Relative Motion Estimation and Control Strategy for a Spacecraft to an Uncooperative Object

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(Received May 7th, 2008)

In this paper, a control strategy for a spacecraft (“Pursuer”) to rendezvous with an uncooperative object (“Target”) is dealt with. This strategy consists of a feedforward optimal control phase (phase 1) and a feedback control phases (phase 2 and 3). In the feedforward optimal control phase, the target’s future attitude motion is predicted from the result of motion estimation using image data in order to determine the final state for the maneuver. It is expected that there will be errors in the result of motion estimation for the target and error compensation is expected to be necessary. For this purpose, the pursuer estimates and predicts target’s motion during maneuver, and recomputes the optimal control input using it. A quasi-optimal control input is proposed and named “Adjusted Control Input” in order to adapt this error compensation. And its performance is investigated through a numerical simulation of the pursuer’s position control.

Key Words: Optimal Control, Feedforward control, Feedback Control

Nomenclature

- \(\mathbf{r}\) : 3D position of pursuer
- \(\mathbf{v}\) : 3D velocity of pursuer
- \(\mathbf{x}\) : 1D position of pursuer
- \(\mathbf{v}\) : 1D velocity of pursuer
- \(m\) : mass of the pursuer
- \(t\) : time
- \(f\) : thrust force of pursuer
- \(\lambda\) : Lagrange multiplier

Subscripts

- \(0\) : initial
- \(f\) : final
- \(\text{max}\) : maximum
- \(\text{op}\) : optimal
- \(\text{ad}\) : adjusted
- \(s\) : switching

1. Introduction

In this paper, a spacecraft attempting to contact with moving objects such as asteroids or large pieces of space debris is called a “pursuer” and the object is called an “uncooperative target”. Examples of pursuers are shown in Fig. 1. The top image shows JAXA’s Hayabusa asteroid sample return probe, which successfully touched down on the asteroid Itokawa in 2005 and is now on its way back to the Earth. The touchdown maneuver was performed using a laser range finder (LRF) for vertical position and relative attitude measurement and images of a target released onto the asteroid’s surface for horizontal position/velocity measurement. The middle image shows the SUMO satellite servicing space robot proposed by DARPA. This captures ordinary satellites and carries out...
servicing such as orbit change, refueling, and repair. The bottom image shows a space robot proposed by JAXA to investigate satellite failures and possibly repair or remove the failed satellite.

In order to perform its mission, a pursuer must control its motion relative to the target. There has been some research into feedback motion control algorithms for this purpose. For example, sliding-mode control using relative motion measurement as feedback signals is proposed\(^1\). But feedback control does not guarantee optimality with respect to fuel consumption, time and other practical constraints. Considering this, a motion control strategy consisting of a feedforward optimal control phase (phase 1) and two feedback control phases (phase 2 and phase 3) is proposed and a quasi-optimal control input is applied for phase 1 in this paper.

2. Strategy for Motion Control of a Pursuer Toward a Target

2.1. Motion control applying FF and FB control

Fig. 2 shows our proposed strategy consisting of three control phases. In phase 1, the relative motion (position, velocity, attitude, attitude rate) between target and pursuer is measured and estimated by an algorithm composed of stereo matching algorithm, Iterative Closest Point (ICP) algorithm and Extended Kalman Filter\(^2\). As a result from it, status of “current” motion for the target is obtained. Moreover, it is possible to predict the target’s “future” motion by integrating the attitude dynamical model of the target using current motion estimates.

In phase 1, using this prediction, the motion control input for the feedforward control is given by solving an optimal control problem with constraints such as the equation of motion of the pursuer and saturation of the control input.

During phase 2, the pursuer attempts to follow the fly-around trajectory to maintain constant relative position and attitude while continuing motion estimation. Then, in phase 3, the pursuer gets away the fly-around trajectory and performs its final approach to the target. During phases 2 and 3, feedback control is applied to control the 6 DOF motion of the pursuer around the target, especially for soft contact with the target, such as sliding-mode control using the relative motion (attitude and position) as feedback signals. Phase 1 is the characteristic part of this strategy, so this paper focuses on the control input design for phase 1.

