寄 書

成層構造に対する ray
theory の適用 (14)

北大理地球物理 田 治 米 鏡 二
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An Application of the Ray Theory to
Stratified Layers (14).

Kyozi TAZIME

Department of Geophysics, Faculty of
Science, Hokkaido University
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§ 19. 一般の 2 層構造に対する M 波の特性方程式の解法. (6.4) の j を 1 及び 2 とすれば, (6.6) と (6.7) とを使つて, 第 1 層と第 2 層との間の反射係数を次のごとく表現することができる.

$$\left. \begin{aligned} A' &= A_1/B_1 = -A'_A/A', \\ B' &= C_1/B_1 = A'_B/A', \\ C' &= A_1/D_1 = -A'_C/A', \\ D' &= C_1/D_1 = -A'_D/A'. \end{aligned} \right\} \quad (19.1)$$

ここでは M 波の考察を主にしているので, § 5. で述べた記号法に従つて次のごとくおくことにする.

$$\alpha_2/\xi = -i\hat{\alpha}_2/\xi, \quad \beta_2/\xi = -i\hat{\beta}_2/\xi. \quad (19.2)$$

更に

$$\left. \begin{aligned} P &= \left(\frac{\hat{\beta}_2^2}{\xi^2} + 1 \right)^2 - 4 \frac{\hat{\alpha}_2}{\xi} \frac{\hat{\beta}_2}{\xi}, \\ Q &= \left(\frac{\hat{\beta}_2^2}{\xi^2} + 1 \right) - 2 \frac{\hat{\alpha}_2}{\xi} \frac{\hat{\beta}_2}{\xi}, \\ R &= 1 - \frac{\hat{\alpha}_2}{\xi} \frac{\hat{\beta}_2}{\xi}, \\ S &= \left(\frac{c}{v_{s1}} - \frac{c}{v_{s2}} \right)^2 \end{aligned} \right\} \quad (19.3)$$

とおけば, (19.1) の行列は次のごとく表わされる.

$$\left. \begin{aligned} A' &= \left(\frac{\mu_2}{\mu_1} \right)^2 P \left(1 + \frac{\alpha_1}{\xi} \frac{\beta_1}{\xi} \right) + 2 \left(\frac{\mu_2}{\mu_1} \right) Q \left\{ \left(\frac{\beta_1^2}{\xi^2} - 1 \right) \right. \\ &\quad \left. - 2 \frac{\alpha_1}{\xi} \frac{\beta_1}{\xi} \right\} + R \left\{ \left(\frac{\beta_1^2}{\xi^2} - 1 \right)^2 + 4 \frac{\alpha_1}{\xi} \frac{\beta_1}{\xi} \right\} \\ &\quad - i \left(\frac{\mu_2}{\mu_1} \right) S \left(\frac{\alpha_1}{\xi} \frac{\hat{\beta}_2}{\xi} + \frac{\beta_1}{\xi} \frac{\hat{\alpha}_2}{\xi} \right), \end{aligned} \right\}$$

$$\left. \begin{aligned} A'_A &= \left(\frac{\mu_2}{\mu_1} \right)^2 P \left(1 - \frac{\alpha_1}{\xi} \frac{\beta_1}{\xi} \right) + 2 \left(\frac{\mu_2}{\mu_1} \right) Q \left\{ \left(\frac{\beta_1^2}{\xi^2} - 1 \right) \right. \\ &\quad \left. + 2 \frac{\alpha_1}{\xi} \frac{\beta_1}{\xi} \right\} + R \left\{ \left(\frac{\beta_1^2}{\xi^2} - 1 \right)^2 - 4 \frac{\alpha_1}{\xi} \frac{\beta_1}{\xi} \right\} \\ &\quad - i \left(\frac{\mu_2}{\mu_1} \right) S \left(\frac{\beta_1}{\xi} \frac{\hat{\alpha}_2}{\xi} - \frac{\alpha_1}{\xi} \frac{\hat{\beta}_2}{\xi} \right), \\ A'_D &= \left(\frac{\mu_2}{\mu_1} \right)^2 P \left(1 - \frac{\alpha_1}{\xi} \frac{\beta_1}{\xi} \right) + 2 \left(\frac{\mu_2}{\mu_1} \right) Q \left\{ \left(\frac{\beta_1^2}{\xi^2} - 1 \right) \right. \\ &\quad \left. + 2 \frac{\alpha_1}{\xi} \frac{\beta_1}{\xi} \right\} + R \left\{ \left(\frac{\beta_1^2}{\xi^2} - 1 \right)^2 - 4 \frac{\alpha_1}{\xi} \frac{\beta_1}{\xi} \right\} \\ &\quad + i \left(\frac{\mu_2}{\mu_1} \right) S \left(\frac{\beta_1}{\xi} \frac{\hat{\alpha}_2}{\xi} - \frac{\alpha_1}{\xi} \frac{\hat{\beta}_2}{\xi} \right), \\ A'_B &= 2(\alpha_1/\xi)N, \quad A'_C = 2(\beta_1/\xi)N, \\ N &= \left(\frac{\mu_2}{\mu_1} \right)^2 P + \left(\frac{\mu_2}{\mu_1} \right) Q \left\{ \left(\frac{\beta_1^2}{\xi^2} - 1 \right) - 2 \right\} \\ &\quad - 2R \left(\frac{\beta_1^2}{\xi^2} - 1 \right). \end{aligned} \right\} \quad (19.4)$$

