Attitude Determination Concept for QSAT

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In Space Systems Dynamics Laboratory at Kyushu University, the mini-satellite QSAT is being developed. This satellite aims at investigating the plasma physics in the aurora zone in order to better understand spacecraft charging and at conducting a comparison of Field-Aligned-Currents observed in orbit with ground-based observations. In order to achieve the mission objectives, the spacecraft attitude must be determined. The attitude determination concept of QSAT is based on a combination of the Weighted-Least-Square and Linearized-Kalman filter estimation methods. The Weighted-Least-Square method produces the optimal attitude-angle observations at one point in time by using the Sun sensor and magnetometer measurements. The recursive Linearized-Kalman filter combines the angular observations with the attitude rate measured by the gyros to produce the optimal attitude solution.

Key Words: Small Satellite, Weighted-Least-Square, Linearized-Kalman-Filter

Nomenclature

- \(a_x, a_y\) : constant parameters defined by the moments of inertia, Eq. (19)
- \(h\) : orbit angular momentum
- \(i\) : inclination
- \(I_x, I_y, I_z\) : components of moments of inertia
- \(I\) : identity matrix
- \(N\) : power spectral density of system noise
- \(r\) : position vector’s magnitude
- \(T_{TDB}\) : Julian centuries of barycentric dynamical time
- \(T_{TTU}\) : Julian centuries of universal time
- \(u\) : argument of latitude
- \(\tau\) : torque component
- \(\Psi\) : transformation matrix from orbit frame to body frame
- \(\Omega\) : right ascension of the ascending node

Superscripts
- \(O\) : reference orbit frame
- \(B\) : body frame

Subscripts
- \(LS\) : Least-Square process
- \(KF\) : Kalman Filter process
- \(TDB\) : barycentric dynamical time

1. Introduction

1.1. Overview of QSAT

Nowadays, there are many micro-satellite projects by universities all over the world. Especially in Europe and the U.S., these activities have been remarkable. Even in Japan, the number of universities that develop small satellites has been increasing in this decade. Kyushu University initiated the mini-satellite project QSAT in 2006 (Fig. 1.). This satellite aims at investigating the plasma physics in the aurora zone in order to better understand spacecraft charging. Furthermore, it allows comparing Field-Aligned-Currents (FAC) observed in orbit with ground-based observations. These observations contribute to prevent the satellite from space hazard such as charging of the electricity, and it is needful information for the Earth-orbiting satellites. Thus, it proves that the university can develop the useful science mission satellite.

The payload instruments of QSAT contain two plasma probes to observe the plasma environment ambient to the satellite\(^{1}\). One of these probes must be located on the wake side of the satellite’s motion in its orbit. In addition to the plasma probes, a flux-gate magneto-meter is used to measure the geomagnetic field due to the FAC. The QSAT system parameters are summarized in Table 1.

QSAT is controlled to be pointing to the Earth by the three-axis magnetorquer. The gravity gradient torque induced by the extension boom also helps the satellite’s attitude to keep pointing to the Earth. In addition, the location of the plasma probe makes the attitude constrained to stay within a 45 degrees angle from the velocity vector. Thus, the QSAT’s attitude needs the three-axis control capability.

Fig. 1. Polar plasma observation satellite

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### 1.2. Attitude determination subsystem

In order to achieve the mission objectives, there are several subsystems in the QSAT satellite system. In these subsystems, the Attitude Determination Subsystem (ADS) is part of the Attitude Determination and Control Subsystem (ADCS) as shown in Fig. 2.

