Roll Angular Momentum Enhancement of a Tethered Satellite System for Transfer in Different Orbit Plane

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This paper discusses the in-planar and out-of-planar dynamics of a Tethered Satellite System (TSS), which aims to inject a satellite into a different orbit plane. The orbital transfer is achieved by only varying the tether length in a gravity gradient field. A previous work treating in-plane orbital transfer induces the system’s pitch motion, and proposes a control procedure. For a transfer in a different orbit plane, the system’s roll motion must be controlled as well as its pitch motion. First, this paper shows the system’s governing equations of motion, and compares the features of the roll motion with its pitch motion. Then, a control method is proposed to increase the angular momentum of the roll motion. Finally, the effectiveness of the proposed method is demonstrated by numerical simulations.

\textbf{Key Words:} Tethered Satellite System, Non-holonomic System, Out-of-plane Orbital Transfer

\textbf{Nomenclature}

\begin{itemize}
\item \(r_e\) : orbital radius
\item \(\theta\) : true anomaly
\item \(\psi\) : pitch angle
\item \(\delta\) : roll angle
\item \(l\) : tether length
\item \(m\) : mass
\item \(e\) : eccentricity
\item \(p\) : semi-latus rectum
\item \(\mu\) : planet’s gravitational constant
\end{itemize}

Subscripts

\begin{itemize}
\item \(0\) : initial value
\item \(f\) : final value
\item \(1\) : satellite 1
\item \(2\) : satellite 2
\end{itemize}

1. Introduction

Control thrusters are most commonly used for orbital transfer of satellite systems. However, alternative methods which do not require any propellant consumption may allow additional instruments/fuel on board. Thus, as one of the alternative methods, orbital transfer methods utilizing a Tethered Satellite System (TSS) have been attracting many researchers\textsuperscript{[4,5,7-10]}. Reference 11 deals with in-planar motion of a TSS, and shows a procedure to generate a tether length profile to achieve designated final state at a specified position in orbit. The study utilizes the nonholonomic constraint of the equation of the pitch motion to control three state variables (pitch angle, pitch angular rate, and tether length) at the final position. That is, although the pitch angular momentum is nonintegrable and changes arbitrarily, the time-average of the momentum shows a constant value when the angular velocity becomes fast enough. The three state variables are adjusted to their target values in three consecutive controls by only tether length variation; 1) control for the angular momentum of the pitch motion, which implies to adjust the tether length, 2) control for the pitch angle, and 3) control for the pitch angular rate.

The purpose of this study is to achieve a precise “out-of-plane transfer” of a satellite into a designated orbit belonging to a different orbital plane. For this final purpose, the out-of-planar and in-planar velocity of the satellite must precisely be controlled at the final position. This implies the control problem considered here is extremely challenging. This is because five state variables (including roll angle and roll angular rate) must be controlled simultaneously by only one control input.

Thus, this study utilizes a speculation deduced from Reference 11: in analogy to the pitch motion, the time-average of the component of the system’s angular momentum about the roll axis (which is referred “roll angular momentum” in this paper) becomes constant when the roll angular rate becomes
2. Mission Outline and System's Dynamics

2.1. Orbital transfer mission using a TSS

For a huge space system, the gravity forces of a planet acting to elements of the system cannot be considered constant. Thus, the magnitude and direction of the forces change according to the system's attitude and the distance from the system's center of mass. This means adversely, tether length variation of a TSS can change the attitude motion of the system.

Figure 1 illustrates a TSS in orbit, and shows its mission outline considered in this study. The mission starts at a certain point in the orbit, and its attitude motion is induced through gravity gradient force by tether length variation. When a satellite obtains the desired velocity increment at a prescribed position in the orbit, the system cuts its tether holding a satellite. Consequently, the satellite can be injected into a desired new orbit.

This transfer method requires very sophisticated control technique, but can show an attracting feature: no fuel consumption is necessary for the orbital transfer, because a reel mechanism can vary the tether length by using electric energy. This means saving propellant consumption and allowing additional instruments/fuel on board.