(19.4) で $\mu_2/\mu_1 = 0$ とおくと, (6.9)~(6.13) と比較して次の関係を得る.

$$A' = A, \quad B' = -B, \quad C' = -C, \quad D' = A. \quad (19.5)$$

これは plate の両側共空気である場合の既出の関係式と一致する.

また (19.4) の第 3 式と (6.10), (6.12) とから, (6.18) の第 2 式を容易に求めることもできる.

(19.4) は更に次のように書き換えておいた方が今後の計算がしやすい.

$$\left. \begin{aligned} A' &= K \\ &\quad + L \left(\frac{\alpha_1}{\xi} \frac{\beta_1}{\xi} \right) - i \left(\frac{\mu_2}{\mu_1} \right) S \left(\frac{\alpha_1}{\xi} \frac{\hat{\beta}_2}{\xi} + \frac{\beta_1}{\xi} \frac{\hat{\alpha}_2}{\xi} \right), \\ A'_A &= K \\ &\quad - L \left(\frac{\alpha_1}{\xi} \frac{\beta_1}{\xi} \right) - i \left(\frac{\mu_2}{\mu_1} \right) S \left(\frac{\beta_1}{\xi} \frac{\hat{\alpha}_2}{\xi} - \frac{\alpha_1}{\xi} \frac{\hat{\beta}_2}{\xi} \right), \\ A'_D &= K \\ &\quad - L \left(\frac{\alpha_1}{\xi} \frac{\beta_1}{\xi} \right) + i \left(\frac{\mu_2}{\mu_1} \right) S \left(\frac{\beta_1}{\xi} \frac{\hat{\alpha}_2}{\xi} - \frac{\alpha_1}{\xi} \frac{\hat{\beta}_2}{\xi} \right). \end{aligned} \right\} \quad (19.6)$$

但し,

$$\left. \begin{aligned} K &= \left(\frac{\mu_2}{\mu_1} \right)^2 P + 2 \left(\frac{\mu_2}{\mu_1} \right) Q \left(\frac{\beta_1^2}{\xi^2} - 1 \right) \\ &\quad + R \left(\frac{\beta_1^2}{\xi^2} - 1 \right)^2, \\ L &= \left(\frac{\mu_2}{\mu_1} \right)^2 P - 4 \left(\frac{\mu_2}{\mu_1} \right) Q + 4R. \end{aligned} \right\} \quad (19.6)$$

このようなおき方をすると, (19.3), (19.4) 及び (19.6) から次の関係を求めておくことができる.

$$Q^2 - PR = \frac{\hat{\alpha}_2}{\xi} \frac{\hat{\beta}_2}{\xi} \left(\frac{c}{v_{s2}} \right)^4, \\ N^2 = KL + \left(\frac{\mu_2}{\mu_1} \right)^2 S^2 \frac{\hat{\alpha}_2}{\xi} \frac{\hat{\beta}_2}{\xi}. \quad (19.7)$$

更に (19.4) と (19.7) とから次の関係を求めておくことと具合がよい.

$$A' A D' D + A' B A' C = A' A^*, \quad (19.8)$$

但し,

$$A^* = K + L \left(\frac{\alpha_1}{\xi} \frac{\beta_1}{\xi} \right) + i \left(\frac{\mu_2}{\mu_1} \right) S \left(\frac{\alpha_1}{\xi} \frac{\beta_2}{\xi} + \frac{\beta_1}{\xi} \frac{\alpha_2}{\xi} \right), \quad (19.9)$$

であつて, これは (19.5) の A' の共軛複素数である.

