Multi-Constrained Optimal Control of 3D Robotic Arm Manipulators

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This paper presents a generic method for optimal motion planning for three-dimensional 3-DOF multi-link robotic manipulators. We consider the operation of the manipulator systems, which involve constrained payload transportation/capture/release, which is a subject to the minimization of the user-defined objective function, enabling for example minimization of the time of the transfer and/or actuation efforts. It should be stressed out that the task is solved in the presence of arbitrary multiple additional constraints. The solutions of the associated nonlinear differential equations of motion are obtained numerically using the direct transcription method. The direct method seeks to transform the continuous optimal control problem into a discrete mathematical programming problem, which in turn is solved using a non-linear programming algorithm. By discretizing the state and control variables at a series of nodes, the integration of the dynamical equations of motion is not required. The Chebyshev pseudospectral method, due to its high accuracy and fast computation times, was chosen as the direct optimization method to be employed to solve the problem. To illustrate the capabilities of the methodology, maneuvering of RRR 3D robot manipulators were considered in detail. Their optimal operations were simulated for the manipulators, bined to move their effectors along the specified 2D plane and 3D spherical and cylindrical surfaces (imitating for example, welding, tooling or scanning robots).

Key Words: 3D Robotic Manipulators, Optimal Path Planning, Pseudospectral Method, Nonlinear Multiple Constraints

1. Introduction

Development of new methods of real-time simulation and optimal control of robotic manipulators enables to perform complex tasks of ever increasing complexity.

In the previous publications by the author, dynamics and optimal control of various systems of robotic manipulators have been considered, including the following cases: collision-free trajectory planning for 2D and 3D robotic arm manipulators in the presence of morphing (i.e. having variable shape) mobile (wandering) obstacles 1); optimal motion planning for the composite systems of multiple robotic manipulators, working cooperatively 2); tracking control 3).

In the current paper we further develop these models, considering operation of the 3D rigid link robotic manipulators which motion are confined to the prescribed 3D surfaces. Among infinite number of possible trajectories along the constrained surfaces, we determine the optimal path and associated actuator moments, ensuring that the user-defined objective function is minimized during the determined optimal maneuver. Note, that the task is solved in view of the bounded actuator moments. Remarkable, that the method developed for the described problems can be also applied to the more complex cases with the additional constraints in form of multiple obstacles, like "no-go" zones, where we ensure with the optimal solution that the payload/effector and/or robotic links are not entering these zones at any time and never collide with the obstacles.

In order to illustrate the effectiveness of the method, we present three illustrative examples, in which the motion of

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the end tip of the 3-DOF robotic arm (also called an effector) is constrained to a 2D plane, to a cylinder and to a sphere.

2. Optimal Path Planning for 3D Robotic Manipulators

2.1. Motivation

Some of the major tasks performed by the robotic arms include transfer of payloads, inspection/tracking of the 3D surfaces or operation on these surfaces, for example welding or polishing, etc. For the same initial and final conditions for the effector, it is possible to find various implementation scenarios of the effector’s transfer in space, which however may have quite different efficiency measures. In order to shift the resting payload from one point to another point (ensuring its zero final velocity) using a robotic arms, there may be many different combinations of actuation torques applied to its vertical base, and two consecutively pinned links.

In this case the practical realization question is to find the best combination, satisfying the optimality criteria. In one case scenario this criteria (optimality requirement) may be the least integrated actuation torque.

The other example of the cost function may involve a requirement of the minimum time to complete the maneuver or minimized integrated acceleration of the payload. In the following Sections we will develop and illustrate the method of the path planning satisfying these additional requirements for single robotic manipulators.

2.2. Specific dynamical constrains supplementing the task of the optimal path planning

Some of the most important tasks of the optimal motion planning for the robotic arms involve a requirement of the minimum actuation efforts and minimum time to complete the maneuver or minimization of the integrated acceleration of the payload.

More importantly the method presented below allows to solve the optimization task in view of multiple additional nonlinear constraints the robotic systems user may impose, based on the mission requirements. These additional constraints may for example involve path constraints on the system, prohibiting the members to enter a specified space or, in contrary, prescribing the system to follow the desired trajectory or not to leave the “corridors” for the members of the robotic systems. More specifically, the emphasis in this paper is in solving the optimal path planning tasks in view of the prescribed compulsory requirement for the robot’s end effector to follow along the specified surface.