2.2. “Quasi-Optimal” control for phase 1

The final position and velocity for the pursuer in phase 1 is given by the target’s future motion prediction results based on the motion estimation results. It is expected that there will be errors in the result of motion estimation and prediction results, these errors should be compensated for the successful maneuver. For this purpose, the following strategy is proposed.

“The pursuer iteratively estimates and predicts target’s motion during maneuver and recomputes the optimal control input using it.”

It is expected and assumed that estimation/prediction error is a function of relative distance and will become smaller as the pursuer approaches to the target. The strategy of iterative estimation/prediction during maneuver is expected to improve performance of phase 1 maneuver. For this strategy, an algorithm obtaining control input with low calculation cost is required.

Lagrange’s method of undetermined multiplier is often used for solving optimal control problems\(^3\)\(^4\), but this algorithm have to solve an increasing number of simultaneous equations as the number of constraints grows and causes increased calculation cost, as well as, it is not clear whether a feasible solution is obtainable. To overcome these problems, it is proposed that firstly an optimal solution is obtained by analytically solving the optimal control problem ignoring the control input and secondly the obtained optimal solution is adjusted to satisfy the control input constraint, resulting in a feasible “quasi-optimal” solution.

3. Feedforward Optimal Control for Phase 1

3.1. Formulation of the control problem

Phase 1 in the proposed strategy is defined as the maneuver of the pursuer from the initial station-keeping state to the fly-around trajectory. The fly-around trajectory is regarded as the locus of a “target point” fixed relative to the target and is determined using the result of the motion estimation and prediction.

The initial and final states of phase 1 are defined as follows,

\[
\begin{align*}
\text{Initial state:} \quad \mathbf{x}(t_0) &= \left( \mathbf{r}_0, \mathbf{v}_0, t_0 \right) \\
\text{Final state:} \quad \mathbf{x}(t_f) &= \left( \mathbf{r}_f, \mathbf{v}_f, t_f \right)
\end{align*}
\]

the final state is on the fly-around trajectory. The initial state and final state are called “boundary conditions”, and a problem defined as a combination of boundary conditions is called “boundary value problem”. The problem of designing the optimal control input for phase 1 can be solved as two-point boundary value problem using Lagrange’s multipliers.
3.2. Optimal control input for phase 1

The equation of motion of the pursuer is described simply as follows ignoring gravitational forces:

$$\frac{d}{dt} \begin{bmatrix} r \\ \dot{r} \end{bmatrix} = \begin{bmatrix} 0_{3x3} & U_{3x3} \\ 0_{3x3} & 0_{3x3} \end{bmatrix} \begin{bmatrix} r \\ \dot{r} \end{bmatrix} + \frac{1}{m} \begin{bmatrix} 0_{3x1} \\ f \end{bmatrix}$$  \hspace{1cm} (3)

It can be seen from Eq. (3) that the motion in each axis \(x, y, z\) is independent and so it is possible to focus the controller design on single degree-of-freedom motion as follows:

$$\frac{d}{dt} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \frac{1}{m} f_x$$  \hspace{1cm} (4)

The cost function is provided as

$$J = \frac{1}{2} \int_{t_0}^{t_f} \left( \frac{f(t)}{m} \right)^2 \, dt$$  \hspace{1cm} (5)

Following the algorithm of Lagrange multipliers, the optimal control input is given as follows.

$$f_{op} = v_1(t - t_0) - (v_1(t_f - t_0) + v_2)$$  \hspace{1cm} (6)

The Lagrange multipliers \(v_1\) and \(v_2\) are determined by the boundary conditions:

$$v_1 = \frac{6m}{(t_f - t_0)^2} \left( (v_f + v_0)(t_f - t_0) - 2(x_f - x_0) \right)$$  \hspace{1cm} (7)

$$v_2 = -\frac{2m}{(t_f - t_0)^2} \left( 2v_f + v_0)(t_f - t_0) - 3(x_f - x_0) \right)$$  \hspace{1cm} (8)

Fig. 3 shows the result of numerical simulation of single degree-of-freedom motion control of point mass applying the optimal control input defined as Eq. 6. The initial state of the point mass of 5kg is \((r_0, v_0, t_0)=(-3000m, 10m/s, 0s)\) and the final state is \((r_f, v_f, t_f)=(-300m, 70m/s, 50s)\). These states are properly selected so that the maximum magnitude of the control input is not too large for the evaluation of proposed control input design.