さて α_1 及び β_1 は何れも実数または純虚数なので, 位相速度の大きさに応じて § 14. におけると同様に α_1 と β_1 との 3 通りの組合せ方が生じる. しかし何れの場合にも P, Q, R, S, K, L, N は実数である.

$$(i) \quad c \geq v_{s1} > v_{s2}; \quad \alpha_1 = \bar{\alpha}_1, \quad \beta_1 = \bar{\beta}_1.$$

$$\left. \begin{aligned} A' &= K \\ &+ \frac{\bar{\alpha}_1}{\xi} \frac{\bar{\beta}_1}{\xi} L - i \left(\frac{\mu_2}{\mu_1} \right) S \left(\frac{\bar{\beta}_1}{\xi} \frac{\hat{\alpha}_2}{\xi} + \frac{\bar{\alpha}_1}{\xi} \frac{\hat{\beta}_2}{\xi} \right), \\ A^* &= K \\ &+ \frac{\bar{\alpha}_1}{\xi} \frac{\bar{\beta}_1}{\xi} L + i \left(\frac{\mu_2}{\mu_1} \right) S \left(\frac{\bar{\beta}_1}{\xi} \frac{\hat{\alpha}_2}{\xi} + \frac{\bar{\alpha}_1}{\xi} \frac{\hat{\beta}_2}{\xi} \right), \\ A' A &= K \\ &- \frac{\bar{\alpha}_1}{\xi} \frac{\bar{\beta}_1}{\xi} L - i \left(\frac{\mu_2}{\mu_1} \right) S \left(\frac{\bar{\beta}_1}{\xi} \frac{\hat{\alpha}_2}{\xi} - \frac{\bar{\alpha}_1}{\xi} \frac{\hat{\beta}_2}{\xi} \right), \\ A' D &= K \\ &- \frac{\bar{\alpha}_1}{\xi} \frac{\bar{\beta}_1}{\xi} L + i \left(\frac{\mu_2}{\mu_1} \right) S \left(\frac{\bar{\beta}_1}{\xi} \frac{\hat{\alpha}_2}{\xi} - \frac{\bar{\alpha}_1}{\xi} \frac{\hat{\beta}_2}{\xi} \right). \end{aligned} \right\} \quad (19.10)$$

$$\left. \begin{aligned} \therefore A^*/A' &= \exp(2i\varepsilon), \\ \tan \varepsilon &= \left(\frac{\mu_2}{\mu_1} \right) S \left(\frac{\bar{\beta}_1}{\xi} \frac{\hat{\alpha}_2}{\xi} + \frac{\bar{\alpha}_1}{\xi} \frac{\hat{\beta}_2}{\xi} \right) \\ &\quad / \left(K + \frac{\bar{\alpha}_1}{\xi} \frac{\bar{\beta}_1}{\xi} L \right), \\ A' D / A' A &= \exp(2i\varepsilon'), \\ \tan \varepsilon' &= \left(\frac{\mu_2}{\mu_1} \right) S \left(\frac{\bar{\beta}_1}{\xi} \frac{\hat{\alpha}_2}{\xi} - \frac{\bar{\alpha}_1}{\xi} \frac{\hat{\beta}_2}{\xi} \right) \\ &\quad / \left(K - \frac{\bar{\alpha}_1}{\xi} \frac{\bar{\beta}_1}{\xi} L \right), \\ A' A / A' &= \Gamma e^{i(\varepsilon - \varepsilon')}, \\ A' D / A' &= \Gamma e^{i(\varepsilon + \varepsilon')}. \end{aligned} \right\} \quad (19.11)$$

但し,

$$\Gamma = \left\{ \left(K - \frac{\bar{\alpha}_1}{\xi} \frac{\bar{\beta}_1}{\xi} L \right)^2 + \left(\frac{\mu_2}{\mu_1} \right)^2 S^2 \left(\frac{\bar{\beta}_1}{\xi} \frac{\hat{\alpha}_2}{\xi} - \frac{\bar{\alpha}_1}{\xi} \frac{\hat{\beta}_2}{\xi} \right)^2 \right\}^{\frac{1}{2}} / \left\{ \left(K + \frac{\bar{\alpha}_1}{\xi} \frac{\bar{\beta}_1}{\xi} L \right)^2 + \left(\frac{\mu_2}{\mu_1} \right)^2 S^2 \left(\frac{\bar{\beta}_1}{\xi} \frac{\hat{\alpha}_2}{\xi} + \frac{\bar{\alpha}_1}{\xi} \frac{\hat{\beta}_2}{\xi} \right)^2 \right\}^{\frac{1}{2}} \\ = \left| \frac{K - (\bar{\alpha}_1/\xi)(\bar{\beta}_1/\xi)L}{K + (\bar{\alpha}_1/\xi)(\bar{\beta}_1/\xi)L} \cdot \frac{\cos \varepsilon}{\cos \varepsilon'} \right|. \quad (19.12)$$