For the illustration purposes the current paper considers three particular cases of the path constrains imposed on the robotic system, constraining the trajectory of the robot's effector to the (1) plane, (2) cylinder and (3) sphere correspondingly. Each of these three illustration cases has its practical relevance. For example, the experimental module platform of the Japanese space research laboratory KIBOU is flat, and it may be of practical interest to consider an optimal payload transfer task requesting the tip of the effector not to leave the surface of the plane platform at any time during the transfer. The second case with the requirement of the effector's trajectory to be assigned to the spherical surface can be illustrated with the Ajisai satellite. This satellite launched by NASA (now JAXA) in 1986 on board of H-I launch vehicle has a spherical shape. Therefore, should the task of surveying the surface of the satellite like Ajisai arise, the robotic arm path planning should be very specific in this case and should ensure that the effector is always following the inspected surface. At last, the third case can be linked to the case of the robotic manipulator handling the spin stabilized satellite and requiring the tip of the effector to follow the cylindrical surface. More complicated cases of the constrained surfaces can be decomposed to the above shapes or can be expressed by combination of the above shapes.

The necessity to comply with additional path constraints (including the described path constraints) significantly complicates the operation of the manipulators. In the sense of the additional constraints, the method is general and is not restricted to the listed examples of the cost functions and additional constraints.

2.3. Brief outline of the solution strategy using the pseudospectral method

We propose to solve the wide class of optimal motion planning for the robotic arms (including the problems formulated above) using the Chebyshev-pseudospectral method, which belongs to the so called direct methods. This section presents a brief outline of the application of the method and its implementation. Interested readers can refer to Refs. 4-5) for details of the actual discretization process.

The idea of direct methods is to convert the continuous optimal control problem into a discrete problem. The question that arises is: what is the best way to do this? Many different approaches have been suggested in the literature, however the pseudospectral approach appears to offer many advantages over other known methods.

The essence of the pseudospectral method is to assume a polynomial representation for the system state and control variables for the entire time interval of interest. It is convenient to choose polynomials of arbitrary order, i.e., order N. The most general representation of an arbitrary function is the Lagrange form. The drawback of using interpolating polynomials is that they suffer from the Runge phenomenon if equidistant grid points are used for the approximation. Approximation theory shows that for any given function, there is an optimal spacing for the grid points. Generally, the best distribution is to cluster the grid points quadratically near the end points of the interval. The best points to choose are the roots of the derivatives of orthogonal polynomials, as well as the end points of the interval. Classical orthogonal polynomials such as Legendre or Chebyshev polynomials are often used. In our work the state equations are enforced as dynamic constraints by differentiating the approximating
Chebyshev polynomials at the corresponding Chebyshev points. (Our specific choice of Chebyshev polynomials offers advantages in implementation because the collocation points can be expressed in closed form, which is explained in detail in the next section.) Hence, the time consuming task of integrating the state equations is avoided. To complement the optimization procedure, the performance index is approximated using an appropriate numerical quadrature. In this paper we are employing the Gauss-Lobatto quadrature. Therefore the grids formed using the described and employed pseudospectral method we will also called Chebyshev-Gauss-Lobatto grids because of their use in high-order quadrature rules.

Once the original optimal control problem is discretised and formulated as a nonlinear programming problem, the structure of the discretised problem resembles that of the original continuous problem. The discretisation procedure has been implemented in the MATLAB-based package DIRECT that automates the solution of general single-phase optimal control problems, as well as parameter estimation problems. This software, developed by Paul Williams \(^{6}\) from RMIT University, is based on direct transcription of the continuous problem to a discrete nonlinear programming problem. The main feature of DIRECT is that it implements a wide variety of different transcription methods. DIRECT was inspired by the software DIDO \(^{7}\) developed by I. M. Ross at the Naval Postgraduate School, which is now available for public release with the TOMLAB solver suite \(^{8}\). DIRECT employs SNOPT \(^{9}\) to solve the resulting constrained nonlinear programming problem.

