Development of Oscillation-Free Attitude Maneuvering System for Spinning Solar Sail

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(Received April 25th, 2008)

Solar sail is one of the promising propulsion systems for future deep space exploration missions as it does not require any fuel to acquire propulsive force. Although folding method and deployment mechanism of the sail have been intensively developed, attitude control of the solar sail, which is necessary for the orbital control by the solar sail, has not been much studied. This paper discusses the attitude dynamics and the control method of a spinning type solar sail spacecraft. The spinning type solar sail, where the sail equipped around the spacecraft hub is to be deployed and extended by centrifugal force, has no rigid structure supporting its membrane. This type of mechanism has the advantage in its simple and lightweight structure, however, the attitude control is difficult due to the flexibility of the membrane. In this paper, we introduced a mathematical dynamics model including first oscillation mode of the membrane which can handle coupled motion of a rigid spacecraft and a flexible membrane, and analytically developed a controller that can avoid unnecessary oscillatory motion. The performance of the controller was verified by numerical simulations using more precise multi-particle numerical model.

Key Words: Spinning solar sail, Large flexible membrane, Oscillation-free attitude control

1. Introduction

Institute of Space and Astronautical Science at Japan Aerospace Exploration Agency (ISAS/JAXA) has been studying the “solar sail” propulsion system as a promising way of orbital control for future deep space explorations. An interplanetary solar sail mission is being planned by ISAS/JAXA in 2010s. The mission is to demonstrate the solar sailing technology and other various technologies required for future interplanetary explorations. A spinning type solar sail has been proposed for the mission, where the sail equipped around the spacecraft hub is to be deployed and extended by centrifugal force. This type of mechanism has the advantage in its simple and lightweight structure.

As to the development of the spinning solar sail, the mechanism to deploy huge and flexible membrane has been extensively investigated by ground experiments or experiments using balloons and sounding rockets. However, the attitude control system of the solar sail, which controls the direction of the sail and the spacecraft, has not been much studied, although this is necessary for the control of solar radiation propulsive force by the sail.

Several methods of attitude control for solar sail have been proposed so far, such as the heliogyro or the control vanes, but most of them require some kind of mechanical structure which is not verified for long-term interplanetary mission over years. Alternatively, this paper introduces a control method which uses conventional reaction control system (RCS) equipped on the center spacecraft hub. Here, the reaction control device is conventional chemical jet propulsion system. As the spinning type solar sail does not have any rigid structure to support its membrane, the impulsive torque by RCS can introduce oscillatory motion of the membrane. Thus, an oscillation free attitude controller is needed to take into account the flexibility of the membrane.

This paper first introduces two dynamics model of spinning solar sail in Section 2. The first one is a simplified analytical model which is to be used for the controller design. The other one is a precise numerical model incorporating the flexibility of the membrane and is to be used for performance verification of the controller.

In section 3, several attitude controllers are designed using the analytical model. In designing the attitude control system, flexibility of the large membrane is taken into consideration. The performance of the controllers is verified by numerical simulations using the precise numerical model. Finally, conclusions are given in Section 4.

2. Dynamics Model of Spinning Solar Sail

This section introduces two dynamics model of spinning solar sail. The first one is a simplified analytical model (section 2.1) and the other one is a precise numerical model incorporating the flexibility of the membrane (section 2.2).

2.1. Analytical dynamics model considering first mode of oscillation

In this section, an analytical dynamics model of the sail is derived considering first mode of oscillation of the membrane.

Here we consider circular spinning solar sail
configuration shown in Fig. 1. The spinning solar sail consists of a center spacecraft hub and a large membrane connected with the hub, which rotate around the Z-axis at a rate of $\Omega$. Subscript B indicates body-fixed coordinate and subscript I indicates inertial coordinate.

![Fig. 1. Configuration of solar sail spacecraft](image)

In this research, we adopted the dynamic model considering the first mode of membrane oscillation proposed by Nakano, et al.\(^3\) The conservation laws of angular momentum and the equations of motion are as follows.