![Fig. 2. Overview of QSAT Subsystems](image)

The function of the ADS is to estimate the satellite attitude within the specified accuracy as defined by the requirements from the payload instruments and from the other subsystems. The attitude is determined by estimation algorithms using measurements from the attitude sensors. For the benefit of the mission instruments mentioned in the previous section, the ADS must meet the requirements listed in Table 2:

<table>
<thead>
<tr>
<th>Description</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accuracy of On-board Attitude Determination</td>
<td>&lt; 5 deg</td>
</tr>
<tr>
<td>Accuracy of On-ground Attitude Determination</td>
<td>&lt; 1.5 deg</td>
</tr>
</tbody>
</table>

In this study, we focus on the first requirement, namely to establish the on-board estimator which produces an accuracy of less than 5 degrees. It is important to limit the amount and complexity of the on-board calculations for cost and reliability reasons. In order to establish an efficient and straightforward estimator, we combine the Weighted-Least-Square (WLS) estimator with the sequential Linearized-Kalman Filter (LKF). The details of this estimation concept and its utility will be presented in the chapter 3. This concept is based on using the three types of sensors that are available in QSAT: Sun sensors, the three-axis magnetometer and the gyro rate sensors.

### 2. Sensors

#### 2.1. Sun sensor

QSAT has two Sun sensor units to measure the Sun vector which is used for the attitude determination. Each of QSAT’s two Sun sensors has three Position Sensing Devices (PSD’s) which are the sensing parts of the Sun sensors. Each of the six PSD’s are attached to the corners of each body panel as shown in Fig. 3. In order to catch the Sun light in any attitude orientation, each PSD has a limited field of view with 50 degrees cone angle.

![Fig. 3. Location of Sun sensor in QSAT’s body](image)

When the Sun light projects onto a PSD, it senses the barycenter of the Sun’s light spot. The injection of the Sun light into these PSD’s induces the photovoltaic effect. Then it generates the electric charge proportional to the light intensity. The voltage of this electric charge can be transformed to the coordinates of the PSD by on-board computation. By using the coordinates of the Sun’s barycenter in the PSD coordinate frame, the Sun vector \( \mathbf{S} \) is calculated as:

\[
\mathbf{S} = \frac{1}{\sqrt{X^2 + Y^2 + Z^2}} (-X, -Y, L)^T \tag{1}
\]

![Fig. 4. Geometry of Sun vector on PSD](image)
2.2. Magnetometer

Because the three-axis attitude, in theory, cannot be determined by one reference vector but requires at least two vectors, another vector is needed in addition to the Sun vector. QSAT is equipped with a three-axis magnetometer (Fig. 5.) to measure the geomagnetic field vector as the second inertial reference vector required for the three-axis attitude determination.

The magnetometer senses the strength and the direction of the geomagnetic field in three components. The measured geomagnetic field vector $B$ is normalized before its use in the attitude determination process:

$$ B = \frac{1}{\sqrt{B_x^2 + B_y^2 + B_z^2}} (B_x, B_y, B_z)^T $$

(2)

2.3. Gyro rate sensor

In principle, the three-axis gyro rate sensor senses the angular rates relative to the inertial coordinate system. This means that the gyro rate sensor senses not only the relative error rates of the satellite body but also the orbital motion, because the attitude normally points to the Earth, see Fig. 6. Therefore, the nominal angular rate of the satellite about the orbit-normal equals the constant orbital rate for a circular orbit. The angular rates of the actual attitude orientation are normally much smaller than the nominal attitude.

The measurements from the three-axis gyro rate sensor are used for the attitude propagation in the Kalman filter estimator. The measurement vector $g$ includes the gyro drift and the measurement noise:

$$ g = \omega + d + m $$

(3)

where $\omega$ is the actual angular rate vector, $d$ is the drift vector and $m$ is the measurement noise vector.

Fig. 6. Reference orbit frame and body frame

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Fig. 7. shows the gyro rate sensor unit of QSAT.

3. Attitude Determination Concept

3.1. Environment

3.1.1. Orbit characteristics

In order to be able to fulfill its mission objectives, QSAT will be launched into a Sun-synchronous orbit. Its altitude is in the range from 600 km to 800 km. At this altitude, the satellite can pass through the aurora region in which the high density of the plasma and the FACs exist.