故に (19.1), (19.8) と (19.11) とから

$$\left. \begin{aligned} (A' D' - B' C')^{\frac{1}{2}} &= e^{i\varepsilon}, \\ \text{また} \\ (A A')^{\frac{1}{2}} &= (-A \Gamma)^{\frac{1}{2}} e^{i \frac{\varepsilon - \varepsilon'}{2}}, \\ (D D')^{\frac{1}{2}} &= (-A \Gamma)^{\frac{1}{2}} e^{i \frac{\varepsilon + \varepsilon'}{2}}, \\ (A D A' D')^{\frac{1}{2}} &= -A \Gamma e^{i\varepsilon}, \\ B' C &= -\{(1 - A^2)(1 - \Gamma^2)\}^{\frac{1}{2}} e^{i\varepsilon}. \end{aligned} \right\} \quad (19.13)$$

(19.13) を (12.2) に代入すると

$$\left. \begin{aligned} X^2 - Y^2 &= l^2 \\ X &= \sin \left\{ \left(\frac{\bar{\alpha}_1}{\xi} + \frac{\bar{\beta}_1}{\xi} \right) \frac{\xi H}{2} - \frac{\varepsilon}{2} \right\}, \\ Y &= (-A \Gamma)^{\frac{1}{2}} \sin \left\{ \left(\frac{\bar{\beta}_1}{\xi} - \frac{\bar{\alpha}_1}{\xi} \right) \frac{\xi H}{2} - \frac{\varepsilon'}{2} \right\}, \\ l^2 &= \frac{1}{2} \{ 1 - (-A \Gamma) \pm (1 - A^2)^{\frac{1}{2}} (1 - \Gamma^2)^{\frac{1}{2}} \}, \\ B' C &\leq D. \end{aligned} \right\} \quad (19.14)$$

$$(ii) \quad v_{s1} \geq c \geq v_{s2}; \quad \alpha_1 = -i\hat{\alpha}_1, \quad \beta_1 = -i\hat{\beta}_1.$$

$$\left. \begin{aligned} A' &= K - \left(\frac{\mu_2}{\mu_1} \right) S \frac{\hat{\alpha}_1}{\xi} \frac{\hat{\beta}_2}{\xi} \\ &- i \left\{ \frac{\hat{\alpha}_1}{\xi} \frac{\bar{\beta}_1}{\xi} L + \left(\frac{\mu_2}{\mu_1} \right) S \frac{\bar{\beta}_1}{\xi} \frac{\hat{\alpha}_2}{\xi} \right\}, \\ A^* &= K + \left(\frac{\mu_2}{\mu_1} \right) S \frac{\hat{\alpha}_1}{\xi} \frac{\hat{\beta}_2}{\xi} \\ &- i \left\{ \frac{\hat{\alpha}_1}{\xi} \frac{\bar{\beta}_1}{\xi} L - \left(\frac{\mu_2}{\mu_1} \right) S \frac{\bar{\beta}_1}{\xi} \frac{\hat{\alpha}_2}{\xi} \right\}, \\ A' A &= K + \left(\frac{\mu_2}{\mu_1} \right) S \frac{\hat{\alpha}_1}{\xi} \frac{\hat{\beta}_2}{\xi} \\ &+ i \left\{ \frac{\hat{\alpha}_1}{\xi} \frac{\bar{\beta}_1}{\xi} L - \left(\frac{\mu_2}{\mu_1} \right) S \frac{\bar{\beta}_1}{\xi} \frac{\hat{\alpha}_2}{\xi} \right\}, \\ A' D &= K - \left(\frac{\mu_2}{\mu_1} \right) S \frac{\hat{\alpha}_1}{\xi} \frac{\hat{\beta}_2}{\xi} \\ &+ i \left\{ \frac{\hat{\alpha}_1}{\xi} \frac{\bar{\beta}_1}{\xi} L + \left(\frac{\mu_2}{\mu_1} \right) S \frac{\bar{\beta}_1}{\xi} \frac{\hat{\alpha}_2}{\xi} \right\}. \end{aligned} \right\} \quad (19.15)$$