\[
I_x \ddot{\phi} + I_y \ddot{\psi} + I_z (\Omega \dot{\Omega} + \dot{\phi}) = 0 \tag{1}
\]

\[
I_y \ddot{\psi} + I_x \ddot{\phi} + I_z (\Omega \dot{\Omega} + \dot{\psi}) = 0 \tag{2}
\]

\[
I_x \ddot{\phi} + (J - I) \Omega \dot{\Omega} x + \frac{1}{2} I_z (\dot{\phi} + \Omega^2 \phi) = 0 \tag{3}
\]

\[
I_y \ddot{\psi} - (J - I) \Omega \dot{\Omega} z + \frac{1}{2} I_z (\dot{\psi} + \Omega^2 \psi) = 0 \tag{4}
\]

where

$I_x$: moment of inertia (MOI) of membrane
$I_y$: MOI of center spacecraft around x or y axis
$I_z$: MOI of center spacecraft around z axis
$I$: MOI of overall spacecraft around X,Y-axis ($=I_x+I_y$)
$J$: MOI of overall spacecraft around Z-axis ($=I_x+I_z$)
$\omega_x$, $\omega_y$, $\Omega$: angular velocity around three axes

and $I_z$ is defined as

\[
I_z = 2\pi \rho h \int_{r_i}^{r_o} r^2 (r - r_o) dr \tag{5}
\]

where

$\rho$: density of membrane [kg/m\(^3\)]
$h$: thickness of membrane [m]
$r_i$: inner radius of membrane [m]
$r_o$: outer radius of membrane [m]

For the condition of numerical simulation, we adopt a value $1.4$ [g/cm\(^3\)] for $\rho$ and $7.5\times10^{-6}$ [m] for $h$, referring to the typical polyimide film. $\phi$ and $\psi$ constitutes the first-order mode of the out-of-plane membrane deformation $w$ as follows.

\[
w(r, \theta, t) = (r - r_o) \phi(t) \sin \theta - (r - r_o) \psi(t) \cos \theta \tag{6}
\]

where $\theta$ corresponds to the phase of the spin motion of the center spacecraft hub.

### 2.2. Multi particle model

Here, we introduce another dynamics model to precisely simulate higher mode of oscillation of the flexible membrane. This model is called multi-particle model (MPM).\(^4\) MPM is a model which substitutes the elements of membrane for particles connected by springs and dashpots, which enables faster and more stable analysis compared with Finite Element Model (FEM). The configuration of particles we adopted is shown in Fig. 2.

The circular membrane is approximated by decagon that is constructed by 200 mass points. The membrane is connected to the center spacecraft hub with 10 tethers. The parameters used in the model are shown in Table 1, which are calculated based on ref. 5), assuming typical polyimide film. Unless otherwise stated, these parameters are used in this paper.

<table>
<thead>
<tr>
<th>Item</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shape of membrane</td>
<td>decagon</td>
</tr>
<tr>
<td>Spring constant of membrane</td>
<td>4804.7 [N/m]</td>
</tr>
<tr>
<td>(longer element)</td>
<td>129.8 [N/m]</td>
</tr>
<tr>
<td>(shorter element)</td>
<td>500000 [N/m]</td>
</tr>
<tr>
<td>Damping factor of membrane</td>
<td>0.01</td>
</tr>
<tr>
<td>Damping factor of tether</td>
<td>0</td>
</tr>
<tr>
<td>Number of mass points</td>
<td>200</td>
</tr>
<tr>
<td>Inner radius of membrane</td>
<td>1.0 [m]</td>
</tr>
<tr>
<td>Outer radius of membrane</td>
<td>7.0 [m]</td>
</tr>
<tr>
<td>$I_1/I_1$</td>
<td>1.11</td>
</tr>
<tr>
<td>$I_1$</td>
<td>164.9 [kgm(^2)]</td>
</tr>
</tbody>
</table>

(Parameters of center spacecraft hub)

<table>
<thead>
<tr>
<th>Item</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_x$, $I_y$</td>
<td>50 [kgm(^2)]</td>
</tr>
<tr>
<td>$I_z$</td>
<td>80 [kgm(^2)]</td>
</tr>
<tr>
<td>Radius</td>
<td>0.75 [m]</td>
</tr>
</tbody>
</table>

Table 1. Parameters of MPM
3. Spin Axis Maneuver with Conventional Reaction Control System

In this section, dynamical property of the solar sail is discussed (sections 3.1). Based on the results, several attitude controllers are designed in section 3.2. In section 3.3, performance of the controllers is verified by numerical simulations using MPM.

3.1. Dynamical property of the sail

Here, we analyze the attitude dynamics of the solar sail.