2.4. Details on Chebyshev pseudospectral method used for the optimal trajectory planning

The general formulation of the constrained optimization task for the robotic arm (which includes the surface-constrained optimization for the effector, described above) requires minimization of the performance index

\[
J = \int_{t_0}^{t_f} [\dot{x}(t), \ddot{x}(t), x(t), u(t), t] + \int_{t_0}^{t_f} L(\dot{x}(t), \ddot{x}(t), x(t), u(t), t) \, dt
\]

where \( t \in \mathbb{R}, x \in \mathbb{R}^n \) and \( u \in \mathbb{R}^m \) are subject to the dynamical constraints

\[
f[\dot{x}(t), \ddot{x}(t), x(t), u(t), t] = 0
\]

and boundary conditions

\[
\Psi_0[\dot{x}(t_0), x(t_0)] = 0
\]

\[
\Psi_p[\dot{x}(t_f), x(t_f)] = 0
\]

where \( \Psi_0 \in \mathbb{R}^p \) and \( \Psi_p \in \mathbb{R}^q \) with \( p \leq n \) and \( q \leq n \), and the state and control constraints

\[
g[\dot{x}(t), \ddot{x}(t), x(t), u(t), t] \leq 0
\]

Note that this problem is an extension of the traditional Bolza problem in optimal control literature by the inclusion of penalty terms on the system accelerations in the performance index, Eq. (1). The Bolza problem is obtained simply by omitting the second derivative of the states from the performance index.

In pseudospectral methods, globally orthogonal polynomials such as Chebyshev or Legendre polynomials, which belong to the class of Jacobi polynomials, are used as the basis for interpolating polynomials. The approaches are very similar and differ only in the specific properties of the polynomial. The choice of Chebyshev polynomials offers advantages in implementation because the collocation points can be expressed in closed form. Legendre and general Jacobi polynomials, on the other hand, require advanced techniques from linear algebra to determine the collocation points and corresponding weights.

The Chebyshev-Gauss-Lobatto points lie in the interval [-1, 1] and are located at the extrema of the \( N \)-th order Chebyshev polynomial \( T_N(\tau) \). The \( j \)-th order Chebyshev polynomial of the first kind \( T_j(\tau) \) is defined by

\[
T_j(\tau) = \cos(j \cos^{-1}(\tau)), \quad j = 0, \ldots, N
\]

The extreme values of Eq. (6) occur at the points \( \tau_j \) defined by

\[
\tau_j = -\cos(j / N), \quad j = 0, \ldots, N
\]

Since Chebyshev polynomials are defined over the interval [-1, 1], a linear transformation is necessary to express the problem on the original time interval \([t_0, t_f]\)

\[
t = \frac{1}{2} \left( (t_f - t_0) \tau + t_f + t_0 \right) / 2
\]

We seek finite length polynomials for approximating the state and control vectors as follows

\[
x_{\hat{t}}(\tau) = \sum_{j=0}^{N} \hat{x}_j \phi_j(\tau)
\]

\[
u_{\hat{t}}(\tau) = \sum_{j=0}^{N} \hat{u}_j \phi_j(\tau)
\]

where \( \hat{x}_j, \hat{u}_j, j = 0, N \) are the coefficients of the interpolating polynomial. If it is desired that the coefficients, \( \hat{x}_j = \mathbf{x}_j(\tau_j) \), \( \hat{u}_j = \mathbf{u}_j(\tau_j) \), then it is clear that \( \phi_j(\tau) \) are the Lagrange interpolating polynomials

\[
\phi_j(\tau) = \frac{(-1)^{j-1} (1-\tau^2)^j \hat{T}_j(\tau)}{c_j N^2 (\tau - \tau_j)}
\]

where \( c_j = 1 \) for \( 1 \leq j \leq N-1 \) and \( c_0 = c_N = 2 \) for \( j = 0, N \).