2.2. Equations of motion of a TSS

Figure 2 depicts the orbital parameters of the TSS and its in-planar and out-of-planar attitude angles. In the figure, "c.m." indicates the center of mass for the system. To simplify the problem, this paper takes a dumbbell-type system model: the system's mass is concentrated to the two end-points satellites. Thus, the tether's mass is negligible, and we do not consider the flexible deformation of the tether. Moreover, we assume that the perturbation sources are also ignorable, such as heterogeneity of the planet, gravities of the other celestial bodies, air drag, and so on.

With these assumptions, the equations of motion of the TSS can be derived through the Lagrange's procedure as follows:

\[ \ddot{r}_c = \frac{\mu}{r_c^3} - a_r, \quad \ddot{\theta} = \frac{2\dot{r}_c}{r_c} + \frac{1}{r_c} a_\theta \]

\[ \ddot{\psi} = -\dot{\delta} - 2(\dot{\theta} + \psi \dot{\theta}) \left( \frac{i}{l} - \delta \tan \delta \right) - \frac{3 \mu}{2 r_c^3} \sin 2\psi \]

\[ \ddot{\delta} = -2 \frac{i}{l} \dot{\delta} - \left[ (\dot{\theta} + \psi)^2 + \frac{3 \mu}{r_c^3} \cos^2 \psi \right] \sin \delta \cos \delta \]

where, \( a_r \) and \( a_\theta \) denote the accelerations originating from the gravity-gradient force in the radial direction and in the transverse direction, respectively. These accelerations imply the perturbation of the orbital motion.

However, this study neglects these accelerations, because they are functions of \((l/r_c)^2\) and \((l/r_c)^2 \leq O(10^{-6})\) for a realistic tether length of several kilometers. Consequently, the orbital motion can be defined as a Keplerian orbit, and the orbital parameters are described as follows:

\[ r_c = \frac{p}{1 + e \cos \theta} \]

\[ \dot{\theta} = \sqrt{\frac{\mu p}{r_c^3}} \left( 1 + e \cos \theta \right)^{3/2} \]

For Keplerian orbits, time variable \( t \) has one-to-one correspondence to the true anomaly \( \theta \). Thus the time derivative in the equations of motion can be transformed into \( \theta \)-derivative, denoted by \( * = d * / d \theta \), as follows:

\[ * = \dot{\theta} \theta' + \ddot{\theta} \theta'^2 + \dddot{\theta} \theta'^3 \quad \text{(7)} \]

Consequently, the governing equations for the pitch motion (Eq. (3)) and the roll motion (Eq. (4)) of the system can be expressed as follows:

\[ \psi' = 2(1 + \psi') \left( \frac{\sin \theta}{1 + e \cos \theta} + \delta' \tan \delta - \frac{l}{l'} \right) = \frac{3}{2} \sin 2\psi \quad \text{(8)} \]

\[ \delta' = -2 \left( \frac{l}{l'} \frac{\sin \theta}{1 + e \cos \theta} \right) \theta' = \frac{3 \cos^2 \psi}{1 + e \cos \theta} \sin \delta \cos \delta \quad \text{(9)} \]

It should be noted that these two equations are highly nonlinear and interact with each other.
The terms $C$ and $K$ can be considered as the damping and the stiffness coefficients in vibration respectively, although they are functions of the pitch motion.

Furthermore, when the pitch angular rate is fast enough, the damping coefficient $C$ can be expressed from the result in Reference 11 as follow.

$$ C(\psi, \psi', \psi^*, \theta) = \frac{H'_{\text{pitch}}}{H_{\text{pitch}}} - \frac{\psi^*}{1 + \psi'} $$

This implies that the roll motion is a libration with no damping when the pitch angular rate is fast enough. This is because the pitch angular momentum $H'_{\text{pitch}}$ and the pitch angular acceleration $\psi^*$ become 0.