$$\left. \begin{aligned}
 A'/A^* &= e^{2i\varepsilon'}, \\
 \tan \varepsilon' &= \left\{ \frac{\hat{\alpha}_1}{\xi} \frac{\bar{\beta}_1}{\xi} L - \left(\frac{\mu_2}{\mu_1} \right) S \frac{\bar{\beta}_1}{\xi} \frac{\hat{\alpha}_2}{\xi} \right\} \\
 &\quad / \left\{ K + \left(\frac{\mu_2}{\mu_1} \right) S \frac{\hat{\alpha}_1}{\xi} \frac{\hat{\beta}_2}{\xi} \right\}, \\
 A'_D/A' &= e^{2i\varepsilon}, \\
 \tan \varepsilon &= \left\{ \frac{\hat{\alpha}_1}{\xi} \frac{\bar{\beta}_1}{\xi} L + \left(\frac{\mu_2}{\mu_1} \right) S \frac{\bar{\beta}_1}{\xi} \frac{\hat{\alpha}_2}{\xi} \right\} \\
 &\quad / \left\{ K - \left(\frac{\mu_2}{\mu_1} \right) S \frac{\hat{\alpha}_1}{\xi} \frac{\hat{\beta}_2}{\xi} \right\}, \\
 A^*/A' &= \Gamma e^{i(\varepsilon - \varepsilon')}.
 \end{aligned} \right\} \quad (19.16)$$

但し,

$$\begin{aligned}
 \Gamma &= \left\{ \left(K + \frac{\mu_2}{\mu_1} S \frac{\hat{\alpha}_1}{\xi} \frac{\hat{\beta}_2}{\xi} \right)^2 + \left(\frac{\hat{\alpha}_1}{\xi} \frac{\bar{\beta}_1}{\xi} L \right. \right. \\
 &\quad \left. \left. - \frac{\mu_2}{\mu_1} S \frac{\bar{\beta}_1}{\xi} \frac{\hat{\alpha}_2}{\xi} \right)^2 \right\}^{\frac{1}{2}} / \left\{ \left(K - \frac{\mu_2}{\mu_1} S \frac{\hat{\alpha}_1}{\xi} \frac{\hat{\beta}_2}{\xi} \right)^2 \right. \\
 &\quad \left. + \left(\frac{\hat{\alpha}_1}{\xi} \frac{\bar{\beta}_1}{\xi} L + \frac{\mu_2}{\mu_1} S \frac{\bar{\beta}_1}{\xi} \frac{\hat{\alpha}_2}{\xi} \right)^2 \right\}^{\frac{1}{2}} \\
 &= \frac{K + (\mu_2/\mu_1) S(\hat{\alpha}_1/\xi)(\hat{\beta}_2/\xi)}{K - (\mu_2/\mu_1) S(\hat{\alpha}_1/\xi)(\hat{\beta}_2/\xi)} \cdot \frac{\cos \varepsilon}{\cos \varepsilon'}.
 \end{aligned} \quad (19.17)$$

故に (19.1), (19.8) と (19.16) とから,

$$\left. \begin{aligned}
 (A'D' - B'C')^{\frac{1}{2}} &= \Gamma^{\frac{1}{2}} e^{i\frac{\varepsilon - \varepsilon'}{2}}, \\
 \text{また} \\
 (AA')^{\frac{1}{2}} &= (-A\Gamma)^{\frac{1}{2}} e^{i\frac{\varepsilon + \varepsilon'}{2}}, \\
 (DD')^{\frac{1}{2}} &= (-A)^{\frac{1}{2}} e^{i\varepsilon}, \\
 (ADA'D')^{\frac{1}{2}} &= -A\Gamma^{\frac{1}{2}} e^{i\varepsilon} e^{i\frac{\varepsilon + \varepsilon'}{2}}, \\
 (B'C)^2 &= (1 - A^2)\Gamma e^{2i\varepsilon} \{ e^{-i(\varepsilon + \varepsilon')} - e^{-i(\varepsilon + \varepsilon')} \}.
 \end{aligned} \right\} \quad (19.18)$$