State equation

State equations of the system (Eqs. (1)-(4)) can be described as the following equation.

\[
\frac{dx}{dt} = Ax 
\]

where

\[
x = \begin{bmatrix} \dot{\phi} \\ \dot{\psi} \\ \dot{\theta} \\ \dot{\psi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} 
\]

\[
A = \begin{bmatrix} 0 & 0 & -B_1 \Omega & 0 & 0 & \frac{1}{2} (N_\omega - 1) \Omega \\ 0 & 0 & 0 & -B_1 \Omega & -\frac{1}{2} (N_\omega - 1) \Omega & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{2} (\beta - 1) \Omega & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} 
\]

\[
B = \frac{I'}{2I_c} 
\]

\[
I' = I_x + \frac{1}{2} (I_1 - I_2) 
\]

\[
N_\omega = \frac{I_1}{I_2} - 1 
\]

It can be said that the dynamical property of the system is determined by the following three parameters which can be calculated by the moment of inertia of the center spacecraft hub and the membrane.

\[
\frac{I_1}{I_2}, \beta, N_\omega 
\]

The characteristic equation of the system can be derived as the following equation.

\[
det(sI - A) = s^3(s^2 + \beta \Omega^2)^2 + \Omega^2(N_\omega s^2 + \Omega^2(\beta + \frac{1}{2}(N_\omega - 1))^2) = 0 
\]

Here, supposing that \( I_1/I_2 \) is close to 1.0, the characteristic equation is simplified to the following expression. This assumption is appropriate for the typical solar sail that have large membrane relative to a center hub.

\[
det(sI - A) = (s^2 + \Omega^2) \{s^4 + (N_\omega^2 + 2 \beta - 1) \Omega^2 s^2 + (N_\omega + \beta - 1)^2 \Omega^4\} = 0 
\]

Finally, it is found that the following three modes of oscillation constitute the nutation of the solar sail.

\[
\Omega, B_1 \Omega, B_2 \Omega 
\]

where

\[
-B_1^2 = -\beta - (N_\omega - 1) \frac{N_\omega + 1 + \sqrt{(N_\omega + 1)^2 + 4(\beta - 1)}}{2} 
\]

\[
-B_2^2 = -\beta - (N_\omega - 1) \frac{N_\omega + 1 + \sqrt{(N_\omega + 1)^2 + 4(\beta - 1)}}{2} 
\]

This means that the nutation angular velocity of the spacecraft has three oscillation modes as in Eq. (14). The numerical expression of \( B_1 \) and \( B_2 \) is illustrated in Fig. 3 and Fig. 4, where \( \beta \) and \( N_\omega \) are independent parameters. For a practical range of \( \beta \) and \( N_\omega \), \( B_1 \) is larger than 1 and \( B_2 \) can be both larger and less than 1.

Fig. 2. Multi particle model

Fig. 3. Value \( B_1 \) for \( \beta \) and \( N_\omega \)
**Nutation mode**

Laplace transform of nutation angular velocity of the center spacecraft hub $\omega_2(t)$ with respect to initial condition $\omega_2(0)$ can be obtained from the state equation (7) and the amplitude of each oscillation mode is given by factorization to partial fraction of the Laplace transform, which is expressed as

$$
\frac{W_2(\Theta)}{\omega_2(0)} = \frac{(\beta - 1)^2 s}{(B_1^2 - 1)(B_2^2 - 1) s^2 + \Omega^2} + \frac{(\beta - B_1^2)^2 s}{(B_1^2 - 1)(B_1^2 - B_2^2) s^2 + B_1^2 \Omega^2}
$$

$$
+ \frac{(\beta - B_2^2)^2 s}{(B_2^2 - 1)(B_2^2 - B_1^2) s^2 + B_2^2 \Omega^2}.
$$

(17)

The attitude motion of spinning spacecraft with large membrane has three modes of oscillation in contrast to the rigid spacecraft which has only one mode of nutation motion. In particular, there exists a mode equivalent to the spin rate of the spacecraft, which represents the relative motion of membrane itself.

By numerically evaluating the amplitude of each oscillation mode, mode $B_2\Omega$ is always smaller than the other two modes, and the relation of the magnitude of $\Omega$ mode and $B_2\Omega$ mode depends on $\beta$ and $N_{sc}$. $\beta$ is a parameter that expresses the mass property of the sail relative to the center spacecraft. If $\beta$ is extremely small, for example its theoretical minimum value 1, nutation frequency becomes that of rigid spacecraft. In this case, $\beta$ is 1 and then $B_2$ becomes equal to $N_{sc}$. Therefore, the dominant nutation mode will be $B_2\Omega$ (= $N_{sc} \Omega$; nutation mode of rigid spacecraft). On the other hand, if $\beta$ is large, that is, the sail is extremely large or heavy, the nutation motion of the center spacecraft hub becomes equal to the spin rate $\Omega$, and therefore in this case $\Omega$ mode will be dominant. Fig. 5 shows the ratio of the amplitude of each oscillation mode and the area where each oscillation mode is dominant compared with the other mode.

