}} - \omega_{\text{Rli}}
\]

then Eq. (8) is transformed as follows:

\[
T(t) = -\frac{1}{8} \pi^2 \sigma^4 \delta \left[ 3B^2 - \begin{array}{c} B_x(t)^2 \\ B_y(t)^2 \\ B_z(t)^2 \end{array} \right] (\omega_{\text{in}} - \omega_{\text{Rli}})
\]

\[
= -\frac{1}{8} \pi^2 \sigma^4 3B^2 (\omega_{\text{in}} - \omega_{\text{Rli}})
\]

The elements of \( b(t) \) are averaged in cycle of rotation, and vary from 5/2 to 6/2 with the relative angular velocity of magnetic field. This equation is used for the numerical simulation, though Eq. (6) is used for the design of the control law.

### 3. Control law

3DRW is required to generate target torque around arbitrary axes. The output torque of 3DRW varies with magnitude, rotation axis, and angular velocity of the magnetic field. In this study, the rotation axis and the angular velocity of the magnetic field are used as the control input, and the magnitude of the magnetic field \( B \) is kept constant. The control law is designed in mind the effect of the damper loss torque of the air bearing.

#### 3.1. Equation for the control

The generated torque of rotational magnetic field is formulated as Eq. (6). The output torque of 3DRW is the sum of the generated torque and the damper loss torque proportional to the angular velocity of the spherical mass. To design the control law, develop Eq. (6) as follows:

\[
T_{\text{out}} = I \frac{d\omega}{dt} = k_e (\Omega(1-c_d) - \omega) - k_o \omega
\]

The coefficient of input torque \( k_e \) is expressed as follows:

\[
k_e = \frac{3}{8} \pi^2 \sigma^5 B^2
\]

In general, the loss torque of the rotor is larger than the disturbance torque. To compensate the disturbance torque efficiency, 3DRW controls the angular velocity of the rotor. The block diagram of the 3DRW is shown in Fig. 3. 3DRW receives a torque command \( T_{\text{in}} \) and outputs control torque \( T_{\text{out}} \).
3.2. Configuration of the motor

The magnetic field $\Omega$ is generated by electromagnets aligned on three orthogonal axes. Suppose that electromagnets are aligned and generate magnetic field along three axes as shown in Fig. 4. From Eq. (11), the magnetic field rotates around $\Omega$ is expressed as follows:

\[
B = B \begin{bmatrix} 
\cos(\Omega t + \phi) & \cos \phi \sin \theta & -\sin \theta \\
\cos \phi \cos \theta & \cos \phi \sin \theta & \cos \theta \\
-\sin \phi & 0 & 0 
\end{bmatrix} + \sin(\Omega t + \phi) \begin{bmatrix} 
-\sin \theta \\
\cos \theta \\
0 
\end{bmatrix}
\]

\[
= B \begin{bmatrix} 
\cos^2 \phi \cos^2 \theta + \sin^2 \theta \cos \Omega t + \phi + \tan^{-1} \left( \frac{-\sin \theta}{\cos \phi \cos \theta} \right) \\
\cos^2 \phi \sin^2 \theta + \sin^2 \theta \cos \Omega t + \phi + \tan^{-1} \left( \frac{-\sin \theta}{\cos \phi \sin \theta} \right) \\
-\sin \phi \cos(\Omega t + \phi)
\end{bmatrix}
\]

(18)

The input magnetic field is expressed as follows:

\[
\Omega = \Omega \begin{bmatrix} 
S_x & S_y & S_z 
\end{bmatrix} = \Omega \begin{bmatrix} 
\sin \phi \cos \theta \\
\sin \phi \sin \theta \\
\cos \phi
\end{bmatrix}
\]

(19)

From Eq. (18) and (19), the magnetic field each electromagnet should generate is determined in following equation.

\[
B = B \begin{bmatrix} 
\sqrt{S_x^2 + S_y^2} \cos(\Omega t + \phi + \tan^{-1} \left( \frac{-S_y}{S_x S_z} \right)) \\
\sqrt{S_x^2 + S_z^2} \cos(\Omega t + \phi + \tan^{-1} \left( \frac{-S_z}{S_x S_y} \right)) \\
\sqrt{S_y^2 + S_z^2} \cos(\Omega t + \phi + \pi)
\end{bmatrix}
\]

(20)

4. Experiment of rotation control

4.1. Experimental setup

To investigate the characteristics of control by 3DRW, some experiments are carried out. Fig. 5. indicates the schematic diagram of the experimental system. An FPGA board counts the pulse from the photo sensors and sends the information to main CPU. The main CPU calculates the angular velocity of the spherical mass and returns the control input to the FPGA board. The control input is changed into three sine waves by the FPGA board and amplified by a power amplifier. The amplified current generates the magnetic flux by three electromagnets and rotates the spherical mass.

The steel bearing sphere is used as the spherical rotor. The moment of inertia is $2.115 \times 10^{-2}$ kgm$^2$ and the weight is 0.173kg. This scale of rotor is aimed at from a few kilograms to tens of kilograms satellites. To investigate the dynamics of the sphere without mechanical contact with stator, an air levitation system is used. To measure the angular velocity of the rotor, photo diodes are used as the sensors. The sphere is ticked so that the sensors could easily detect the pulse every rotation. The overview of experimental setup is shown in Fig. 6.