In order to approximate the system dynamics, it is necessary to find expressions of the first and second derivatives of the interpolating polynomials at the CGL nodes. The derivatives of the approximating functions can be obtained by directly differentiating Eq. (9) to obtain the Chebyshev differentiation matrices

\[
d_x = \hat{x}(\tau_j) = \sum_{j=0}^{N} \hat{x}_j \dot{\phi}_j(\tau_j) = \sum_{j=0}^{N} D_{\hat{x}j} \hat{x}_j
\]

\[
d_{x}^{(2)} = \hat{x}(\tau_j) = \sum_{j=0}^{N} \hat{x}_j \ddot{\phi}_j(\tau_j) = \sum_{j=0}^{N} D_{\hat{x}j}^{(2)} \hat{x}_j
\]

where the coefficients \( D_{\hat{x}j} \) and \( D_{\hat{x}j}^{(2)} \) are entries of...
\((N+1)\times(N+1)\) differentiation matrices. The derivatives of the interpolating polynomials given in Eq. (12) and (13) are derivatives with respect to the parameter \(\tau\) and not the independent variable in the original problem, \(t\). Derivatives with respect to \(t\) are obtained are follows

\[
\dot{x}^N(t) = \frac{2}{t_f - t_0} \ddot{x}^N(\tau)
\]

\[
\ddot{x}^N(t) = \frac{2}{t_f - t_0} \dot{x}^N(\tau)
\]

The integral in Eq. (1) may be approximated either by the Gauss-Lobatto or Curtis-Clenshaw formulae. In the end of the process, the original control problem can now be formulated as the following nonlinear programming problem of finding the coefficients

\[
X = [x_0, \ldots, x_N, u_0, \ldots, u_N]
\]

that minimize the performance index

\[
J^N = \phi \left[ \frac{4}{(t_f - t_0)^2} \ddot{d}^{(2)}, \frac{2}{(t_f - t_0)^2} \dot{d}, x_N, t_f \right] + \sum_{k=0}^{N} \left[ \frac{4}{(t_f - t_0)^2} \ddot{d}^{(2)}, \frac{2}{(t_f - t_0)^2} \dot{d}, x_k, t \right] w_k
\]

subject to

\[
f = \frac{4}{(t_f - t_0)^2} \ddot{d}^{(2)}, \frac{2}{(t_f - t_0)^2} \dot{d}, x_k, t = 0
\]

\[
\psi_0 \left[ \frac{2}{(t_f - t_0)} \dot{d}, x_0, t_f \right] = 0
\]

\[
\psi_f \left[ \frac{2}{(t_f - t_0)} \dot{d}, x_N, t_f \right] = 0
\]

\[
g \left[ \frac{2}{(t_f - t_0)} \ddot{d}, x_k, u_k \right] \leq 0
\]

Equations (17) through (21) illustrate the advantages that direct discretization of the original second-order system of equations offers over the conventional reduction to state-space form. The proposed approach is considerably simpler and retains the original structure of the problem without algebraic simplifications or reductions.

3. Mathematical Model of a Rigid Three-link Robotic Manipulator

3.1. Equations of motion

In order to describe the motion of the system shown in Fig. 1, we firstly derive dynamic equations for a single manipulator using the method of Lagrange's Equations of Motion \(^1, 10, 11\).

\[
M(\theta)\dddot{\theta} + V(\theta, \dot{\theta}) + G(\theta) = u_i \quad (i = 1, 2, 3)
\]

where complex expressions on the matrix \([M]\) and vectors \([V]\) and \([G]\) are calculated using the mathematical package MAPLE.

3.2 Simulation computer code

The differential equations for the motion of various multi-link robotic arm manipulators and their combinations were programmed by the author in MATLAB. Having obtained an adequate universal analysis tool, it is possible to perform a numerical study focusing on robotic dynamics through the simulation of the optimal trajectories for the illustrative robotic systems under various conditions and non-linear geometric and force constraints.

Developed MATLAB-based computer software enables for not only the simulation of various optimized maneuvers of the compound systems of cooperative robots, but also allows to demonstrate the process using animations. For enhanced realism of the animations, an additional set of computer programs was developed in MATLAB which generates Virtual Reality animation files for playing them under Internet Explorer.

4. Optimal Trajectory Planning for Rigid-link Manipulators: Cases Study Results

4.1. Payload is constrained to a 2D plane

An example of a 3-DOF robotic arm manipulator is given in Fig. 1. For the presented simulation case the following parameters were used: \(L_1 = L_2 = L_3 = 1\), \(m_1 = m_2 = m_3 = 1\) (in non-dimensional units).