(19.18) と (14.14) の A とを (12.2) に代入すると

$$\left. \begin{aligned}
 X &= \left(\Gamma^{-\frac{1}{4}} e^{\frac{\hat{\alpha}_1 H}{2}} \right) e^{\frac{i}{2}(\bar{\beta}_1 H - \delta - \varepsilon)} \\
 &\quad - \left(\Gamma^{\frac{1}{4}} e^{-\frac{\hat{\alpha}_1 H}{2}} \right) e^{-\frac{i}{2}(\bar{\beta}_1 H + \delta + \varepsilon')}, \\
 Y &= \left(\Gamma^{\frac{1}{4}} e^{-\frac{\hat{\alpha}_1 H}{2}} \right) e^{\frac{i}{2}(\bar{\beta}_1 H + \delta + \varepsilon')} \\
 &\quad - \left(\Gamma^{-\frac{1}{4}} e^{\frac{\hat{\alpha}_1 H}{2}} \right) e^{-\frac{i}{2}(\bar{\beta}_1 H - \delta - \varepsilon)}, \\
 l^2 &= 4i \left[\sin \left(\delta + \frac{\varepsilon + \varepsilon'}{2} \right) \right. \\
 &\quad \left. + \left\{ \sin^2 \left(\delta + \frac{\varepsilon + \varepsilon'}{2} \right) - \sin^2 \left(\delta - \frac{\varepsilon + \varepsilon'}{2} \right) \right\}^{\frac{1}{2}} \right], \\
 X^2 - Y^2 &= l^2.
 \end{aligned} \right\} \quad (19.19)$$

しかし上式のままでは X , Y 及び l は何れも複素量なので理解しにくい, また数値計算を行なうためにも不便である. 故に,

$$\left. \begin{aligned}
 X &= \bar{X} + i\dot{X}, \quad Y = -\bar{X} + i\dot{X}, \\
 \dot{X} &= (\Gamma^{-\frac{1}{4}} e^{-\frac{\hat{\alpha}_1 H}{2}}) \cos \frac{1}{2}(\bar{\beta}_1 H - \delta - \varepsilon) \\
 &\quad - (\Gamma^{\frac{1}{4}} e^{-\frac{\hat{\alpha}_1 H}{2}}) \cos \frac{1}{2}(\bar{\beta}_1 H + \delta + \varepsilon'), \\
 \dot{X} &= (\Gamma^{-\frac{1}{4}} e^{-\frac{\hat{\alpha}_1 H}{2}}) \sin \frac{1}{2}(\bar{\beta}_1 H - \delta - \varepsilon) \\
 &\quad + (\Gamma^{\frac{1}{4}} e^{-\frac{\hat{\alpha}_1 H}{2}}) \sin \frac{1}{2}(\bar{\beta}_1 H + \delta + \varepsilon'), \\
 l^2 &= \sin \left(\delta + \frac{\varepsilon + \varepsilon'}{2} \right) + \left\{ \sin^2 \left(\delta + \frac{\varepsilon + \varepsilon'}{2} \right) \right. \\
 &\quad \left. - \sin^2 \left(\delta - \frac{\varepsilon + \varepsilon'}{2} \right) \right\}^{\frac{1}{2}}.
 \end{aligned} \right\} \quad (19.20)$$

とおくと, (19.19) の第 4 式は次のごとくなる.

$$\bar{X}\dot{Y} = l^2. \quad (19.21)$$

(19.20), (19.21) の組合せは (14.16) と全く同形である. 故に (19.21) の $\bar{X}$, $\dot{X}$ 軸をそれぞれ 45° だけ傾斜させた座標軸を更めて U , V 軸とすれば, (14.17) と同形の式を得る.

$$\left. \begin{aligned}
 U^2 - V^2 &= l^2, \\
 U &= (\Gamma^{-\frac{1}{4}} e^{\frac{\hat{\alpha}_1 H}{2}}) \cos \frac{1}{2} \left(\bar{\beta}_1 H - \delta - \varepsilon - \frac{\pi}{2} \right) \\
 &\quad - (\Gamma^{\frac{1}{4}} e^{-\frac{\hat{\alpha}_1 H}{2}}) \cos \frac{1}{2} \left(\bar{\beta}_1 H + \delta + \varepsilon' + \frac{\pi}{2} \right), \\
 V &= (\Gamma^{-\frac{1}{4}} e^{\frac{\hat{\alpha}_1 H}{2}}) \sin \frac{1}{2} \left(\bar{\beta}_1 H - \delta - \varepsilon - \frac{\pi}{2} \right) \\
 &\quad + (\Gamma^{\frac{1}{4}} e^{-\frac{\hat{\alpha}_1 H}{2}}) \sin \frac{1}{2} \left(\bar{\beta}_1 H + \delta + \varepsilon' + \frac{\pi}{2} \right).
 \end{aligned} \right\} \quad (19.22)$$