**Divergent condition**

In Eq. (17), the term $B_2^2 - 1$ can be zero when the following relation is satisfied:

$$
\beta - 1 = 2(1 - N_{sc})
$$

(18)

Under the condition, the amplitude of the modes $\Omega$, $B_2\Omega$ diverges and the system becomes unstable as illustrated in Fig. 7. This figure also shows that the dynamic model agrees very well with the numerical simulation. The combination of ($\beta$, $N_{sc}$) which satisfies the above equation should be avoided when we design a spinning solar sail. For initial conditions other than $\omega_2(0)$, such as $\omega_2(0)$ or $\phi(0)$, the dynamics has the same tendency regarding the amplitude profile, and actually the amplitude of angular velocity is the sum of these terms derived from each initial condition. So, the divergent condition also remains under any initial condition.
3.2. Controller design

The most important operation regarding attitude control of a solar sail is the spin axis maneuver, which is required for the control of solar radiation propulsive force by the sail. For attitude control device, we suppose the use of typical Reaction Control System (RCS).

First, we investigate the motion when external torque is applied on the center spacecraft in order to maintain or change the attitude. When torque $u_x$ is applied, the transfer function is given by

$$W_x = \frac{\beta - B_1^2}{B_1^2 - B_2^{-2}} s + \frac{\beta - B_2^2}{B_1^{-2} - B_2^{-2}} s^2 + B_2^{-2} \Omega^2$$

(19)

According to numerical evaluation, the amplitude of $B_2 \Omega$ mode is dominant compared with that of $B_1 \Omega$ mode. Fig. 8 shows an example of numerical simulation when external torque is applied at $t=4.0$ [s]. At first the mode $\Omega$ is dominant, and after the external torque is applied, the mode $B_2 \Omega$ becomes larger. In addition, we can conclude from the transfer function that the RCS system mounted on the spacecraft hub is almost unable to decrease the rotating motion at the rate of $\Omega$ and it should be considered to adopt some damper to reduce the oscillation of mode $\Omega$.

Impulsive control input excites $B_2 \Omega$ nutation mode. In order to control the spin axis direction in the inertial frame, impulsive torque should be periodically applied synchronizing with the spin rate $\Omega$, so that the direction of total angular momentum gradually tilted towards the target direction. If $B_2 \Omega$ is close to $\Omega$ (this situation can possibly happen), the impulse timing and nutation motion can synchronize and the controller can continually excite the nutation. Therefore, the controller is needed that does not excite nutation and also actively damps it in order to keep suppressing the nutation as much as possible.

Spin axis direction of conventional spinning rigid spacecraft is generally controlled by Rhumb Line Controller (RLC). However, applying RLC to spinning spacecraft with large membrane requires consideration into the disturbance of the flexible membrane. As an efficient controller, the authors have developed a control logic “Flex-RLC” for spinning solar sail which can control the spin axis while damping the nutation. Flex-RLC is a logic in which the impulsive torque around X-axis is applied when the sign of angular velocity about the X-axis is negative. The upper part of Fig. 9 illustrates the control logic. As is imagined from the governing equations, applying the torque at that situation enables more stable reorientation of a spacecraft compared with simple RLC.

In this paper, we newly propose an improved version of Flex-RLC logic. In contrast to the conventional Flex-RLC logic where the duration of the applied impulsive torque is constant, the improved method adopts time-variant duration to enable faster and more stable attitude control.