Let us consider its transfer from the specified initial point \(A_0\) to the final point \(A_f\), which requires the initial and final velocities for the free end (tip) of the manipulator (also called effector) to be zero. A particular feature of this case is in imposing set bounds for the actuation moment \(u_1\) and \(u_2\). The “efficiency” of each of the simulated cases will be assessed based on the following cost function:

\[
\int 0 \rightarrow t_f \sum_{i=1}^{3} \sum_{j=1}^{3} 
\]
The solution will be accepted as optimal if the cost function is minimized against all other feasible solutions. The resulting problem was programmed in MATLAB and solved using the DIRECT 6 MATLAB-based computer package, which transcribes the optimal control problem into a nonlinear programming problem. Examination of the exit flag and constraint file returned by SNOPT indicated the optimization was successful; the optimal solution was found with all target states being met. The optimal results of the simulated case are presented in Figs. 2 to 7. Fig. 2 shows the Cartesian \( x \) and \( y \) positions of the effector. The feature of the case is constraining the motion of the tip effector in the plane.

The time histories for the angles \( \theta_1, \theta_2 \) and \( \theta_3 \) and their time rates are shown in Figs. 3 and 4. They are all a reflection of the actuator moments, \( u_1, u_2 \) and \( u_3 \), determined during solution of the described nonlinear, multi-constraint optimization problem and given in Fig. 5. Because in this example the control actuator values were not bounded, they are represented with the smooth plots.

It is interesting to note that the effector’s transfer from the initial to the final point is not given by a straight line. This is because that the performance index is selected as the least integrated actuation torques, minimizing the moment of inertia around the link-1, that is, folding the arm toward the origin is effective in reducing the control effort.

\[ J = \sum_{i=1}^{3} \int_{0}^{T} u_i^2(t) dt \]  

(23)
Fig. 6. Time history for the squared combined effort of the robot’s three actuators.

Fig. 7. Time history for the minimized cost function of the robotic manipulator.

Fig. 8. The optimal trajectory for the 3D robotic arm constrained to slide its payload along the sphere.

Fig. 9. Time history for position angles of the robot’s manipulator.

Fig. 6 shows the squared combined efforts of three actuators. It can be useful in determining the most critical stages of the operation of the robotic system, enabling to see the most “effort-consuming” time segments, during which the maximum power is required. For example, for the illustration case shown, at approximately non-dimensional times equal to 0.4 and 0.6, the robotic system should develop its maximum combined power.

Fig. 7 shows the time history of the accumulative cost function defined by the Eq. (23) and can be used to rank the performance of the robotic arm against any other, non-optimal solution. The other use of the plot is to see the expenditure of the power during the operation scenario.

4.2 Payload is constrained to a sphere

Second example for the same robotic system introduces a few complications. Firstly, the prescriptive surface for the effector is taken of a more complex, 3D shape, namely as a perfect sphere. Secondly, the actuators control input is bounded as follows: $|u_j| \leq 24$.

Initial and final position angles are equal to: $\theta_{1o}=\theta_{2o}=0[\text{deg}]$, $\theta_{3o}=-30[\text{deg}]$, $\theta_{4o}=0[\text{deg}]$, $\theta_{5o}=30[\text{deg}]$, and $\theta_{6o}=0[\text{deg}]$.

The results of the solution of the optimal task are presented on the Figs. 8 to 13.

It is interesting to note that the solution complies with all requirements for actuator torques: In particular, the solution for actuation moments $u_1, u_2$ and $u_3$ which is plotted in Fig. 11 reflects the superimposed requirement of the absolute values of the maximum torques not to exceed 24 conventional units, used in this illustration example.

Similar to the plane case, the optimal trajectory for the robotic manipulator transferring its effector from the initial point to the final point is not along the shortest curve, but rather along a curved line completely confined to the sphere. We attribute this result to the non-linear nature of the system, described by the non-linear differential equations and subject to the non-linear constraints and the cost function. It is evident that if the shortest curve on the sphere was chosen for the transfer of the robot’s effector, the links 2 and 3 would have moved upward, and the motion would have resulted in the larger control effort, which was not desired in minimization of the integrated control effort.
4.3. Payload is constrained to a cylinder

The last example for the robotic system in Fig. 1 prescribes the end effector to slide along the cylindrical surface. In this case the actuators control input is bounded as follows: $|u_1| \leq 11$, $|u_2| \leq 10$, and $|u_3| \leq 9$.