$$\left. \begin{aligned}
 \text{(iii)} \quad v_{p1} &> v_{s1} \geq c; \quad \alpha_1 = -i\hat{\alpha}_1, \quad \beta_1 = -i_1\hat{\beta}_1 \\
 A' &= K \\
 &\quad - \frac{\hat{\alpha}_1}{\xi} \frac{\hat{\beta}_1}{\xi} L - \left(\frac{\mu_2}{\mu_1} \right) S \left(\frac{\hat{\alpha}_1}{\xi} \frac{\hat{\beta}_2}{\xi} + \frac{\hat{\beta}_1}{\xi} \frac{\hat{\alpha}_2}{\xi} \right), \\
 A^* &= K \\
 &\quad - \frac{\hat{\alpha}_1}{\xi} \frac{\hat{\beta}_1}{\xi} L + \left(\frac{\mu_2}{\mu_1} \right) S \left(\frac{\hat{\alpha}_1}{\xi} \frac{\hat{\beta}_2}{\xi} + \frac{\hat{\beta}_1}{\xi} \frac{\hat{\alpha}_2}{\xi} \right),
 \end{aligned} \right\}$$

$$\left. \begin{aligned} A'_A &= K \\ &+ \frac{\hat{\alpha}_1}{\xi} \frac{\hat{\beta}_1}{\xi} L + \left(\frac{\mu_2}{\mu_1} \right) S \left(\frac{\hat{\alpha}_1}{\xi} \frac{\hat{\beta}_2}{\xi} - \frac{\hat{\beta}_1}{\xi} \frac{\hat{\alpha}_2}{\xi} \right), \\ A'_D &= K \\ &+ \frac{\hat{\alpha}_1}{\xi} \frac{\hat{\beta}_1}{\xi} L - \left(\frac{\mu_2}{\mu_1} \right) S \left(\frac{\hat{\alpha}_1}{\xi} \frac{\hat{\beta}_2}{\xi} - \frac{\hat{\beta}_1}{\xi} \frac{\hat{\alpha}_2}{\xi} \right). \end{aligned} \right\} \quad (19.23)$$

$$\left. \begin{aligned} \therefore A^*/A' &= \exp(2\varepsilon), \\ \tanh \varepsilon &= \left(\frac{\mu_2}{\mu_1} \right) S \left(\frac{\hat{\alpha}_1}{\xi} \frac{\hat{\beta}_2}{\xi} + \frac{\hat{\beta}_1}{\xi} \frac{\hat{\alpha}_2}{\xi} \right) \\ &\quad / \left(K - \frac{\hat{\alpha}_1}{\xi} \frac{\hat{\beta}_1}{\xi} L \right), \\ \text{or} &= -\exp(2\varepsilon_*), \\ \coth \varepsilon_* &= \left(\frac{\mu_2}{\mu_1} \right) S \left(\frac{\hat{\alpha}_1}{\xi} \frac{\hat{\beta}_2}{\xi} + \frac{\hat{\beta}_1}{\xi} \frac{\hat{\alpha}_2}{\xi} \right) \\ &\quad / \left(K - \frac{\hat{\alpha}_1}{\xi} \frac{\hat{\beta}_1}{\xi} L \right), \\ A'_A/A'_D &= \exp(2\varepsilon'), \\ \tanh \varepsilon' &= \left(\frac{\mu_2}{\mu_1} \right) S \left(\frac{\hat{\alpha}_1}{\xi} \frac{\hat{\beta}_2}{\xi} - \frac{\hat{\beta}_1}{\xi} \frac{\hat{\alpha}_2}{\xi} \right) \\ &\quad / \left(K + \frac{\hat{\alpha}_1}{\xi} \frac{\hat{\beta}_1}{\xi} L \right), \\ A'_A/A' &= \Gamma, \quad A'_D/A' = \Gamma e^{-2\varepsilon'}, \\ \Gamma &= \left\{ K + \frac{\hat{\alpha}_1}{\xi} \frac{\hat{\beta}_1}{\xi} L + \left(\frac{\mu_2}{\mu_1} \right) S \left(\frac{\hat{\alpha}_1}{\xi} \frac{\hat{\beta}_2}{\xi} \right. \right. \\ &\quad \left. \left. - \frac{\hat{\beta}_1}{\xi} \frac{\hat{\alpha}_2}{\xi} \right) \right\} / \left\{ K - \frac{\hat{\alpha}_1}{\xi} \frac{\hat{\beta}_1}{\xi} L \right. \\ &\quad \left. - \left(\frac{\mu_2}{\mu_1} \right) S \left(\frac{\hat{\alpha}_1}{\xi} \frac{\hat{\beta}_2}{\xi} + \frac{\hat{\beta}_1}{\xi} \frac{\hat{\alpha}_2}{\xi} \right) \right\} = e^{2\varepsilon'}. \end{aligned} \right\} \quad (19.24)$$