This satisfies the initial and final state of the pursuer (boundary conditions), but the control input immediately exceeds the maximum thruster output (10N) at the beginning of the control. This control input is therefore infeasible.

A method to adjust the obtained optimal solution to satisfy the ignored control input constraint is described in next section. And a feasible “quasi-optimal” solution obtained by this method, “Adjusted Control Input” is proposed.

3.3. Adjusted control input

In Fig. 4, the red line shows the relationship between time and velocity of the pursuer (the “t-v curve”) under the optimal control input. Since the control input is a linear function of time, its velocity is a function of the square of time. In addition, both ends of the t-v curve are fixed by the boundary conditions (the initial and final state of the pursuer), and the area enclosed by the t-v curve, the x-axis and the vertical lines at \(t=t_0\) and \(t=t_f\) (the area of the light blue region in Fig. 4) is equal to the distance traveled by the pursuer.

The optimal control problem in this case is therefore restated as the problem of modifying the t-v curve to minimize the cost function while keeping both ends of the t-v curve fixed and the area of the light blue region constant.

Now, consider the “feasible region” parallelogram (the shaded area in Fig. 4) defined by opposite corner points \((t_0, v_0), (t_f, v_f)\) and lines with gradient \(\pm(f_{max}/m)\). This region is uniquely determined by the boundary conditions and the maximum thruster output constraint. Only the case with the curve being convex upward is handled here for simplification but the idea is the same with the curve being convex downward. Since the gradient of the t-v curve must not lie outside of \(\pm(f_{max}/m)\), it can be considered that the statement “The t-v curve drawn by a certain control input must lie entirely within the parallelogram between \(t_0\) and \(t_f\)” is the necessary and sufficient condition to obtain a control input which is feasible with respect to the boundary conditions and the maximum thruster output constraint. Therefore, the feasibility of a given control input can be investigated using t-v curve. Utilizing this “feasible region”, it is possible to make the control input satisfy the constraint by “pushing” the curve drawn by the input into the feasible region.
The “Adjusted Control Input” is defined as follows using “switching time” \( t_s \).

\[
f_{ad} = \begin{cases} f_{\text{max}} & (t_0 \leq t \leq t_s) \\ v'_1(t-t_s) - (v'_1(t_f-t_s) + v'_2) & (t > t_s) \end{cases}
\]

(9)

The adjusted control input is the maximum thrust value from \( t_0 \) and switches to the optimal control input defined as the bottom of Eq. (9) at switching time \( t_s \) (Fig. 5 top). This optimal control input is recomputed by using \( t_s \) and the position and the velocity of the pursuer at \( t_s \) as renewed initial state (time, position, and velocity). \( v'_1 \) and \( v'_2 \) are recomputed Lagrange multipliers.

Increasing \( t_s \) from \( t_0 \) while keeping the distance moved by the pursuer constant (that is, keeping the area of the light blue region made by \( t-v \) curve in Fig. 4 constant) corresponds to the “adjustment” of the \( t-v \) curve into the feasible region, as shown in Fig. 5. It is therefore possible to obtain a control input satisfying both the maximum thrust constraint and the boundary conditions by increasing \( t_s \) from initial value \( t_0 \), if a feasible solution is obtainable from the conditions.

There are a certain number of values of \( t_s \) that satisfy the above feasibility condition. By estimating the fuel consumption for each discrete value of \( t_s \), a one-dimensional search for \( t_s \) from \( t_0 \) to \( t_f \) is performed and the value of \( t_s \) that gives the minimum fuel consumption is regarded as the “optimal switching time” for the optimal adjusted control input.