When $B_2 \Omega$ mode is assumed to be dominant, the Laplace transform of nutation angular velocity is calculated as the following equation.

$$W_x(T + \Delta t) = \frac{(\beta - B_2^{-2})^2}{(B_2^{-2} - B_1^{-2})(B_1^{-2} - 1)} s s + B_2^{-2} \Omega^2 \omega_x(T) + \frac{\beta - B_2^{-2}}{B_1^{-2} - B_2^{-2}} s^2 + B_2^{-2} \Omega^2 U_x$$

$$+ \frac{(\beta - B_2^{-2})^2}{(B_2^{-2} - B_1^{-2})(B_1^{-2} - 1)} \Omega^2 \omega_x(T)$$

(20)

where $\Delta t$ is the duration of the thruster impulse. The inverse Laplace transform is given by

$$\omega_x(T + \Delta t) = \left(\frac{\beta - B_2^{-2}}{B_1^{-2} - B_2^{-2}} \omega_x(T) - U_x\right) \cos B_2 \Omega \Delta t + \left(\frac{\beta - B_2^{-2}}{B_1^{-2} - B_2^{-2}} \omega_x(T) \sin B_2 \Omega \Delta t \right)$$

(21)

The second term is negligible under the condition where $\Delta t \ll 1$, and then the optimal angular impulse that makes the nutation angular velocity zero can be calculated as the following equation.

$$U_x = -\frac{\beta - B_2^{-2}}{B_1^{-2} - B_2^{-2}} \omega_x(T)$$

(22)
3.3. Simulation results and discussions

Fig. 10 to Fig. 15 are the results of numerical simulation of each control logic where nutation velocity and precession angle of the z-axis of the center spacecraft hub are described. The spin rate of the spacecraft was 2.5 [rpm] and the torque generated by the thruster was 0.05 [Nm]. It is assumed that the center spacecraft hub and the membrane are initially spinning at the same spin rate and no deformation of the membrane exists.

These figures show that the nutation velocity initially increases for all control logics but quickly decreases for Flex-RLC method and improved Flex-RLC method, and the nutation level keeps the same level until the control goal is achieved. On the other hand, the conventional RLC logic shows much larger nutation velocity and spin axis precession angle. In some other cases, Flex-RLC or improved Flex-RLC could change the spin axis smoothly while the simple RLC method caused the divergence of nutation motion.

As to the control speed, conventional Flex-RLC requires longer time than simple RLC. This is because Flex-RLC does not necessarily apply control torque for every spin while simple RLC blindly utilizes every chance of control input. Compared with conventional Flex-RLC, the proposed improved RLC can achieve the control goal in shorter time, and it also shows comparable performance in the control time compared with simple RLC.

Therefore, it can be said that the improved Flex-RLC can achieve fast control while suppressing nutation motion at the same time.
The dominancy of each nutation mode was analytically and numerically discussed and it was also found that a specific relation between the moment of inertia of membrane and that of center spacecraft causes resonance of nutation motion, which has to be avoided when we design a solar sail.

Then, we discussed the spin axis maneuver control using conventional RCS. It was analytically shown that continual impulsive torque synchronizing the spin rate can excite nutation velocity and that a controller is needed to damp the nutation while controlling the spin axis at the same time. The authors proposed new controller named Flex-RLC and the improved one. Their effectiveness was verified by numerical simulations using precise multi-particle numerical model which can express higher order oscillatory motion of the flexible membrane, and it was found that the proposed method can control the attitude of spinning solar sail while drastically reduces the nutation velocity compared with conventional control logic. As the conclusion, the proposed method is promising fast and stable controller for spinning solar sail.

4. Conclusion

In this paper, the attitude motion and attitude control strategy of spinning solar sail are discussed. As the spinning type solar sail does not have any rigid structure to support its membrane, the impulsive torque by the RCS can introduce oscillatory motion of the membrane. Thus, an “oscillation free” attitude controller is needed, which takes into account the flexibility of the membrane and avoid unnecessary oscillatory motion.

First, the dynamics model was introduced, and based on the analysis of the dynamics of torque-free motion, it was shown that a spinning solar sail has three oscillation modes of nutation, one of which is equal to the spinning rate of the spacecraft. The dominancy of each nutation mode was analytically and numerically discussed and it was also found that a specific relation between the moment of inertia of membrane and that of center spacecraft causes resonance of nutation motion, which has to be avoided when we design a solar sail.

Then, we discussed the spin axis maneuver control using conventional RCS. It was analytically shown that continual impulsive torque synchronizing the spin rate can excite nutation velocity and that a controller is needed to damp the nutation while controlling the spin axis at the same time. The authors proposed new controller named Flex-RLC and the improved one. Their effectiveness was verified by numerical simulations using precise multi-particle numerical model which can express higher order oscillatory motion of the flexible membrane, and it was found that the proposed method can control the attitude of spinning solar sail while drastically reduces the nutation velocity compared with conventional control logic. As the conclusion, the proposed method is promising fast and stable controller for spinning solar sail.

References