故に, (19.1), (19.8) と (19.24) とから

$$\left. \begin{aligned} (A'D' - B'C')^{\frac{1}{2}} &= e^\varepsilon \text{ or } ie^{\varepsilon_*}, \\ \text{また} \\ (AA')^{\frac{1}{2}} &= (-A\Gamma)^{\frac{1}{2}}, \\ (DD')^{\frac{1}{2}} &= (-A\Gamma)^{\frac{1}{2}} e^{-\varepsilon'}, \\ (ADA'D')^{\frac{1}{2}} &= (-A\Gamma) e^{-\varepsilon'}, \\ (B'C')^2 &= (1 - A^2)(e^{2\varepsilon} - \Gamma^2 e^{-2\varepsilon'}). \end{aligned} \right\} \quad (19.25)$$

(19.25) を (12.2) に代入すれば,

$$\left. \begin{aligned} X^2 - Y^2 &= l^2 \\ X &= e^{-\frac{1}{2}(\delta + \delta' - \frac{\varepsilon + \varepsilon'}{2})} \\ &\quad \cdot \sinh \left\{ \left(\frac{\hat{\alpha}_1}{\xi} + \frac{\hat{\beta}_1}{\xi} \right) \frac{\xi H}{2} - \frac{\varepsilon}{2} \right\}, \end{aligned} \right\}$$

$$\left. \begin{aligned} Y &= -e^{\frac{1}{2}(\delta + \delta' - \frac{\varepsilon + \varepsilon'}{2})} \\ &\quad \cdot \sinh \left\{ \left(\frac{\hat{\alpha}_1}{\xi} - \frac{\hat{\beta}_1}{\xi} \right) \frac{\xi H}{2} - \frac{\varepsilon'}{2} \right\}, \\ l^2 &= \sinh \left(\delta + \delta' - \frac{\varepsilon + \varepsilon'}{2} \right) \\ &\quad - \left\{ \sinh^2 \left(\delta + \delta' - \frac{\varepsilon + \varepsilon'}{2} \right) \right. \\ &\quad \left. - \sinh^2 \left(\delta - \delta' + \frac{\varepsilon + \varepsilon'}{2} \right) \right\}^{\frac{1}{2}}. \end{aligned} \right\} \quad (19.26)$$

δ 及び ε の代りに δ_* または ε_* を用いる場合には (14.24) におけるがごとく, l^2 の符号は負になることもある. この場合には l^2 の表現式内の $\sinh$ を $\cosh$ に変えなければならない.

AIRY-phase の ray theory 的解釈

北大理地球物理 田 治 米 鏡 二

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Ray-theoretical Interpretation on
the AIRY-phase

Kyozi TAZIME

Department of Geophysics, Faculty of
Science, Hokkaido University.

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§ 1. 地表面へ入射する SH 平面波の変位振幅を $A_1 \exp(i\eta_1 z)$ とすると, 反射波の変位振幅は $A_1 \exp(-i\eta_1 z)$ となり, 両者の和は

$$\psi_1 = 2A_1 \cos \eta_1 z \quad (1)$$

で表わされる. かくのごとき簡単な関係は, 地表面での反射係数が 1 であることによる.

地表面での変位振幅は (1) に $z = 0$ を代入して

$$\psi_1(z = 0) = 2A \quad (2)$$

となる. 但し, (1) 式を求めるに当つては, 入射角 θ を固定している. 故に θ を変えると A_1 もそれに応じて変ることが考えられる.

以上の事柄を ray theory 的に考えてみよう. 今平面波を単に 1 つの ray で代表させることを止めて, 第 1 図の A, A' 間に ray の flux を考えること