Fig. 6 shows the result of a numerical simulation of single degree-of-freedom motion using the adjusted control input. The constraints are same as the motion control simulation in subsection 3.2. (Fig. 3.), and the constraint of maximum thruster output (10N) is applied. It seems that as the result of adjustment of control input by switching time \( t_s=33 \)sec, the control input satisfies the thrust constraint.

4. Numerical Simulation of Three Degrees-of-Freedom Position Control

The Adjusted Control Input is applied to the numerical simulation of position control of the pursuer. Adjusted control input is compared with the other optimal control algorithm, Sequential Conjugate-Gradient Restoration Algorithm (SCGRA) to investigate its performance. SCGRA is a numerical method proposed by A. K. Wu and A. Miele. It consists of two phases, one is “Conjugate-Gradient Phase”, reducing the cost function, and the other is “Restoration Phase”, restoring the feasibility of constraints. SCGRA performs these two phases iteratively and alternately to obtain optimal solution.

From the reason that SCGRA can easily manage constraints such as the maximum thruster output, this algorithm is applied for the comparison with the Adjusted Control Input in this simulation. But it seems that SCGRA requires relatively larger computational load than the Adjusted Control Input.
4.1. Model for numerical simulation

It is assumed that the centroid of the target is fixed to the origin of the inertial reference frame and the target performs attitude motion (nutation). To evaluate basic properties of proposed algorithm, it is assumed that the pursuer controls only its position and velocity and does not perform attitude motion. The pursuer is at (0m, 0m, -800m) in the inertial coordinate system and the relative velocity is (0m/s, 0m/s, 0m/s) at t=0 (initial time \( t_0 \)). The mass of the pursuer is 67kg, and maximum thrust output is 19.2N. The target point (defined in subsection 3.1) is at (0m, 0m, -300m) at initial time, and its position and velocity at final time (\( t_f = 200 \text{ sec} \)) are determined by the predicted future attitude of the target by integrating the equation of motion applying Runge-Kutta method using estimated current attitude motion of the target as the initial state. The position and the velocity of the target point becomes the final state of the pursuer.

It is assumed that this simulation contains two kinds of errors. One is pursuer’s position error, the other is prediction error of the target state.

The pursuer’s position error is supposed to be due to thrust force error and Gaussian white noise with variance 1 and average 0 is applied to control input as thrust force error. The prediction error of the target state is regarded as the one caused by motion measurement error, and depend on relative distance between the pursuer and the target. So it is assumed that measurement of the target’s attitude motion has error proportional to the square of relative distance in this simulation.

Measurement error is defined as below.

\[
\begin{bmatrix}
q'_{4x1} \\
\omega'_{4x1}
\end{bmatrix} = \begin{bmatrix}
q_{4x1} \\
\omega_{4x1}
\end{bmatrix} + C_{7x1} d^2 w 
\]  

(10)

\( q \) and \( \omega \) denote true relative attitude quaternion and angular velocity of the target, \( q' \) and \( \omega' \) denote measured ones, \( d \) denotes relative distance, \( w \) denotes Gaussian white noise with variance 1 and average 0. And \( C \) denotes coefficient vector determined as follows.

\[
C_{7x1} = 10^{-9} \times \begin{bmatrix}
0.662 \\
1.21 \\
1.54 \\
3.71 \\
2.43 \\
0.502 \\
3.38
\end{bmatrix}
\]  

(11)

It is determined by the result of experiment of motion estimation[3].

Considering error compensation, two cases were simulated. In case A, the Adjusted Control Input is used. Since it is expected that the Adjusted Control Input requires very low calculation cost, the pursuer is expected to be able to recompute the control input during maneuver continuing estimation and prediction of target’s motion. So it is assumed that the pursuer continues to estimate target’s “current” motion during maneuver, and update relative position and velocity every 5 seconds. And the pursuer also continues to predict target’s “future” motion using target’s motion estimates, and update it every 10 seconds. Using these updated estimates and predicts, the pursuer recomputes control input iteratively.

In case B, the control input is calculated applying SCGRA with the cost function defined by Eq. (5). It is expected that SCGRA is difficult to apply iterative calculation during maneuver because of its calculation cost, so no error compensation was carried out in this B.