In order to simulate the environment surrounding the satellite, more concrete information about the orbit is needed. Thus, the orbital elements of QSAT’s orbit are defined as in Table 3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semi-major Axis</td>
<td>7044 (km)</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>1.0e-7</td>
</tr>
<tr>
<td>Inclination</td>
<td>98.1 (deg)</td>
</tr>
<tr>
<td>Right Ascension of Ascending Node</td>
<td>354.7 (deg)</td>
</tr>
<tr>
<td>Argument of Perigee</td>
<td>0.001 (deg)</td>
</tr>
<tr>
<td>Initial Mean Anomaly</td>
<td>0.0 (deg)</td>
</tr>
</tbody>
</table>

Table 3. Orbital elements of assumed QSAT’s orbit

In the normal mode, the satellite attitude is controlled to have one-axis pointing to the Earth by the magnetorquers. In addition, a boom helps to maintain this attitude by means of gravity gradient effect[2]. It is convenient to take the orbit frame as the nominal reference frame (Fig. 6.) of the attitude, because the angle between the body frame and the orbit frame must always remain small during the mission.

3.1.2. Geomagnetic field

The Earth’s magnetic filed extends around the Earth even up to the altitude of the QSAT orbit. In the attitude determination algorithm, the geomagnetic field vector sensed by the magnetometer must be compared with the predicted reference vector. In other words, we need a realistic geomagnetic field model to be able to determine the attitude.

The International Geomagnetic Reference Field (IGRF) 10th generation model[3] was established in 2005 based on the actual measurements by a few orbiting satellites. This model is used widely for several applications related to the geomagnetic field. It provides the intensity (Fig. 8.) and the direction of the geomagnetic field. The harmonic function is used in the calculation of this model, and its order dominates the accuracy of its output vector. For the on-board attitude estimation of QSAT, the 4th order model is used to limit computation time.

Fig. 7. Picture of gyro rate sensor

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3.1.3. Sun ephemeris

Similar to the geomagnetic field vector, we need the reference model for the Sun vector. There exist very precise ephemerides of the Sun with respect to the Earth. However, it is often more convenient to use a compact mathematical algorithm\(^4\) based on a less precise model, especially for on-board calculations. The Sun vector in the inertial frame \(IJK\) can be described as:

\[
r_{\text{Sun}} = (r_{\text{Sun}} \cos \lambda_e, r_{\text{Sun}} \cos \varepsilon \sin \lambda_e, r_{\text{Sun}} \sin \varepsilon \sin \lambda_e)^T \tag{4}
\]

with \(r_{\text{Sun}}\) the distance to the Sun, \(\lambda_e\) the ecliptic longitude of the Earth and \(\varepsilon\) the obliquity of the ecliptic. This obliquity is defined by \(T_{\text{DB}}\) with constant numbers:

\[
\varepsilon = 23.480 - 0.0130 \, T_{\text{DB}} \tag{5}
\]

The ecliptic longitude of the Earth can be expressed in the mean longitude of the Sun \(\lambda_M\) and the mean anomaly of the Sun \(M_{\text{Sun}}\):

\[
\lambda_e = \lambda_M + 1.91^\circ \sin(M_{\text{Sun}}) + 0.0199^\circ \sin(2M_{\text{Sun}}) \tag{6}
\]

These parameters can be obtained from the time parameters, respectively.

\[
\lambda_M = 280.460^\circ + 36,000.770 \, T_{\text{UT1}} \tag{7}
\]

\[
M_{\text{Sun}} = 357.527^\circ + 35,999.050^\circ T_{\text{DB}} \tag{8}
\]

This model will be used for the Sun reference vector of the attitude estimation.