3.2. Angular momentum for the roll motion

From Eqs. (6) and (7), the system’s angular momentum of the roll motion, $H_{\text{roll}}$, is expressed as follows:

$$ H_{\text{roll}} = m^* \Gamma \dot{\delta} = m^* \left[ \mu \left( 1 + \cos \varphi \right)^2 \right] \Gamma \dot{\delta} $$

This equation implies that the roll angular momentum increases for the monotonic tether length expansion. However, for the precise orbital transfer, the tether length must be adjusted to the prescribed value at the final position in orbit. Thus, the tether length can not be expanded arbitrarily.

By substituting Eq. (9) into Eq. (15), we can express the roll angular momentum variation as a function of the variables for the system’s pitch and roll motions as follows:

$$ \frac{H'_{\text{roll}}}{H_{\text{roll}}} = -2 \frac{e \sin \theta}{1 + e \cos \varphi} + 2 \frac{l'}{l} + \frac{\delta^*}{\delta} $$

Integrating the above equation over a duration $[\theta_0, \theta_1]$ gives us the follow expression:

$$ \ln \frac{H'_{\text{roll}}(\theta_1)}{H_{\text{roll}}(\theta_0)} = -2 \int_{\theta_0}^{\theta_1} \left( (1 + \psi^*)^2 + \frac{3 \cos^2 \psi}{1 + e \cos \theta} \right) \frac{\sin \delta \cos \delta}{\delta} d\theta $$

On the other hand, Reference 11 has shown the following relationship on the system’s angular momentum for the in-planar pitch motion.

$$ \ln \frac{H'_{\text{pitch}}(\theta_1)}{H_{\text{pitch}}(\theta_0)} = -2 \int_{\theta_0}^{\theta_1} \left( (1 + \psi^*)^2 + \frac{3 \cos^2 \psi}{1 + e \cos \theta} \right) \frac{\sin 2\delta}{2\delta} d\theta $$

This relationship is applicable to induce the pitch motion of the system considered in this study, when the roll motion is still zero. And Eq. (17) can be regarded as the governing equation after the roll motion is induced.

Eqs. (17) and (18) have common features as follows. 1) Both equations impose nonholonomic constraints, because the
integrals are nonintegrable. Thus, various tether length profiles induce different attitude motions, and result in different final angular momenta. 2) When the roll/pitch angular rate is fast enough, the time average of the roll/pitch angular momentum becomes constant. This is because the integral in Eqs. (17) and (18) are averaged according to the rotational angle.

Eqs. (17) and (18), however, show an essentially different feature to increase the angular momentum. For the pitch motion, the pitch angle can be held in any value by applying a properly designed tether length expansion or retrieval. Thus, Eq. (18) indicates that keeping the pitch angle at $\psi = 135$ [deg] can increase the pitch angular momentum most effectively.

However, this strategy cannot be applied to increase the roll angular momentum, because the roll motion has no equilibrium states except $\delta = 0$[deg]. This can be easily verified from the following thought. If we assume $\delta = 0$ in Eq. (9), $\delta' = 0$ requires

$$\left(1 + \psi^2 \right) + \frac{3 \cos^2 \psi}{1 + \cos \theta} = 0 \quad \text{or} \quad \sin \delta \cos \delta = 0$$

The first equation cannot always be satisfied, because the pitch motion varies with time. The second one indicates the equilibrium states, $\delta = n\pi/2$[rad] where $n$ expresses an integer. Thus, the increase of the roll angular momentum requires a different technique than the method for the pitch angular momentum increment. This difference between the pitch and the roll angular momenta is caused by the orbital motion. The orbital motion influences the pitch motion, because these two motions occur in the same plane. On the other hand, the roll motion has no influence because it is defined in the normal direction to the orbital plane.