These two cases are compared for calculation time and fuel consumption. The overall simulation is executed by Simulink®, and for the comparison of the calculating time for the control input the program coded in C language is developed on a personal computer (CPU: Pentium 4, 3.00GHz; RAM: 1GB). The fuel consumption is evaluated by delta-V defined as follows.

\[
\Delta V = \frac{1}{m} \int_0^{t_f} (|f_x(t)| + |f_y(t)| + |f_z(t)|) dt \]  

(12)

4.2. The result of numerical simulation

Table 1 shows the initial prediction error of the position and velocity of the target point at final time. This error is assumed by Eq. (10). The pursuer calculates thrust input profile by using this prediction at initial time.

Fig. 7 shows the thrust input profile in case A (Adjusted Control Input), and Fig. 8 shows that in case B (SCGRA). In Fig. 7, it is realized that the control input in z-axis is adjusted at switching time \( t_s = 44 \text{ sec} \) in case A. And it is realized that the control input becomes discontinuous especially after 160sec, because the control input is recomputed.

The result of comparison of control inputs is shown in Table 2. The delta-V of case B is smaller than that of case A, while case B requires longer time to calculate the control input.

Fig. 9 shows the result of the pursuer’s position control simulation. The blue line shows the trajectory of pursuer’s centroid in case A, applying the Adjusted Control Input with proposed error compensation. The red line shows that in case B, applying SCGRA with no error compensation. And the green line shows that of the target point. Table 3 shows the final position of the pursuer’s centroid in each case. In case A, as the effect of reducing the target point prediction error and the pursuer’s position estimation error, the final position error at the final time of the pursuer is reduced to 1.938m. While in case B, since the prediction error compensation is not carried out, the pursuer is destined for the target point predicted at initial time containing relatively large prediction error (bottom in Table 1). Moreover, as the result of no position error compensation during flight, the final error from the target point predicted at initial time became 28.45m. Our proposed compensation scheme therefore worked properly and minimized the final position error.
5. Concluding Remarks and Future Work

A strategy for position control of a spacecraft applying feedforward optimal control is proposed. For feedforward control phase of this strategy, Adjusted Control Input is proposed and its basic properties are investigated by numerical simulation comparing with SCGRA.

This paper showed typical feasibility of proposed control algorithm using simple example for the numerical simulation. Applying proposed algorithm for specific missions such as space exploration at deep space and space debris removal in a Low Earth Orbit is likely to be the next step of this research.

Table 1. Initial prediction error

<table>
<thead>
<tr>
<th>final position of the target point [m]</th>
<th>final position of the target point [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>true value</td>
<td>predicted at initial time</td>
</tr>
<tr>
<td>(255.6, -80.11, -135.8)</td>
<td>(269.2, -92.04, -95.31)</td>
</tr>
</tbody>
</table>

Table 2. Comparison of control inputs

<table>
<thead>
<tr>
<th>Adjusted Control Input</th>
<th>ΔV [m/s]</th>
<th>calculation time [sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>case A</td>
<td>68.1</td>
<td>0.0062</td>
</tr>
<tr>
<td>SCGRA</td>
<td>57.4</td>
<td>48.3</td>
</tr>
</tbody>
</table>

Table 3. Final state of the pursuer's centroid

<table>
<thead>
<tr>
<th>final position of the pursuer's centroid [m]</th>
<th>final velocity of the pursuer’s centroid [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>case A</td>
<td>(255.6, -81.27, -134.3)</td>
</tr>
<tr>
<td></td>
<td>(-4.178, 9.966, -13.73)</td>
</tr>
<tr>
<td>case B</td>
<td>(271.8, -104.7, -69.97)</td>
</tr>
<tr>
<td></td>
<td>(-1.559, 9.2, -13.17)</td>
</tr>
</tbody>
</table>

Fig. 7. Control input profile of case A (Adjusted Control Input)

Fig. 8. Control input profile of case B (SCGRA)

Fig. 9. Result of numerical simulation of position control of pursuer

References