3.1.4. Simulator

In fact, the attitude estimator cannot be verified on-ground or in an aircraft, because we cannot produce the same environment as on-orbit. Therefore, we have developed a dedicated simulator\(^5\) to produce the imitated space environment. The simulator cannot only model the dynamics of the satellite such as; the orbit, attitude, perturbing torques, effects of the actuator, and the power consumption, but it also produces realistic sensor outputs by adding the noise and biases. In Table 4, the characteristics of the simulator are summarized:

<table>
<thead>
<tr>
<th>Category</th>
<th>Model Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numerical Integrator</td>
<td>8th Dormand-Prince</td>
</tr>
<tr>
<td>Orbit Propagator</td>
<td>SGPS(^6)</td>
</tr>
<tr>
<td>Sun Position</td>
<td>VSOP87(^7)</td>
</tr>
<tr>
<td>Geomagnetic Field</td>
<td>IGRF Model(^3)</td>
</tr>
<tr>
<td>Atmospheric Drag</td>
<td>NRLMSISE-00(^8)</td>
</tr>
<tr>
<td>Gaussian White Noise</td>
<td>L’Ecuyer &amp; Box-Muller Method(^9)</td>
</tr>
</tbody>
</table>

3.2. Attitude estimation algorithms

The attitude determination concept of QSAT is based on a combination of the Weighted-Least-Square (WLS) and Linearized-Kalman Filter (LKF) estimation methods as shown in Fig. 9. The WLS method produces the optimal attitude-angle observations at one point in time by using the Sun sensor and magnetometer measurements. The recursive LKF combines the WLS observations with the attitude rate measurements from the gyros. It produces the optimal attitude estimate at the time \(t_k\) by incorporating the a priori attitude estimate.

\[
y = [M_{\text{LS}}] a + v \tag{9}
\]

with the state vector \(a = (\phi_{\text{LS}}, \theta_{\text{LS}}, \psi_{\text{LS}})^T\) consisting of the Tait-Bryan angles as defined in Fig. 10. The measurement vector \(y = (S, B)^T\) contains the Sun vector \(S\) and the magnetic field vector \(B\), and \([M_{\text{LS}}]\) denotes the measurement matrix.

For QSAT attitude determination, the rotation sequence from the reference orbit frame to the body frame is chosen as 3→2→1. The Tait-Bryan angles are defined in Table 5.
Table 5. Definition of Tait-Bryan angles

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Dynamics on Orbit</th>
<th>Rotation Axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>φ</td>
<td>Yaw about Earth-pointing axis</td>
<td></td>
</tr>
<tr>
<td>θ</td>
<td>Roll about normal axis to 1st &amp; 3rd axis</td>
<td></td>
</tr>
<tr>
<td>ψ</td>
<td>Pitch about orbit normal axis</td>
<td></td>
</tr>
</tbody>
</table>

In this case, the transformation matrix from the reference orbit frame to the body frame is expressed as:

\[ [\Psi] = \begin{bmatrix}
    c\theta c\psi & c\theta s\psi & -s\theta \\
    s\theta c\psi & s\theta s\psi & c\theta \\
    -c\psi & s\psi & 0
\end{bmatrix} \tag{10} \]

with \( c = \cos \) and \( s = \sin \). If we assume that the Tait-Bryan angles are sufficiently small, this matrix can be approximated as:

\[ [\Psi] \approx \begin{bmatrix}
    1 & -\psi & -\theta \\
    -\psi & 1 & \phi \\
    \theta & -\phi & 1
\end{bmatrix} \tag{11} \]

By using this approximation, the transformation of the measurement vector from the reference orbit frame to the body frame can be linearized:

\[ \mathbf{y}^b = [\Psi] \mathbf{y}^O = [M_{LS}] \mathbf{a} + \mathbf{y}^O \tag{12} \]

If we define the measurement at a given instant of time as \( \mathbf{y} = \mathbf{y}^b - \mathbf{y}^O \), the measurement equation can be written as:

\[ \mathbf{y} = \mathbf{y}^b - \mathbf{y}^O = [M_{LS}] \mathbf{a} \tag{13} \]

Therefore, the measurement matrix can be expressed in the components of the Sun and the geomagnetic field vectors:

\[ [M_{LS}] = \begin{bmatrix}
    0 & -S^O_z & S^O_y \\
    S^O_z & 0 & -S^O_y \\
    -S^O_y & S^O_z & 0 \\
    0 & -B^O_z & B^O_y \\
    B^O_z & 0 & -B^O_y \\
    -B^O_y & B^O_z & 0
\end{bmatrix} \tag{14} \]