Initial and final conditions for the task are: $\theta_{10}=-90[\text{deg}]$, $\theta_{20}=60[\text{deg}]$, $\theta_{30}=-60[\text{deg}]$, $\theta_{1f}=0[\text{deg}]$, $\theta_{2f}=60[\text{deg}]$, and $\theta_{3f}=-60[\text{deg}]$.

Optimal solution for the task is shown in Fig. 14, where the optimal path of the robot’s effector is shown in 3D space using the virtual reality graphical tools. The reason why the trajectory was not the shortest curve connecting the initial point and the final one is the same as for the sphere case, presented in Section 4.2. The time histories for the angles $\theta_i$, $\theta_j$, and $\theta_k$ and their rates are shown in Fig. 15 and Fig. 16, respectively. They are all a reflection of the actuator moments $u_1$, $u_2$, and $u_3$ determined during solution of the described nonlinear, multi-constraint optimization problem and given in Fig. 17.

Fig. 18 represents the squared combined efforts of three actuators and can be useful in determining the most critical stages of the operation of the robotic system, enabling to see the most “effort-consuming” time segments, during which the maximum power is required.

For example, for the illustration case shown, during the non-dimensional time intervals 0.2-0.35 and 0.6-0.8, the robotic system should develop its maximum combined power. Fig. 19 shows the time history of the accumulative cost function for cylindrical surface case.
Fig. 14. The optimal trajectory for the 3D robotic arm constrained to slide its payload along the cylinder.

Fig. 15. Time history for position angles of the robot’s manipulator.

Fig. 16. Time history for the rates of the position angles of the robot’s manipulator.

Fig. 17. Optimal time history of the bounded actuation moments controlling the motion of the 3D robotic arm.

Fig. 18. Time history for the squared combined effort of the robot’s three actuators.

Fig. 19. Time history for the cost function of the robotic manipulator.
5. Main Results and Conclusions

This paper presented new results on optimal path planning and optimal control of single robotic manipulators. For these systems the associated non-linear optimization problems were formulated in this paper and solved using the Chebyshev-pseudospectral method.

It should be stressed out that, the method presented in the paper allows not only to minimize the specified arbitrary non-linear cost function, but also allows to solve the optimization task in view of multiple additional non-linear constraints that the user of the robotic systems may choose to impose based on mission requirements or considerations.

In the current paper a procedure of optimal path planning for rigid manipulators performing operations in presence of the path constraining surfaces for the robot’s end effectors, has been proposed and successfully implemented.

The optimal scenarios enable to perform deployment of the effector following the prescribed surfaces and in view of the bounded constraints on the actuator control torques.

It has been demonstrated that the actuator efforts required to perform the task is higher than for the similar cases without the obstructing obstacles. The presented method is general and enables to apply unlimited number of other additional dynamical constraints. Examples of these additional constraints may involve path constraints on the system, prohibiting the members to enter a specified space area or, on the contrary, prescribing the system to follow the desired trajectory or prescribing for the members of the robotic system not to leave the allowed bandwidth’ corridors’. It should be emphasised that the method is generic and is not restricted to the listed examples of the cost functions and additional constraints.

Further work may involve consideration of the operation of a compound system of multiple manipulators working cooperatively towards the common goals, including a payload transfer.

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References

Appendix: Virtual Reality Animated Summary of Optimally Controlled Payload Transfer with the Effector Constrained to the Flat, Spherical and Cylindrical Surfaces

(a) Plane

Time =0/121

Time =20/121

Time =40/121

Time =60/121

Time =80/121

Time =100/121

Time =121/121

(b) Sphere

Time =0/121

Time =20/121

Time =40/121

Time =60/121

Time =80/121

Time =100/121

Time =121/121

(c) Cylinder

Time =0/51

Time =10/51

Time =20/51

Time =30/51

Time =40/51

Time =50/51

Time =51/51


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