4. Control Method to Increase the Roll Angular Momentum

Eq. (17) indicates that the sign of $H_{roll}/H_{roll}$ depends only on the signs of $\delta$ and $\delta'$, because the bracket in the equation is always positive. Figure 4 illustrates the schematic plot of $H_{roll}/H_{roll}$ according to the profiles of $\delta$ and $\delta'$ during a period of libration. This figure shows us a clue to increase the roll angular momentum. That is, shorting the time intervals of the sections 1 and 3, and lengthening them of the sections 2 and 4 may result in increasing the roll angular momentum. (This idea is verified in the following section.)

The changing of the time intervals can be achieved through varying the roll angular rate by tether length control. That is, tether retrieval increases the angular rate and shortens the time interval, and tether deployment lengthens the time interval. (Figure 5 is the schematic diagram of the tether length control for the roll motion.)

The following tether length control realizes this idea.

$$l \rightarrow l - K \delta'$$

where $K$ is a constant gain. By differentiating this relation, we can get a feedback control method of the tether length as follows:

$$l' = -K \left( \delta' \delta'' + \delta'^2 \right)$$

This feedback control input requires the information of the roll angular acceleration $\delta''$. In a real system, the roll angular acceleration must be measured by instruments on board, or estimated by the following equation. (This equation is obtained by substituting Eq. (21) into Eq. (9).)

$$\delta'' = \frac{1}{1 - \frac{2K}{l} \delta''} \left\{ \frac{2K}{l} \delta'^3 + \frac{2 \cos \theta}{1 + \cos \theta} \delta' \right\}$$

5. Simulation

This section verifies the effectiveness of the proposed method in the previous section to increase the angular momentum of the roll motion by numerical simulation.

We use the same system and consider the same conditions for most of states as the ones used in Reference 11.

The mass of the satellites:

$$m_1 = m_2 = 25 \, [\text{kg}]$$

The orbit of the system:

eccentricity: $e = 0.7268$

height at the perigee: $300 \, [\text{km}]$
For the roll motion, as described above, \( \delta = 0 \) [deg] and \( \dot{\delta} = 0 \) [deg/s] indicate the equilibrium state. Thus, we take the following initial conditions for the roll motion.

\[
\delta = 5 \text{[deg]}, \quad \dot{\delta} = 0 \text{[deg/s]}
\]

(In a real system, even for an initial condition \( \delta = \delta' = 0 \), the system’s state departs from the point soon, because the equilibrium state is unstable. However, since numerical simulations keep the system around the equilibrium state for long, we use the above initial roll angle.) And the feedback gain in Eq. (22) is set as \( K = 0.5 \).

Figure 6 shows the result of the simulation. Figs. 6 (a) and (b) indicate the time histories of the pitch angle and pitch angular velocity. Figs. 6 (c) and (d) are the time histories of the roll angle and roll angular rate. It should be noted that although the magnitude of the roll libration in Figs. 6 (c) becomes smaller at 1.5-orbit than at 0.5-orbit, the roll angular rate in Fig. 6 (d) shows larger amplitude. This is the result that the restoring force due to the centrifugal force becomes larger, because the pitch motion is induced. Thus, the angular momentum in Fig. 6 (e) shows higher amplitudes at 1.5-orbit than 0.5-orbit. Figure 6 (f) is the time history of the tether length controlled by Eq. (21).

To clarify the enhancement of the roll angular momentum, we define the roll energy increment by the following equation:

\[
\Delta E_{\text{roll}} = \delta^2 + \left( 1 + \psi' \right)^2 \frac{3 \cos^2 \psi}{1 + e \cos \theta} \sin \delta \cos \delta
\]

(23)

Figure 7 shows the time history of the energy increment for the roll motion, and clearly indicates the enhancement of the roll angular momentum.

6. Conclusion

This study aims to inject a satellite by a tethered satellite system into a designated out-of-plane orbit. For the out-of-plane orbital transfer, the system’s roll motion must be
controlled as well as its pitch motion. Thus, this paper has analyzed the features of the roll motion, and shown that the time average of the roll angular momentum becomes constant when the roll angular rate becomes fast enough. Moreover, a control method is proposed to increase the angular momentum of the roll motion. Finally, the effectiveness of the proposed method is verified by numerical simulations.

References