Finally, the estimate \( \hat{\mathbf{a}} \) of the Tait-Bryan angles are obtained from Eq. (15) by using the WLS Pseudo-Inverse:

\[ \hat{\mathbf{a}} = [M_{LS}]^T [W_{LS}]^{-1} [M_{LS}]^T \mathbf{y} \tag{15} \]

The process noise vector in Eq. (17) can be calculated by integration over the time step from \( t_{k-1} \) to \( t_k \):

\[ \mathbf{q}_k = \int_{t_{k-1}}^{t_k} [\Phi(\eta)] \mathbf{n}(\eta) d\eta \tag{21} \]

with \( \mathbf{n} \) the process noise in the original continuous differential equation of Eq. (17). From this formula, the covariance matrix can be determined:

\[ [Q]_k = \int_{t_{k-1}}^{t_k} [\Phi(\eta)] [\mathbf{W}] [\Phi(\eta)]^T d\eta \quad (k = 1, 2, ...) \tag{22} \]

By using the transition matrix of Eq. (20), the error covariance extrapolation of LKF is described by:

\[ [P]_{k+1} = [\Phi]_{k+1} [P]_k [\Phi]_{k+1}^T + [Q]_{k+1} \tag{23} \]

The updated state estimates of LKF and also the error covariance matrix can be obtained:

\[ \hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k + [K]_k (\mathbf{y}_k - [M_{KF}] \hat{\mathbf{x}}_k) \tag{24} \]

\[ [P]_k = \hat{K}_k [M_{KF}]_k [P]_k [M_{KF}]_k^T + [R]_k \] \tag{25} \]

with the Kalman gain matrix:

\[ [K]_k = [P]_k [M_{KF}]_k^T [M_{KF}]_k [P]_k [M_{KF}]_k^T + [R]_k \] \tag{26} \]

where \( [R] \) denotes the covariance matrix of the observation noise \( \mathbf{w} \) in Eq. (16).
3.3. Discussion of results

In order to evaluate the estimation algorithm, several simulation runs have been performed using the simulator (see 3.1.4.). The simulation conditions are summarized in Table 6.

Table 6. Simulation condition

<table>
<thead>
<tr>
<th>Category</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orbit Perturbation</td>
<td>None</td>
</tr>
<tr>
<td>Attitude Perturbation</td>
<td>Gravity Gradient</td>
</tr>
</tbody>
</table>

Initial Attitude Angles (φ, θ, ψ) [deg]  (5.0, 5.0, 5.0)
Initial Angular Rates ($\omega_x$, $\omega_y$, $\omega_z$) [deg/sec]  (0, 0, 0.06)

Table 7. Attitude errors from simulation results

<table>
<thead>
<tr>
<th>Category</th>
<th>Angles (WLS)</th>
<th>Angles (LKF)</th>
<th>Angular Rate (LKF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error (RSS)</td>
<td>2.9 [deg]</td>
<td>2.13 [deg]</td>
<td>0.012 [deg/sec]</td>
</tr>
</tbody>
</table>

4. Conclusions

In order to satisfy the QSAT mission requirements, we developed an attitude estimator for the on-board attitude determination subsystem. In this paper, we have shown by realistic simulations that this estimator provides good results for the attitude angles and rates. Furthermore, even under the largest possible bias effects, the estimator is able to determine QSAT’s attitude adequately in accordance with the mission requirements.

The attitude estimator presented here is unique, in particular due to the straightforward linear model for the measurement equations. This model is justified by the control strategy that guarantees that the attitude deviations, relative to its ideal Earth-pointing attitude. It can be applied to any Earth-pointing satellite in low-Earth orbit with relatively modest pointing requirements. It is a new approach for the university satellite to implement two estimation methods and it will provide the straightforward and valid combination of estimators.

References