![Fig. 4. The alignment of electromagnets](image)

![Fig. 5. Schematic diagram of the experimental system](image)

![Fig. 6. Overview of the experimental setup](image)
4.2. Experimental results

To estimate the values of constant numbers in Eq. (15), transient response and damping of angular velocity of the experimental setup is investigated. From the result of transient response, the maximum torque generated on the experimental system is calculated in $1.77 \times 10^{-5}$Nm. The maximum angular velocity is 18Hz. The values of constant numbers in Eq. (15) are determined as shown in Table 1.

Table 1. Determined values of constant number

<table>
<thead>
<tr>
<th>Constant Number</th>
<th>Unit</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_z$</td>
<td>kg.m$^2$/s</td>
<td>$3.33 \times 10^8$</td>
</tr>
<tr>
<td>$k_a$</td>
<td>kg.m$^2$/s</td>
<td>$1.2512 \times 10^{-7}$</td>
</tr>
<tr>
<td>$c_d$</td>
<td>–</td>
<td>0.619</td>
</tr>
</tbody>
</table>

Fig. 7. shows the experimental result of the step response of 3DRW for $K=10$ compared to the model described in Eq. (17). In this figure, the output torque is calculated by the differential of the angular velocity and result in vibrational response. This would because of the high-frequency disturbance torque caused by the compressed air. To avoid this difficulty, consider the angular velocity as the target value by integrating the target torque. Fig. 8. shows the result of the step response for the target value of angular velocity. The response is smoothed by the filter effect. Reflecting this result, in this study, the angular velocity feedback control is performed.

The model described in Eq. (17) is first order system and has the stationary error. To compensate the stationary error, PI-controller is used:

$$ K = \frac{K_p}{I} \left(1 + \frac{1}{T_1 s}\right) $$

(21)

The gains of this controller are determined through some experiments and analysis to obtain the expected response of angular velocity.

Fig. 9. shows the experimental result of angular velocity control. The target angular velocity is set to $\omega=(5.5, 5.5, 5.5)^T$. Gains of PI-controller is set to $K_p=0.1$ and $T_1=0.01$. The result shows that the angular velocity converges to the target value within a margin of error about 5%. Convergence time is about 400sec. From the result, it is expected that the control law designed in Eq. (18) can control the angular velocity of the sphere arbitrary under the maximum rotation velocity of 18Hz. The result of experiment shows that the obtained performance is not sufficient for the practical use, though a method to evaluate a control performance of experimental system.

4.3. Verification by numerical simulation

To verify the formulation of rotation system derived in previous section, the numerical simulation is carried out and compared with the result of experiment.

Fig. 10. shows the result of simulation of the angular velocity feedback control. The result of simulation shows a similar response to the result of the experiment. This denotes that the rotation system is properly expressed by the analytical model of Eq. (15) and the constant values on Table 1. However, some error has also observed by comparing in detail. This is thought to be due to the disturbance torque caused by the compressed air. In actual device, this influence would be ignorable and this error should be improved.

In conclusion, the spacecraft can control it's attitude using 3DRW by this angular velocity feedback control.
5. Application to the attitude control of spacecraft

In this section, the characteristics of the attitude control of spacecraft using 3DRW are investigated through the numerical simulation. The rotational axis of spherical rotor in 3DRW is not constrained relative to the attitude of the spacecraft. If the rotor is kept in a given position relative to the spacecraft by some method, there is no need to count the interference of the attitude motion of the spacecraft.

Suppose that the inertia moment of spacecraft \( I_{SC} \) is 2.0kg·m\(^2\) aimed at microsatellites. The spacecraft detects the error of the attitude and calculates the input torque. The block diagram of this system is shown in Fig. 11. The error of the attitude is described using euler angles. \( C(s) \) is the controller of the spacecraft, and PID controller is designed in this study.

![Block diagram of the attitude control system](image)

As the characteristics of 3DRW, the constant values on Table 1 are used. Constant disturbance torque \( T_d \) is set to 1.0×10\(^{-6}\)Nm around for each axes.

Fig. 12. shows a result of the simulations. \( C(s) \) is designed as follows:

\[
C(s) = 2.0e^{-4} + \frac{2.5e^{-7}}{s} + 5.0e^{-2}s
\]

The attitude converges to the origin, which is the target point. \( C(s) \) should be designed appropriately to prevent the resonance with the time constant of 3DRW.

6. Conclusion

In this paper, the novel attitude control device, 3DRW, was investigated. An experimental system includes an air levitation device was developed and the feasibility of 3-DOF rotation was verified using this system.

The dynamics of spherical rotor in a magnetic field was formulated, and the generated torque was analyzed. Using the analytical model, the angular velocity feedback controller was designed.

To verify the analytical model and obtain the characteristics of the 3DRW, some experiments were carried out. From the result of the experiments, the maximum torque was found to be up to 1.77×10\(^{-5}\)Nm and the maximum angular velocity was 18Hz. The angular velocity of the sphere was controlled by using a feedback controller under the maximum rotation velocity. The result of the experiment of rotation control was verified by numerical simulations, and the similar response was obtained. In conclusion, the analytical model and control law of 3DRW was available for practical use.

The characteristics of the attitude control of small spacecraft using 3DRW were investigated through the numerical simulation. The result shows that the attitude of the spacecraft is converge to the target point by choosing the gain appropriately.

References

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